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Laminar Stratified (Unsteady) Flow over a Porous Bed under the Action of a Body Force

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Abstract: ***In this paper, we have studied the unsteady flow of viscous stratified fluid over a permeable bed under the action of a body force. We have studied the effect of slip parameter α, porosity factor σ and viscous stratification factor η on the flow over a permeable bed on the fractional increase in the mass flow rate $|\phi|$. We have also calculated fractional increase $|\phi|$ in mass flow rate with the assumption of the validity of Darcy Law for laminar flow when $R < 1$. We have seen that for very small body force and for fixed σ and set of η and α have no effect on $|\phi|$ but for large body force $|\phi|$ increases with the increase of σ for each set η and α and $|\phi|$ decreases with the increase of η for each set of σ and α. Thus it can be predicted that the stratification factor is not favourable to the fractional increase in mass flow rate.***

Keywords: Laminar Stratified flow, stratification factor, porosity factor, body force, the fractional increase in mass flow, Navier Stokes equation, Darcy law

1. Introduction

The study of flow through porous media is of principal interest due to its importance in chemical engineering for filtration and water purification purposes, petroleum engineering for studying the movement of natural gas, oil and water through the oil reservoirs and to study the under groundwater in the river bed. Its flow behaviour of fluids in a petroleum reservoir rock depends to a large extent the viscous stratification and also on the porous properties of the rock, a technique of core study that can give new or additional information on the characteristics of the rock which would provide a better understanding of petroleum reservoir performance. Unsteady flow of viscous fluid over the permeable bed has been treated by Hunt(1959). Bhattacharyya (1980) considered the unsteady laminar flow in a channel with a porous bed. Mukherjee et al (1986) considered the unsteady flow 0f viscous stratified fluid in a rotating system. Majumder and Debnath (1987) studied the unsteady motion of an inviscid rotating stratified fluid in different geometries. Channabasappa and Ranganna (1976) and Gupta & Sharma (1978) discussed the stratified viscous flow of variable viscosity between a porous bed and moving impermeable plate under the action of a body force.

In this paper, we have studied the unsteady flow of viscous fluid over a permeable bed under the action of a body force. We have also calculated the fractional increase $|\phi|$ in mass flow rate with the assumption of the validity of Darcy law for laminar flow R<1. We have seen that for very small body force, η,σ and α have no effect on $|\phi|$, but for large body force, $|\phi|$ increases with the increase of σ for each set of values of η and α and $|\phi|$ decreases with the increase of η, for each set of values σ,α. Thus it can be predicted that the stratification factor is not favourable to the fractional increase in the mass of the fluid.

2. Mathematical Formulation

The physical model consists of two zones. In zone-1, from the impermeable upper rigid plate up to the interface, the flow called the free flow, is governed by the usual Navier Stokes equation. In the other zone, below the interface, the flow is governed by the Darcy law. In the following discussion, we shall refer to these zones as zone-1 and zone-2 respectively.

The basic equations for zone-1 are taken as

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + A\, e^{-\lambda y} \qquad \text{.........(1)}$$

$$\text{Where } \mu = \mu_0\, e^{-\beta y}\ ,\ \rho = \rho_0\, e^{-\beta y} \qquad \text{..........(2)}$$

$$\text{and } \frac{\partial P}{\partial x} = -g\rho \qquad \text{..........(3)}$$

Here μ_0 and ρ_0 are the coefficients of viscosity and density respectively at the interface $y = 0$. The term $A\ e^{-\lambda y}$ represents the body force. Also $\beta(>0)$ represents the stratification factor. $\frac{\partial P}{\partial x}$ is the common pressure gradient by which the flow in zone-1 and zone-2 in the direction is driven.

The basic equations for zone-2 are

$$Q = Q_0\, e^{\beta y} \qquad \text{................(4)}$$

$$Q_0 = -\frac{K}{\mu_0}\frac{\partial P}{\partial x} \qquad \text{................(5)}$$

The relevant boundary conditions are

$$u = u_0 \text{ at } y = h \qquad \text{.......... (6)}$$

$$\text{and } \frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{K}}(u_B - Q_0) \text{ at } y = 0 \qquad \text{.......... (7)}$$

Where α is the slip parameter; K, the permeability coefficient, which has the dimension of length square; $Q$is the Darcy velocity and μ_B, the slip velocity at the nomial surface $y = 0$.

