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Maximizing Agricultural Production with Limited Inputs and Fixed Cost

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Abstract: ***We presented a computational procedure to maximize the production of a given agricultural crop with limited inputs (water-nitrogen), and where a fixed cost (or expense) of the inputs is imposed. Theoretically, the procedure is based on the duality theory of quadratic programming and the logarithmic barrier method of nonlinear programming. We tested the procedure in three different numerical scenarios defined in the literature, for the cultures: Lettuce, Oats, Onion and Melon. In each agricultural scenario considered, we verified that the procedure is a reliable alternative in making agribusiness economic decisions.***

Keywords: water-nitrogen, duality, logarithmic barrier, agribusiness

1. Introduction

The economic evaluation of agricultural production involves the quantification of productivity in response to the total of inputs applied. Water and nitrogen are essential for the development of agricultural crops, and when they are correlated with the production obtained, we obtain the function of production or water-nitrogen-culture response.

The use of analytical functions of production and net revenue in the analysis of the results of agricultural experiments is widespread (MOUSINHO et al., 2003; FRIZZONE et al., 2005; MONTEIRO et al., 2006; SILVA et al., 2008; CARVALHO et al., 2009; DELGADO et al., 2010 and TEODORO et al., 2013).

If these functions were known with precision, it would be possible to precisely select the optimum level of water and nitrogen for a given situation. However, such functions are restricted to large variations, making predictions difficult.

Climatic variations, physical attributes related to the soil, the plant, and many other factors, make it difficult to predict crop yields. In practice, linear and/or quadratic regressions are generated to represent “good approximations” of the response or agricultural production functions. The quality of the adjustment, which is the proportion of variation in the function, is indicated by a descriptive unit known as the coefficient of determination (r^2).

Bearing in mind that the rational management of inputs is imperative in maximizing agricultural production, in this work, we will consider that the actual values of inputs, water and nitrogen, are limited above and below. The resulting model presented is a nonlinear programming problem with linear constraints, considering that the objective is a quadratic function in two variables: water and nitrogen.

In this way, a computational procedure resulting from the duality theory and the logarithmic barrier method (BERTSEKA, 2004) is developed, to determine the water depth and nitrogen dose that maximizes the production of a given culture with limited inputs and fixed expenditures on inputs. Some numerical tests with data known in the literature are performed, with the purpose of testing the effectiveness of the proposed procedure.

2. Material and Methods

Let $y(w,n)$ be the nonlinear analytical function of production or response of a given culture $(kg.ha^{-1})$in relation to the water depth $w(mm)$ and nitrogen dose $n(kg)$; $w_l, w_u, n_l, n_u \geq 0$, lower and upper limits of w and n respectively; c_w the cost of a water depth $(R\$.mm^{-1}.ha^{-1})$ and c_n the cost of a dose of nitrogen$(R\$.kg^{-1}ha^{-1})$. Suppose that c_0 represents a fixed cost $(R\$.ha^{-1})$ intended exclusively to cover expenditure on water-nitrogen inputs.

The problem that maximizes agricultural production with limited inputs and spending on fixed inputs, can be written mathematically as the problem of nonlinear programming with linear constraints:

$$\text{(P)} \quad \text{Maximize} \quad y(w,n) \tag{1}$$
$$\text{Subject to: } r(w,n) = c_w w + c_n n = c_0 \tag{2}$$
$$w_l \leq w \leq w_u \tag{3}$$
$$n_l \leq n \leq n_u. \tag{4}$$

In what follows, $y(w,n) = aw^2 + bn^2 + cwn + dw + en + f$ (quadratic form in variables w and n); and where $a,b,c,d,e,f \in \mathbb{R}$ with $a < 0$ and $4ab - c^2 > 0$. Thus $y(w,n)$ is a strictly concave function and therefore reaches its maximum at the intersection of the two-dimensional box $[w_l, w_u] \times [n_l, n_u]$ and the plane $r(w,n) = c_0$. Note that (2) impose a constraint on expenditures on inputs w and n, of $c_0 -$ reais per hectare.

