Analytical Relationship between the Electrical Parameters of the PV Array and External Factors based on the Lambert W-Function

Mamadsho Ilolov[a, c], Khakim Ahmedov[a], Ahmadsho M. Ilolov[b], Anvar S. Qodirov[a], Jamshed Sh. Rahmatov[a], Narimon Sh. Yusufbekov[a]

[a] Center of Innovative Development of Science and New Technologies, National Academy of Sciences of Tajikistan, Dushanbe, 734025, Tajikistan

[b] Tajik Technical University, Dushanbe, 734042, Tajikistan

[c]Tajik State University, Dushanbe, 734042, Tajikistan

[a, c]Corresponding Author, Center of Innovative Development of Science and New Technologies, National Academy of Sciences of Tajikistan
Dushanbe, 734025, Tajikistan
E-mail address: *ilolov.mamadso[at]gmail.com* (M.Ilolov)

Abstract: ***The main goal of this research is to find and to analyze qualitatively, the current/voltage characteristics of the photovoltaic (PV) array. The proposed mathematical model is based on the single diode equivalent circuit; the analytical relationship between the electrical characteristics of the PV module and external environmental factors such as the intensity of solar radiation and temperature are taken into account. Mathematical and computer modeling are of the particular importance under the conditions of a significant spread in the values of external factors which occur in the mountain and remote desert areas in Central Asia. A mathematical analysis of the resulting transcendental equation is carried out using a Lambert W-function which has been the subject of a large number of publications.***

Keywords: Mathematical model, solar cell, Lambert W-functions, maximum power point, photovoltaic module

1. Introduction

Solar energy and its use are of great interest for science research and technological development. According to [1], the total resource of renewable solar radiation is 13.7 TW, which exceeds the current primary energy consumption by almost 6000 times. The main indicator of the energy efficiency of sunlight is the insolation index of the project implementation region. Insolation shows the intensity of irradiation of the surface by sunlight and is measured in kW.h/m^2 for a certain time period (day, month and year). The greater insolation of the region, the greater solar energy that could be converted into electrical and thermal energy.

Transformation of solar energy into electricity requires a device, which takes the energy off sunlight, the flow of elementary particles (photons). The "heart" of a solar module is a semiconductor cell in which a gated photoelectric effect occurs. It consists in the appearance of an electromotive force in p-n transition under the action of photons. The most common semiconductor is silicon with a suitable "forbidden zone" width of 1.12 eV for absorbing solar energy. In this regard, at present, crystalline silicon solar cells constitute about 90% of the world market; the thin film and hybrid perovskite solar cells [2, 3] take the rest.

Note that the PV effect was first observed in an electrolytic cell by Edmond Becquerel in 1839, that is, more than 180 years ago [4]. Over the past centuries, outstanding discoveries have been made in this branch of science and technology. We only note the great achievements noted by the Nobel Prize (Albert Einstein, 1922 [5]; Zhores Alferov, Herbert Kroemer, 2000 [6]). Solar photovoltaics is not born from scratch. Largely due to the development of electronics, laser technology an electric power for spacecraft, a scientific and technological base has been created, which can serve as a starting point for the development of terrestrial solar energy based on semiconductors. We are witnessing a wide investment of funds in this area, corresponding to the significance that solar power will have in future. Along with the analysis of the theoretical foundations of solar energy, great results have been achieved in the development of photovoltaic technologies. In this regard, three main stages can be distinguished. The first important improvements in the efficiency of solar cells were obtained in the early 50s of the last century [7]. This is the development of the technology of crystal growth and diffusion junction, cell refinement and contact design. The second stage of development was achieved in the 70s in the form of shallow junctions, photolithographic metallization, surface textures and anti-reflective coatings [8]. Finally, the third stage occurred in the early 80s, which led to an improvement in surface and contact passivation, bulk lifetimes and light trapping in the cell [9]. An analysis of the current level of photovoltaic technologies is given in [10]. In this paper, we establish an analytical relation between the electric parameters of the photoelectric array and external factors (solar radiation, ambient temperature) based on the special Lambert W-function. This approach, unlike the numerical methods for analyzing the corresponding models, makes it possible to obtain an explicit expression for the values of current and voltage of a photovoltaic device.

