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The Usage of Cyclic Group in the Clock

Miakhel Samimullah

Lecturer at Mathematic Department, Education Faculty, Paktika University, Paktika Afghanistan. Student of Master program in "Algebra and Differential Equations" Department University of Kabardino Balkarian state Russia

Abstract: ***The paper presents the result of research on application of cyclic group in the clock presented. In the research work we used foreign reliable sources and materials. Cyclic groups are common in our everyday life. A cyclic group is a group with an element that has an operation applied that produces the whole set. A cyclic group could be a pattern found in nature, for example in a snowflake, or in a geometric pattern we draw ourselves. Cyclic groups can also be thought of as rotations, if we rotate an object enough times we will eventually return to the original position. Cyclic groups are used in topics such as cryptology and number theory. In this paper we explore further applications of cyclic groups in the 12hours clock.***

Keywords: Cyclic group, 12-hours clock, clock arithmetic, subgroup, whole numbers, addition table

1. Introduction

Let (G, o) be a group and a an element of G. The subgroup $<\{a\}>$ generated by $\{a\}$ is denoted by $<a>$. Thus, $<a> = \{a^n \mid n \in Z\}$

Definition: The subgroup $<a>$ of (G, o) generated by $\{a\}$ is called the **cyclic subgroup generated by a**. A group (G, o) is said to be a **cyclic group** if it is generated by a single element.

Let (G, o) be a group and a ∈ G. Define a map f from Z to G by f (n) = a^n.

$< a > = \{e, a, a^2, \ldots, a^{m-1}\}$[3, p. 134, 135].

Should mention that if we use addition (+), cyclic subgroup generated by a than we are wright

$<a>=\{na|n \in Z\}$.

Example 1. Let $|x| = n; a,b \in Z$,

$x^a = x^b$ if and only if $a \equiv b (\mathrm{mod}\, n)$.

Proof: $x^{a-b} = e$; $a - b \equiv 0 (\mathrm{mod}\ n)$, therefore $a \equiv b\ (\mathrm{mod}\ n)$ [4, p. 31, 32]. Example 2. Let Z will be group and <1>= $\{n^1|n$

1+1=2, 1+1+1=3, 1=1+1+1=4 etc. and -(1+1) = -2, -(1+1+1) = -3, … So that <1>=Z.

Clock Arithmetic

On a clock the numbers cycle from one to twelve. After circulating around the clock we do not go to 13 but restart at one. If it was 6 o'clock, what would it be in 9 hours? 6*am* + 9 =3*pm*. The set of the numbers on a clock are C = {0,1,2,3,4,5,6,7,8,9,10,11}. This set of numbers is a group. The identity element is 0 what we will think of as 12. If we add 12 hours to anywhere on the clock, we will end up in the same position [2, p.17]

Since there are infinitely many whole numbers, it is not possible to right a complete table of addition facts for that set. Such a table, to show the sum every possible pair of whole numbers, would have an infinite number of rows and columns, making it impossible to construct. On the other hand, the 12-hours clock system uses only the whole numbers 0,1,2,3,4,5,6,7,8,9,10, and 11. A table of possible sums for this system requires only 12 rows and 12 columns. The 12-hours clock **addition table** is shown in table 1. Since the 12-hours system is built upon a finite set, it is called a **finite mathematical system** [1, p. 220].

Table: 12-Hours Clock Addition

Z }

+	0	1	2	3	4	4	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	0	1	2	3
5	5	6	7	8	9	10	11	0	1	2	3	4
6	6	7	8	9	10	11	0	1	2	3	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	0	1	2	3	4	5	6	7
9	9	10	11	0	1	2	3	4	5	6	7	8
10	10	11	0	1	2	3	4	5	6	7	8	9
11	11	0	1	2	3	4	5	6	7	8	9	10

2. Conclusion

Human minds are designed for pattern recognition and we can find algebraic structures in common objects and things around us. Cyclic groups are the simplest groups that have an object that can generate the whole set. The object can generate the set by addition, multiplication, or rotations. Cyclic groups are not only common in pure mathematics, but also in patterns, shapes, music, and chaos. Cyclic groups are an imperative part of number theory used with the Chinese remainder theorem and Fermat's theorem. Knowing if a group is cyclic could help determine if there can be a way to write a group as a simple circuit. This circuit could simplify the process of generation to discover the most efficient way to generate the object for use of future applications in mathematics and elsewhere. As we know the human life direct and indirect have close relationship with mathematics, here we mentioned the application of cyclic group in the clock which is match important in our life, 12 – hours clock elements set C={0,1,2,3,4,5,6,7,8,9,10,11}, has the algebraic structure of the group and it is cyclic group.

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Author Profile

Miakhel Samimullah Received the B.S.C. degree in Mathematics from the Parwan University, Parwan Province, Afghanistan in (2008), and at present student of master program in "Algebra and Differential Equations" Department of Kabardino Balkarian state University, Russia. He has been a University Lecturer at Paktika University, Paktika Province, Afghanistan.

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