Note that the problem (P) can be written as the quadratic program:

$$\text{(QP)} \quad \text{Minimize} \quad \frac{1}{2}(w,n)\, Q \begin{pmatrix} w \\ n \end{pmatrix} + (-d,-e)\begin{pmatrix} w \\ n \end{pmatrix} - f \tag{5}$$
$$\text{Subject to:} \quad A \begin{pmatrix} w \\ n \end{pmatrix} \leq \rho, \tag{6}$$

where $Q = \begin{pmatrix} -2a & -c \\ -c & -2b \end{pmatrix}$ is a symmetric positive-definite 2×2 matrix, and where

$$A = \begin{pmatrix} c_w & c_n \\ -c_w & -c_n \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \text{ e } \rho = \begin{pmatrix} c_0 \\ -c_0 \\ -w_l \\ w_u \\ -n_l \\ n_u \end{pmatrix}.$$

As in DELGADO, et al., (2020), the dual problem associated with (QP) is given by:

$$\text{(DQ)} \quad \text{Minimize} \quad \frac{1}{2} u^T H u - \sigma^T u + (g+f)$$

$$\text{Subject to:} \quad u \in \mathbb{R}^6_+,$$

where $H = AQ^{-1}A^T$ is symmetric positive-definite, $\sigma = AQ^{-1}\begin{pmatrix} d \\ e \end{pmatrix} - \rho$, $g = \frac{1}{2}(d,e)Q^{-1}\begin{pmatrix} d \\ e \end{pmatrix}$. Following the logarithmic barrier methodology applied to the dual problem (DQ), we associate with each $\mu > 0$ the function $\varphi_\mu(u) = \frac{1}{2}u^T H u - \sigma^T u + (g+f) + \mu \sum_{j=1}^{6} Ln(u_j)$, and then we solve the unconstrained nonlinear programming problem:

$$\text{Minimize}\, \varphi_\mu(u).$$

Furthermore, u solves the unconstrained problem for each $\mu > 0$, if and only if:

$$\nabla \varphi_\mu(u) = 0.$$

Thus, we seek, $(u, z) > 0$, such that:

$$Hu - \sigma + z = 0,$$

where $z \in \mathbb{R}^6$ is such that $z_j = \mu / u_j$. Then $z_j u_j = \mu$, and now we try to solve the system of non-linear equations:

$$Hu + z = \sigma \qquad (7)$$

$$z_j u_j = \mu, \quad (j = 1,2,3,4,5,6). \qquad (8)$$

Applying Newton's method for the resolution of the non-linear system (7)-(8), a direction is sought: $\Delta u = (\Delta u_1, \Delta u_2, \Delta u_3, \Delta u_4, \Delta u_5, \Delta u_6) \in \mathbb{R}^6$ and $\Delta z = (\Delta z_1, \Delta z_2, \Delta z_3, \Delta z_4, \Delta z_5, \Delta z_6) \in \mathbb{R}^6$ such that:

$$H(u + \Delta u) + (z + \Delta z) = \sigma,$$

$$(z_j + \Delta z_j)(u_j + \Delta u_j) = \mu \quad (j = 1,2,3,4,5,6).$$

Then:

$$H\Delta u + \Delta z = \sigma - Hu - z = \theta$$

$$z_j \Delta u_j + u_j \Delta z_j = \mu - z_j u_j = \tau_j^\mu (j = 1,2,3,4,5,6).$$

Next, we present the computational procedure to solve the problem (1)-(4).

Procedure

DATA: $a, b, c, d, e, f \in \mathbb{R}$; $a < 0$ and $4ab - c^2 > 0$, $w_l, w_u, n_l, n_u \geq 0$; $w_u \geq w_l$ and $n_u \geq n_l$, $\varepsilon, \xi \in (0,1)$, $\mu_0 > 0$, $u \in \mathbb{R}^6_{++}$, that is, $u > 0$.