2. Modeling of a Solar Cell

Our model reflects the mutual correlation of four experimentally measured variables of a solar cell: current (I), voltage (V), energy of illumination (E), and cell temperature (T). A solar cell can be represented by a five-parametric single diode equivalent circuit (IM5P) as shown in Fig. 1. (Note also that there are multi-diode models [11].) The single diode model is simple and it describes fairly accurately the characteristics of a solar cell for most applications.

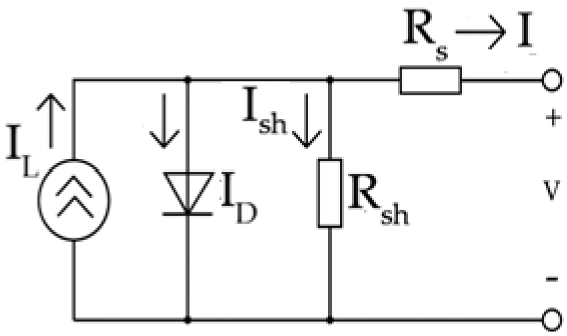

Figure 1: Single diode equivalent circuit (IM5P)

To specify the most popular and practical functioning of a solar cell, authors [12] denoted the representation of losses by the resistors as shown in Fig. 1, having a current source connected in parallel with a diode and two resistors, one in series and other in parallel. Authors [13-15] noted the importance of the five-parameter approach to an equivalent solar circuit.

To fully define the relationship between the output current (I) of photocell and the terminal voltage (V) for the single diode model, the following equations apply:

$$I = I_L - I_D - I_{sh} \tag{1}$$

or

$$I = I_L - I_o\left[\exp\left(\frac{V + IR_s}{a \cdot V_T}\right) - 1\right] - \frac{V + IR_s}{R_{sh}} \tag{2}$$

where I_L – current generated by the incidence of light, I_o – return saturation current, I_D – diode current, R_s – series resistance, R_{sh} - shunt resistance, a – ideality factor of the p-n transition (a = 1, …, 5), V_T = nkT/q – thermal voltage, n – number of cells in series, q – electron charge, k – Boltzmann constant, T – temperature.

The model parameters I_o, R_s and R_{sh} can be calculated on the basis of the passport data of a solar module. Photocurrent I_L is calculated on the basis of illumination and temperature.

Current I_o depends on temperature and is determined by formula

$$I_o = \frac{I_{sc} + k_{I_{sc}}(T - T_0)}{\exp\left(\frac{V_{oc} + k_{V_{oc}}(T - T_0)}{a \cdot V_T}\right) - 1}$$

where I_{sc} – short circuit current, V_{oc} – open circuit voltage, $k_{I_{sc}}$ and $k_{V_{oc}}$ – temperature coefficients of short circuit current and open circuit voltage, respectively, T_0 – absolute temperature under standard conditions ($T_0 = 298K$) [16]. The parameters I_{sc}, V_{oc}, $k_{I_{sc}}$, $k_{V_{oc}}$ are indicated in the passport of a solar module.

Photocurrent (I_L) is directly proportional to the illumination and increases with temperature in solar cells [16]:

$$I_L = \frac{E}{E_0}\left(I_o + k_{I_{sc}}(T - T_0)\right)$$

where E – irradiance, E_0 – irradiance under standard conditions ($E_0 = 1000w/m^2$).

Transcendental equations like (1), (2) are solved by numerical methods. There are many different numerical methods of varying accuracy and complexity (Newton method of tangents, Gauss-Zeidel method, PSpice modeling method).

Our approach in this paper is to apply the universal method, which would allow a sufficiently accurate determination of different electrical characteristics of the solar module through the use of a Lambert W-function. Over the past 20 years the properties of this special function have been studied to a good degree [17-19].