Using the dimensionless quantities,

$$u' = \frac{u}{u_m}\ ;\ t' = \frac{u_m}{\frac{h}{t}}\ ;\ x' = \frac{x}{h}\ ;\ y' = \frac{y}{h}\ ;P' = \frac{P}{\rho_0 u_m^2}\ ;u_0' = \frac{u_0}{u_m}\ ;\ A = \frac{\rho_0 u_m^2}{h}\ ;V'_B = \frac{u_B}{u_m}$$

equation (1) takes the form (dropping all dashes)

$$\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial u}{\partial y} - R\frac{\partial u}{\partial t} = Re^{\eta y}\frac{\partial P}{\partial x} - Re^{(\eta-\lambda)y} \quad \ldots\ldots(8)$$

The boundary conditions in terms of non-dimensional quantities become

$$u = u_0 \quad \text{at} \quad y = 1 \quad \ldots\ldots\ldots\ldots(9)$$

$$\text{and } \frac{\partial u}{\partial y} = \alpha\sigma\left(V_B + \frac{R}{\sigma^2}\frac{\partial P}{\partial x}\right) \text{at } y = 0$$

$$\text{where } \sigma = \frac{h}{\sqrt{K}} \quad \ldots\ldots(10)$$

$$\text{Taking} \quad u = f(y)\, e^{-\lambda t} \quad \ldots\ldots\ldots\ldots(11)$$

$$V_B = u_s\, e^{-\lambda t} \quad \ldots\ldots\ldots\ldots(12)$$

$$\text{and} \quad -\frac{\partial P}{\partial x} = B\, e^{-\lambda t} \quad \ldots\ldots\ldots\ldots(13)$$

The equations (8),(9) and (10) takes the form

$$\frac{d^2 f}{dy^2} - \eta\frac{df}{dy} + R\lambda f = -RBe^{\eta y} - Re^{\lambda t}e^{(\eta-\lambda)y} \quad ..(14)$$

$$f(y) = u_0 e^{\lambda t} \quad \text{at} \quad y = 1 \quad \ldots\ldots\ldots(15)$$

$$\frac{df}{dy} = \alpha\sigma\left(u_s - \frac{RB}{\sigma^2}\right) \quad \text{at} \quad y = 0 \quad \ldots\ldots\ldots(16)$$

The solution of equation (14) using boundary conditions (15) & (16) becomes

$$f(y) = e^{\frac{\eta y}{2}}\left(\xi_i e^{\lambda_1 y} + \xi_j e^{-\lambda_1 y}\right) + \frac{B}{\lambda}e^{\eta y} - \frac{R}{\lambda}e^{\lambda t}\frac{e^{(\eta-\lambda)y}}{R-\eta+\lambda}$$

$$\ldots\ldots\ldots(17)$$

Where

$$\xi_i = \frac{\tau_1 - \tau_2\left(\frac{\eta}{2}-\lambda_1\right)e^{\lambda_1}}{\left(\frac{\eta}{2}+\lambda_1\right) - \left(\frac{\eta}{2}-\lambda_1\right)e^{2\lambda_1}} \quad ; \quad \xi_j = \frac{\tau_2\left(\frac{\eta}{2}+\lambda_1\right)e^{\lambda_1} - \tau_1 e^{2\lambda_1}}{\left(\frac{\eta}{2}+\lambda_1\right) - \left(\frac{\eta}{2}-\lambda_1\right)e^{2\lambda_1}}$$

$$\tau_1 = \alpha\sigma\left(u_s - \frac{RB}{\sigma^2}\right) + \frac{\eta B}{\lambda} + \frac{Re^{\lambda t}}{\lambda}\left(\frac{\eta-\lambda}{R-\eta+\lambda}\right)$$

$$\tau_2 = u_0 e^{\lambda t - \frac{\eta}{2}} + \frac{B}{\lambda}e^{\frac{\eta}{2}} + \frac{Re^{\lambda t}e^{\left(\frac{\eta}{2}-\lambda\right)}}{\lambda(R-\eta+\lambda)}$$