Beginning
DO

$$Q = \begin{pmatrix} -2a & -c \\ -c & -2b \end{pmatrix}, A = \begin{pmatrix} c_w & c_n \\ -c_w & -c_n \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}, \rho = \begin{pmatrix} c_0 \\ -c_0 \\ -w_l \\ w_u \\ -n_l \\ n_u \end{pmatrix}, \mu = \mu_0,$$

$\sigma = AQ^{-1}\begin{pmatrix} d \\ e \end{pmatrix} - \rho$, $Q^{-1} = \frac{1}{c^2 - 4ab}\begin{pmatrix} 2b & -c \\ -c & 2a \end{pmatrix}$, $H = AQ^{-1}A^T$, $z = \mu u^{-1}$

$\theta = \sigma - Hu - z$, $\tau_j^\mu = \mu - z_j u_j$; $j = 1,2,\ldots,6$; $\tau^\mu = (\tau_j^\mu)_{j=1,2,\ldots,6}$

WHILE Max $\{\|\theta\|, \|\tau^\mu\|\} > \varepsilon$

FIND $\Delta u = (\Delta u_1, \Delta u_2, \Delta u_3, \Delta u_4, \Delta u_5, \Delta u_6)$, $\Delta z = (\Delta z_1, \Delta z_2, \Delta z_3, \Delta z_4, \Delta z_5, \Delta z_6)$ such that:

$$\begin{cases} H\Delta u + \Delta z = \theta \\ z_j \Delta u_j + u_j \Delta z_j = \tau_j^\mu \,(j = 1,2,3,4,5,6) \end{cases}$$

DO $\quad \alpha = \min\left\{\frac{\|u\|}{\|\Delta u\|}, \frac{\|z\|}{\|\Delta z\|}\right\}$

$$u = u + 0.999\alpha\Delta u$$

$$z = z + 0.999\alpha\Delta z$$

$$\mu = \xi\mu$$

REEVALUATE θ, τ_μ

DO $\begin{pmatrix} w \\ n \end{pmatrix} = Q^{-1}\left(\begin{pmatrix} d \\ e \end{pmatrix} - A^T u\right)$

End

To test the presented procedure, several numerical tests were performed using information known in the literature. Table 1 presents analytically the responses or production functions of the cultures: Lettuce (SILVA et al., 2008), Oats (FRIZZONE et al., 1995), Onion (BAPTESTINI, JCM, 1982) and Melon (MONTEIRO et al., 2006). According to the data provided by the bibliographic sources, for each crop (Lettuce, Oats, Onion and Melon) it is possible to determine lower and upper limit of the water depth; the lower ones between 100 mm and 600 mm and the upper ones between 400 mm and 600 mm.

For the nitrogen input, a ceiling of 300 $kg.ha^{-1}$ was fixed, and considering that the most common in the literature for these cultures is a minimum dose of 75 $kg.ha^{-1}$, three numerical tests defined by the two-dimensional boxes were performed: $[100,500] \times [0,300]$; $[100,400] \times [75,300]$ and $[100,600] \times [75,300]$.

Table 1: Responses or production functions in quadratic forms in variables w and n for crops: Lettuce, Oats, Onion and Melon

Cultures	Production Function or Response($kg.ha^{-1}$)
Lettuce	$y(w,n) = -1.042\,w^2 - 0.04563\,n^2 + 0.1564\,wn + 388.1w - 6.02\,n - 12{,}490$
Oats	$y(w,n) = -5.6 \cdot 10^{-5}\,w^2 - 5.1 \cdot 10^{-5} n^2 + 3.6 \cdot 10^{-2} w + 1.6 \cdot 10^{-2}\,n$
Onion	$y(w,n) = -2.00 \cdot 10^{-4} w^2 - 2.00 \cdot 10^{-4} n^2 + 3.28 \cdot 10^{-1} w + 9.07 \cdot 10^{-2}\,n$
Melon	$y(w,n) = -0.05781\,w^2 - 0.07612\,n^2 + 70.77509w + 34.16737\,n$

It is important to emphasize that the proper management of the water depth (w) is fundamental, considering that the agricultural sector is the largest consumer of water, and that water resources are essential and strategic in the development of agriculture. Also, considering that currently the costs of nitrogen fertilization, specifically nitrogen (n), are increasingly variable, and that the demand in Brazil grows every day, it is necessary to respect the environmental and soil preservation issues, as a fundamental part for sustainable agriculture. Table 2 shows the costs of a water depth (c_w) and a dose of nitrogen (c_n), for each agricultural crop.

Table 2: Water cost (c_w) and nitrogen cost (c_n) for each crop considered.