3. Lambert W-function and some properties

We define real Lambert W-function for real argument x in the form of solution of the functional equation

$$W(x)\exp\big(W(x)\big) = x \tag{3}$$

In other words, Lambert W-function is an inverse function for $W = x\exp(x)$. It is quite easy to represent the graph of the Lambert function and establish its simplest properties (Fig. 2).

The Lambert function has no expression through the elementary functions and is neither an even nor an odd function. It is defined in the interval $(-e^{-1}, \infty)$, where it takes values from $-\infty$ to ∞.

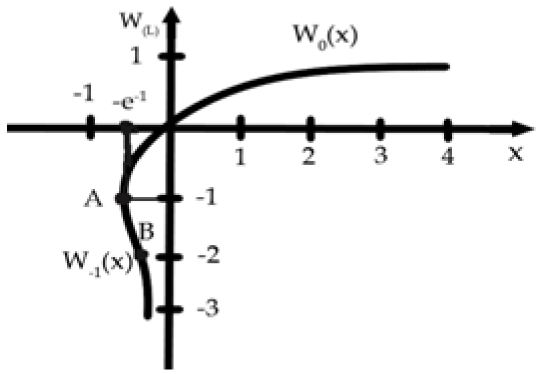

Figure 2: Graph of real branches of Lambert W-function

Point A with coordinates$(-e^{-1}, -1)$ divides the graph of the function into two branches, upper $W_0(x)$ and lower $W_{-1}(x)$, so that both branches at point A have a vertical tangent. The upper branch $W_0(x)$, often called the main branch, passes through the origin and has no more features. The lower branch $W_{-1}(x)$ has a inflection point B with coordinates $(-2/e^2, -2)$ and a vertical asymptote. Other integer values of the index $x \neq 0, -1$ for the function $W_k(x)$ belong to complex-valued branches and are not of interest for our work.

Here are other identities of the Lambert W-function needed for further calculations:

Paper ID: SR201211161232 DOI: 10.21275/SR201211161232

$$\exp(W(x)) = \frac{x}{W(x)} \quad (4)$$

$$\ln W(x) = \ln x - W(x) \quad (5)$$

$$\exp\{W[\exp(W(x))]\} W[\exp(W(x))] W(x) = x \quad (6)$$

4. Mathematical modeling of IM5P via a Lambert W-function

Solar cell is required to be characterized by the short circuit current (I_{sc}), the open circuit voltage (V_{oc}), and ideality factor a. Equation (2) can be rewritten as follows:

for $I = I_{sc}, V = 0$,

$$I_{sc} = I_L - I_0 \left[\exp\left(\frac{I_{sc} R_s}{a \cdot V_T}\right) - 1\right] - \frac{I_{sc} R_s}{R_{sh}}, \quad (7)$$

for $I = 0, V = V_{oc}$,

$$0 = I_L - I_0 \left[\exp\left(\frac{V_{oc}}{a \cdot V_T}\right) - 1\right] - \frac{V_{oc}}{R_{sh}}. \quad (8)$$

For maximum power point $I = I_{mp}, V = V_{mp}$

$$I_{mp} = I_L - I_0 \left[\exp\left(\frac{V_{mp} + I_{mp} R_s}{a \cdot V_T}\right) - 1\right] - \frac{V_{mp} + I_{mp} R_s}{R_{sh}} \quad (9)$$

Considering that the derivative of the solar cell power at the maximum power point is equal to zero, we find

$\frac{\partial P}{\partial V} = V \frac{\partial I}{\partial V} + V = 0.$

Then we obtain

$$-\frac{I_{mp}}{V_{mp}} = -\frac{I_0}{a \cdot V_T}\left(1 - \frac{I_{mp} R_s}{V_{mp}}\right) \exp\left(\frac{V_{mp} + I_{mp} R_s}{a \cdot V_T}\right) - \frac{1}{R_{sh}}\left(1 - \frac{I_{mp} R_s}{V_{mp}}\right). \quad (10)$$

Equations (7-10) represent a system with four unknowns, two of which (R_s and R_{sh}) refer to the equivalent circuit. It is necessary to derive equations for the resistances R_s and R_{sh}.