Therefore

$$u = e^{-\lambda t} f(y)$$

$$u = e^{-\lambda t}\left[e^{\frac{\eta y}{2}}\left(\xi_i e^{\lambda_1 y} + \xi_j e^{-\lambda_1 y}\right) + \frac{B}{\lambda}e^{\eta y} - \frac{R}{\lambda}e^{\lambda t}\frac{e^{(\eta-\lambda)y}}{R-\eta+\lambda}\right]$$

$$\ldots\ldots\ldots(18)$$

If M denotes the dimensionless mass flow rate per unit channel width, then

$$M = \int_0^1 e^{-\eta y} u\, dy$$

$$M = e^{-\lambda t}\int_0^1 e^{-\eta y}\left[e^{\frac{\eta y}{2}}\left(\xi_i e^{\lambda_1 y} + \xi_j e^{-\lambda_1 y}\right) + \frac{B}{\lambda}e\right.$$

$$M = e^{-\lambda t}\left[\left(\xi_i\frac{e^{\lambda_1-\frac{\eta}{2}}}{\lambda_1-\frac{\eta}{2}} - \xi_j\frac{e^{-\left(\lambda_1+\frac{\eta}{2}\right)}}{\lambda_1+\frac{\eta}{2}}\right) - \left(\frac{\xi_i}{\lambda_1-\frac{\eta}{2}} - \frac{\xi_j}{\lambda_1+\frac{\eta}{2}}\right)\right.$$

If the porous bed is replaced by an impermeable rigid plate, then M^*, the dimensionless mass flow rate is obtained as $\left(\sigma = \frac{h}{\sqrt{K}} \to \infty\right)$

$$M^* = e^{-\lambda t}\left[e^{-\frac{\eta}{2}}\left(\frac{\xi_i^* e^{\lambda_1}}{\lambda_1-\frac{\eta}{2}} - \frac{\xi_j^* e^{-\lambda_1}}{\lambda_1+\frac{\eta}{2}}\right) - \left(\frac{\xi_i^*}{\lambda_1-\frac{\eta}{2}} - \frac{\xi_j^*}{\lambda_1+\frac{\eta}{2}}\right)\right] - \frac{B}{\lambda}e^{-\lambda t} + \frac{R\left(e^{-\lambda}-1\right)}{\lambda^2(R-\eta+\lambda)}$$

$$\tau_1^* = \frac{\eta B}{\lambda} + \frac{R(\eta-\lambda)e^{\lambda t}}{\lambda(R-\eta+\lambda)} \quad ; \quad \tau_2^* = \tau_2$$

$$\xi_i^* = \frac{\tau_1^* - \tau_2^*\left(\frac{\eta}{2}-\lambda_1\right)e^{\lambda_1}}{\left(\frac{\eta}{2}+\lambda_1\right) - \left(\frac{\eta}{2}-\lambda_1\right)e^{2\lambda_1}} \quad ; \quad \xi_j^* = \frac{\tau_2^*\left(\frac{\eta}{2}+\lambda_1\right)e^{\lambda_1} - \tau_1^* e^{2\lambda_1}}{\left(\frac{\eta}{2}+\lambda_1\right) - \left(\frac{\eta}{2}-\lambda_1\right)e^{2\lambda_1}}$$

The fractional decrease in the mass flow rate through the channel with a permeable lower wall over what it would be if the wall was impermeable is given by

$$\phi = \frac{M - M^*}{M^*}$$

$$\phi = \frac{e^{-\lambda t}\alpha\sigma\left(u_s - \frac{RB}{\sigma^2}\right)\left(e^{\lambda_1 - \frac{\eta}{2}} - 1\right)}{M^*\left(\lambda_1 - \frac{\eta}{2}\right)}$$

3. Discussion

To study the effect of slip parameter α, porosity factor σ, and viscous stratification factor η on the flow over a permeable bed, we calculate the fractional decrease in the mass flow rate $|\phi|$which has been numerically evaluated for each set of values of α,$\sigma$and η.

From table 1 it follows that for small body force, the porosity factor (σ), stratification factor (η) and slip parameter (α) are not at all favourable to the fractional increase in the mass flow of the fluid.

From table 2 it clear that for a given set of values of $\alpha$and η, $|\phi|$directly varies with σ, and when the flow is governed by a body force $Ae^{-\lambda y}$, we see from table 2 that $|\phi| \infty \sigma$which shows that increase of σ is favourable to a fractional increase in the mass flow.

From table 3 it is seen that in presence of a body force $Ae^{-\lambda y}$, $|\phi|$decreases with the increase of $\eta$for each set of value of α, σ which shows that the stratification is not favourable to the fractional increase in the mass flow.