Cultures	c_w $(R\$.mm^{-1}.ha^{-1})$	c_n $(R\$.kg^{-1}.ha^{-1})$
Lettuce	0.44	2.09
Oats	0.08	0.42
Onion	0.025	1.20
Melon	0.134	2.33

3. Results and Discussion

Tables 3, 4, and 5 show the results obtained for each numerical test implemented.

Table 3: Optimal solution (w^*, n^*) and optimal value of the production $y(w^*, n^*)$ of (P), in the two-dimensional box $[100{,}500] \times [0{,}300]$.

Cultures	w^* (mm)	n^* (kg)	$y(w^*, n^*)$ $(kg.ha^{-1})$	c_0 $(R\$.ha^{-1})$
Lettuce	200.389436	197.046950	39,133.883962	500
Oats	333.140852	174.639948	7,016817	100
Onion	490.609563	156.445630	122.074960	200
Melon	497.189178	185.998562	24,619.474627	500

Table 3 reports that in the two-dimensional scenario $[100{,}500] \times [0{,}300]$ and for a fixed cost of inputs, water-nitrogen (c_0) of R\$ 500, Lettuce reaches its maximum production at the point $(w^*, n^*) = (220.389436, 197.046950)$, Oats for a fixed input cost of R\$ 100, at the point $(w^*, n^*) = (333.140852, 174.639948)$, Onion for a fixed input cost of R \$ 200, at the point $(w^*, n^*) = (490.609563, 156.445630)$ and Melon for a fixed input cost of R \$ 500, at the point $(w^*, n^*) = (497.189178, 185.998562)$.It is possible to graphically show the trajectory of points generated by the implemented procedure, converging to the optimal solution of the problem, for each culture considered. Figure 1, for example, shows for the Oats culture, the sequence of interior points in the two-dimensional box $[100{,}500] \times [0{,}300]$generated by the implemented procedure, and converging to the optimal solution $(333.140852, 174.639948)$. Note that this optimal solution satisfies the plane equation: $r(w, n) = 0.08\, w + 0.42\, n = 100$.

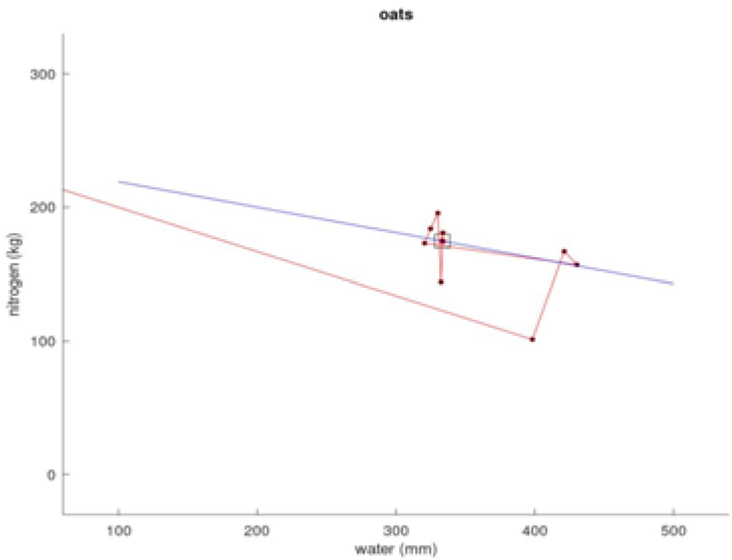

Figure 1: Sequence of points generated by the procedure in the two-dimensional box $[100{,}500] \times [0{,}300]$ for Oats, converging to the point $(333.140852, 174.639948)$.

Table 4 shows the results obtained in the two-dimensional box $[100{,}400] \times [75{,}300]$.

Table 4: Optimal solution (w^*, n^*) and optimal value of the production $y(w^*, n^*)$ of (P), in the two-dimensional box$[100{,}400] \times [75{,}300]$.