After series of transformations we find the following equations:

for R_s

$$\frac{a \cdot V_T V_{mp} (2I_{mp} - I_{sc})}{(V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc}))(V_{mp} - I_{mp} R_s) a \cdot V_T (V_{mp} I_{sc} - V_{oc} I_{mp})} = \exp\left(\frac{V_{mp} + I_{mp} R_s - V_{oc}}{a \cdot V_T}\right), \quad (11)$$

for R_{sh}

$$R_{sh} = \frac{(V_{mp} - I_{mp} R_s)(V_{mp} - R_s(I_{sc} - I_{mp}) a \cdot V_T)}{(V_{mp} - I_{mp} R_s)(I_{sc} - I_{mp}) - a \cdot V_T \cdot I_{mp}} \quad (12)$$

Equation (11) is presented in implicit form with respect to R_s. Such an equation can be written in a form convenient for the further introduction of the Lambert W-function. We rewrite the equation (11) in the form

$$\frac{V_{mp}(2I_{mp} - I_{sc})}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})} \exp\left(-\frac{2V_{mp} - V_{oc}}{a \cdot V_T} + \frac{V_{mp} I_{sc} - V_{oc} I_{mp}}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})}\right) =$$
$$= \left(\frac{I_{mp} R_s - V_{mp}}{a \cdot V_T} + \frac{V_{mp} I_{sc} - V_{oc} I_{mp}}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})}\right) \times$$
$$\times \exp\left(\frac{I_{mp} R_s - V_{mp}}{a \cdot V_T} + \frac{V_{mp} I_{sc} - V_{oc} I_{mp}}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})}\right).$$

Further, using relation (3), we transform equation (11) to a form containing the Lambert W-function:

$$\frac{I_{mp} R_s - V_{mp}}{a \cdot V_T} + \frac{V_{mp} I_{sc} - V_{oc} I_{mp}}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})} =$$
$$= W_{-1}\left(-\frac{V_{mp}(2I_{mp} - I_{sc})}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})} \times \exp\left(-\frac{2V_{mp} - V_{oc}}{a \cdot V_T} + \frac{V_{mp} I_{sc} - V_{oc} I_{mp}}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})}\right)\right) \quad (13)$$

where W_{-1} is the negative branch of the Lambert W-function, since the left side of equation (13) is less than -1 for typical solar cells.

Thus, the explicit form of the equation for the series resistance R_s of solar cell will be as follows

$$R_s = A\left(W_{-1}(B \cdot \exp(C)) - (D + C)\right), \quad (14)$$

where

$$A = \frac{a \cdot V_T}{I_{mp}}$$

$$B = -\frac{V_{mp}(2I_{mp} - I_{sc})}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})}$$

$$C = -\frac{2V_{mp} - V_{oc}}{a \cdot V_T} + \frac{V_{mp} I_{sc} - V_{oc} I_{mp}}{V_{mp} I_{sc} + V_{oc}(I_{mp} - I_{sc})}$$

$$D = \frac{V_{mp} - V_{oc}}{a \cdot V_T}$$

The exact analytical solution of equation (2) in the form $V = f(I)$ can be found also through a Lambert W-function.

This is done by series of transformations. Multiplying both sides of (2) by

$$\frac{R_s}{a \cdot V_T}$$

and introducing the following substitution

$$b = -I \cdot \frac{R_{sh}}{a \cdot V_T} + I_L \frac{R_{sh}}{a \cdot V_T} + I_0 \frac{R_{sh}}{a \cdot V_T} - \frac{N}{a \cdot V_T} \quad (15)$$

we obtain

$$b = I_{sc} \frac{R_{sh}}{a \cdot V_T} \exp\left[\frac{V}{a \cdot V_T} + I \frac{R_{sh}}{a \cdot V_T}\right] \quad (16)$$

To use the Lambert W-function, it is necessary to get equation (14) to the form

$b \exp(b) = f(b)$.

For this, we multiply both sides of (14) by the exponent of the right side of equation (15).