We assume u_0= 1/2, u_s =0.25, λ = 0.1, u_m=1, B=1, R=0.99, A=1, t=0

The fractional increase in the mass flow rate $|\phi|$ as a function of porosity factor (σ) is calculated numerically for different values of α and , shown as under

Table 1

$\alpha = 0.01, \eta = 0.2$

σ	5	10	15	20	25
|ϕ|	0.0000216	0.000046	0.000069	0.000094	0.000121

$\alpha = 0.1, \eta = 0.2$

σ	5	10	15	20	25
|ϕ|	0.000216	0.00046	0.00069	0.00094	0.00121

$\alpha = 0.1, \eta = 0.6$

σ	5	10	15	20	25
|ϕ|	0.000002	0.000005	0.000007	0.0000094	0.00012

$\alpha = 0.01, \eta = 0.6$

σ	5	10	15	20	25
|ϕ|	0.000002	0.000005	0.000007	0.0000094	0.00012

$\alpha = 0.01, \eta = 1$

σ	5	10	15	20	25
|ϕ|	$1x10^{-8}$	$3x10^{-8}$	$4x10^{-8}$	$6x10^{-8}$	$8x10^{-8}$

$\alpha = 1, \eta = 1$

σ	5	10	15	20	25
|ϕ|	$1x10^{-7}$	$3x10^{-7}$	$4x10^{-7}$	$6x10^{-7}$	$8x10^{-7}$

$\alpha = 1, \eta = 0.2$

σ	5	10	15	20	25
|ϕ|	0.00216	0.0046	0.0069	0.0094	0.0121

Now, we assume $u_0 = 1$, $u_s = 2$, $\lambda = 1$, $u_m = 5$, B=1, R=0.01, A=10, t=0 for all calculations of table 2 and table 3. The fractional increase in the mass flow rate $|\phi|$ as a function of porosity factor (σ) is calculated numerically for different values of α and η shown as under

Table 2

$\alpha = 0.01, \eta = 0.2$

σ	5	10	15	20	25
|ϕ|	0.00948	0.01897	0.02845	0.037946	0.04743

$\alpha = 0.01, \eta = 0.4$

σ	5	10	15	20	25
|ϕ|	0.00541	0.01082	0.01624	0.02165	0.02707

$\alpha = 0.01, \eta = 0.6$

σ	5	10	15	20	25
|ϕ|	0.00223	0.00447	0.00670	0.00894	0.011176

$\alpha = 0.01, \eta = 0.8$

σ	5	10	15	20	25
|ϕ|	0.00098	0.00196	0.00295	0.00393	0.004919

$\alpha = 0.1, \eta = 0.2$

σ	5	10	15	20	25
|ϕ|	0.09484	0.18972	0.2845	0.37946	0.47432

$\alpha = 0.1, \eta = 0.4$

σ	5	10	15	20	25
|ϕ|	0.05413	0.10828	0.1624	0.2165	0.2707

$\alpha = 0.1, \eta = 0.6$

σ	5	10	15	20	25
|ϕ|	0.022349	0.044705	0.0670	0.0894	0.11176

$\alpha = 0.1, \eta = 0.8$

σ	5	10	15	20	25
|ϕ|	0.00983	0.01967	0.0295	0.03935	0.049137

The fractional increase in the mass flow rate $|\phi|$ as a function of the stratification factor $(\eta)$is calculated numerically for different values of $\alpha$and σ shown as under:

Table 3

$\alpha = 0.01, \sigma = 5$

η	0.2	0.4	0.6	0.8
|ϕ|	0.00948	0.00541	0.00223	0.00098

$\alpha = 0.01, \sigma = 10$

η	0.2	0.4	0.6	0.8
|ϕ|	0.01897	0.01082	0.00447	0.001967

$\alpha = 0.01, \sigma = 15$

η	0.2	0.4	0.6	0.8
|ϕ|	0.02845	0.01624	0.00670	0.00295

$\alpha = 0.01, \sigma = 20$

η	0.2	0.4	0.6	0.8
|ϕ|	0.037946	0.02165	0.00894	0.00393

$\alpha = 0.1, \sigma = 5$

η	0.2	0.4	0.6	0.8
|ϕ|	0.09484	0.05413	0.022349	0.00983

$\alpha = 0.1, \sigma = 10$

η	0.2	0.4	0.6	0.8
|ϕ|	0.18972	0.10828	0.044705	0.01967

$\alpha = 0.1, \sigma = 15$

η	0.2	0.4	0.6	0.8
|ϕ|	0.2845	0.1624	0.0670	0.0295

$\alpha = 0.1, \sigma = 20$

η	0.2	0.4	0.6	0.8
|ϕ|	0.37946	0.2165	0.0894	0.03935

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