Cultures	w^* (mm)	n^* (kg)	$y(w^*, n^*)$ $(kg.ha^{-1})$	c_0 $(R\$.ha^{-1})$
Lettuce	200.225177	197.081757	39,133.895712	500
Oats	317.590353	177.601837	7,017856	100
Onion	360.198429	159.162527	101.566002	200
Melon	398.854799	191.653847	22,784.233597	500

Table 4 informs that in the two-dimensional scenario $[100,400] \times [75,300]$ and for a fixed input cost of R$ 500, Lettuce reaches its maximum production at the point $(w^*, n^*) = (200.225177\,, 197.081757)$, Oats for a fixed input cost of R$ 100, at point $(w^*, n^*) = (317.590353\,, 177.601837)$, Onion for a fixed input cost of R$ 200, at point $(w^*, n^*) = (360.198429\,, 159.162527)$ and Melon for a fixed cost of inputs of R$ 500, at the point $(w^*, n^*) = (398.854799\,, 191.653847)$.Figure 2 shows for the Onion crop, the sequence of interior points in the two-dimensional box $[100,400] \times [75,300]$ generated by the implemented procedure, and converging to the optimal solution $(360.198429\,, 159.162527)$. Note that this optimal solution satisfies the plane equation: $r(w, n) = 0.025\,w + 1.20\,n = 200$.

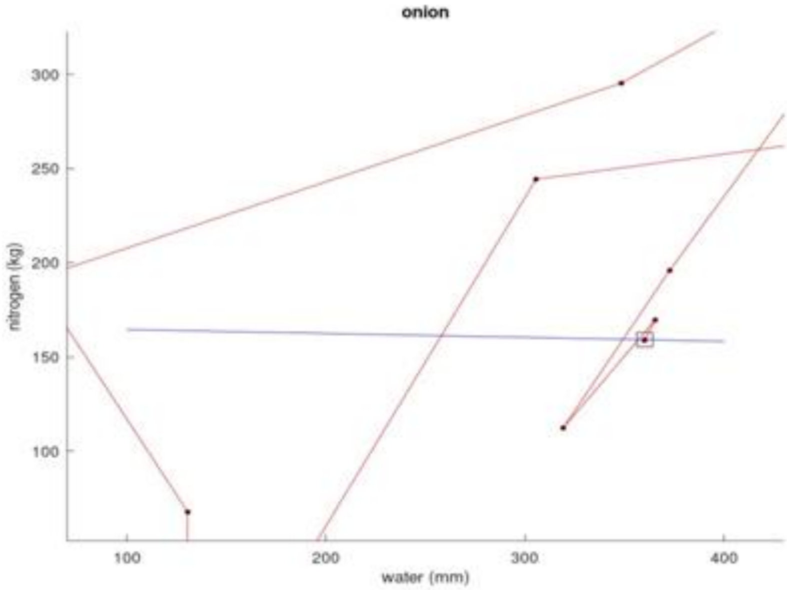

Figure 2: Sequence of points generated by the procedure in the two-dimensional box $[100,400] \times [75,300]$ for Onion, converging to the point $(360.198429\,, 159.162527)$

Comparing the results obtained in the first scenario $[100,500] \times [0,300]$ (Table 1) with those obtained in the second scenario $[100,400] \times [75,300]$ (Table 2), we can observe that the Lettuce culture, for the same fixed cost $c_0 = 500\ R\$.ha^{-1}$, does not show much difference in relation to water depth, nitrogen dose and productivity.

Regarding the Oats, on the other hand, for the same fixed cost $c_0 = 100\ R\$.ha^{-1}$, in the second scenario, a small reduction in relation to the water depth of 15.55 mm can be noted, and a small increase 2.9 kg in relation to the nitrogen dose. However, productivity is practically the same.

As for the Onion crop, for the same fixed cost $c_0 = 200\ R\$.ha^{-1}$, it can be noted that in the second scenario (Table 2), there was a reduction of 130.41mm in relation to the water depth, a slight increase of 2.71 kg in relation to the nitrogen dose, and a drop in productivity of 20.51 $kg.ha^{-1}$.

In the case of Melon, for the same fixed cost $c_0 = 500\ R\$.ha^{-1}$, we obtained a drop of 98.33 mm in relation to the water depth, an increase of 5.655281 kg in relation to the nitrogen dose, and a significant drop in productivity of 1,835.24 $kg.ha^{-1}$.

Finally, Table 5 presents the results obtained for the third scenario performed in the two-dimensional box $[100,600] \times [75,300]$.

Table 5: Optimal solution(w^*, n^*) and optimal value of the production $y(w^*, n^*)$ of (P), in the two-dimensional box $[100,600] \times [75,300]$

Cultures	w^* (mm)	n^* (kg)	$y(w^*, n^*)$ $(kg.ha^{-1})$	c_0 $(R\$.ha^{-1})$
Lettuce	200.005960	197.127783	39,133.817733	500
Oats	318.218870	177.464264	7.018394	100
Onion	595.698057	154.256266	133.649772	200
Melon	596.983192	180.259360	25,333.957910	500

In this numerical scenario, again for a fixed input cost of R$ 500, Lettuce remains almost invariant in relation to the first two scenarios. In the case of Oats and for a fixed cost of inputs of R$ 100, the optimum water depth found (318.218870 mm) is limited below by the optimum water content in the second scenario and above by the optimum water content in the first scenario. The values of the nitrogen dose and the productively optimum found were practically the same as in the previous scenario.

For Onion, as in the previous scenarios, for a fixed cost of inputs of R$ 100, we achieved the highest productivity $(133.649772\ \text{kg.ha}^{-1})$ and the lowest nitrogen dose (154.256266 kg), among the three numerical scenarios considered.

Finally, in relation to Melon, and for a fixed cost of inputs of R$ 500, we achieved the highest productivity $(25{,}333.957910\ \text{kg.ha}^{-1})$ and the lowest nitrogen dose (180.259360 kg), among the three numerical scenarios considered. Figure 3 shows for the Melon culture, the sequence of interior points in the two-dimensional box $[100,600] \times [75,300]$ generated by the implemented procedure, and converging to the optimal solution $(596.983192, 180.259360)$. Note that this optimal solution satisfies the plane equation: $r(w,n) = 0.134\, w + 2.33\, n = 500$.

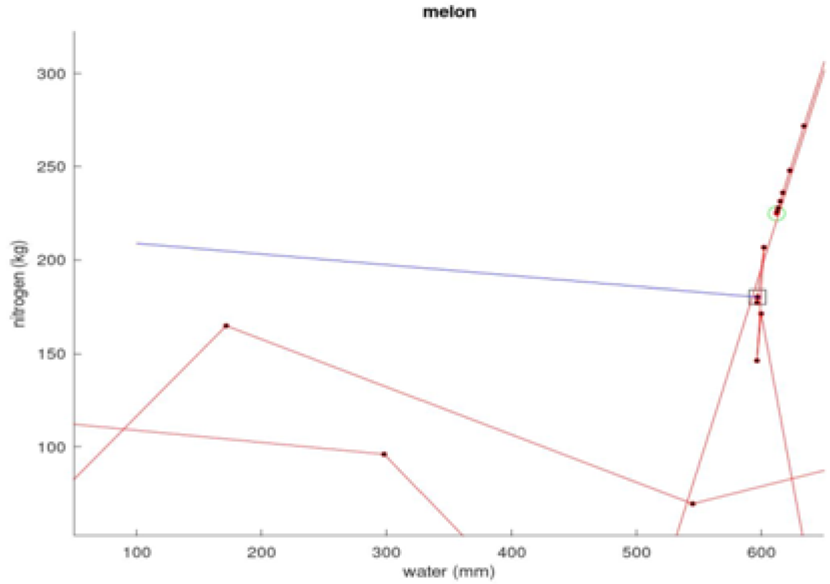

Figure 3: Sequence of points generated by the procedure in the two-dimensional box $[100,600] \times [75,300]$ for the Melon, converging to the point $(596.983192, 180.259360)$

4. Conclusions

- We presented a computational procedure based on the duality theory of quadratic programming and the logarithmic barrier method, which maximizes the production of a certain agricultural crop with limited inputs (water-nitrogen), and a fixed cost (or expense) of the inputs.
- For each agricultural scenario considered, it was possible to confirm that all the optimal solutions of (P) generated by the procedure, satisfy the imposed constraints.
- In the three numerical scenarios presented, Lettuce and Oats were the only agricultural crops considered to have remained almost invariant in relation to the water depth, nitrogen dose and productivity.
- Finally, when making agricultural production decisions (agribusiness), it is important to deal with problems where it is desirable to maximize the production of a given agricultural crop with limited inputs (water-nitrogen), and where there must be a fixed cost for input expenses.

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