After simplification we obtain

$$b \exp(b) = I_0 \frac{R_{sh}}{a \cdot V_T} \exp\left(\frac{R_{sh}}{a \cdot V_T}(-I + I_L + I_0)\right) \quad (17)$$

Solution (15) can be written in the form containing the Lambert W-function

$$b = W\left(I_0 \frac{R_{sh}}{a \cdot V_T}\left(\exp\left(\frac{R_{sh}}{a \cdot V_T}(-I + I_L + I_0)\right)\right)\right) \quad (18)$$

Next, the reverse replacement is introduced (14) instead of b in (15). After further simplification of the expression for V, we obtain the requesting formula for finding the solar cell voltage depending on the current I:

$$V = a \cdot V_T W\left\{I_0 \frac{R_{sh}}{a \cdot V_T} \exp\left[\frac{R_{sh}}{a \cdot V_T}(-I + I_L + I_0)\right]\right\} - \\ -I(R_s + R_{sh}) + R_{sh}(I_L + I_0). \quad (19)$$

In a similar way, current can be expressed as a function of voltage. To do this, both sides of equation (2) are multiplied by
$R_s / a \cdot V_T r_1$,

where
$r_1 = 1 + R_s / R_{sh}$.

After some transformations and application of the Lambert W-function we obtain the following formula

$$I = \frac{a \cdot V_T}{R_s}\left\{I_0 \frac{R_s}{a \cdot V_T r_1} \exp\left[\frac{V + R_s(I_L + I_0)}{a \cdot V_T r_1}\right]\right\} + \frac{I_L + I_0 - \frac{I_0}{R_{sh}}}{r_1} \quad (20)$$

Note that the calculation of the values of the Lambert W-function by formulas (17), (18), (19), (20) for very large values of the argument often leads to numerical overflow. To overcome this difficulty, a generalized Lambert function is introduced in the work [20].

The mentioned function has the form

$$y = g(x) = \log(W(\exp(x))) \quad (21)$$

and it does not lead to numerical overflow or underflow. The function (21) is the solution of the equation
$y + e^y = x$
defined both in a neighborhood of zero and in infinity, which cannot be said about the Lambert W-function.

5. Modeling the parameters calculation using the manufacturer datasheet

In order to evaluate the unknown parameters (I_L, I_0, R_s, R_{sh}) and to simplify the calculations for a specific model we must get the ideality factor according to [13] where it is suggested that for monocrystalline silicon cell a=1.2. Also, we suggest that a is independent of any irradiation and temperature.

For an example, a solar module HS200 of Chinese company HOMSOL with the power of 200W was selected, the parameters of which are shown at Table 1. The module consists of 72 series of connected cells.

Table 1: Specifications of HS200 module

Module characteristics	Specifications	Units
Open circuit Voltage (V_{oc})	43.2	V
Short circuit Current (I_{sc})	6.11	A
Voltage at maximum power point (V_{mp})	36.0	V
Current at maximum power point (I_{sc})	5.55	A
Maximum power (P_{mp})	200	W
Maximum system Voltage	1000	V
Operating temperature range	−40℃~90℃	℃
Normal operating temperature	45℃ ± 2℃	℃
Panel efficiency	15.07%	%

Obtained electrical parameters of the equivalent circuit for the module are given in the Table 2 (the ideality factor is taken to be constant). The currents I_0 and I_L are calculated by formulas (7) and (8) respectively. Resistances R_s and R_{sh} are calculated by formulas (12), (13) and (14). The input parameters from the datasheet at Standard Test Conditions (STC) are incident normal radiance (1000 W/m^2), cell temperature is 25℃ and air mass AM =1.5 g.

Table 2: Calculated parameters of HS200 module

Parameters	Calculated values
I_0	$9.825 \cdot 10^{-8} A$
I_L	8.214 A
R_{sh}	415.45 Ohm
R_s	0.221 Ohm
a	1.2

6. Simulating the solar equivalent circuit

For comparison purposes a program was written in the Python language for solving equation (2) by Newton's numerical method. For five levels of irradiance the current-voltage characteristics (CVC) of the module were calculated. The results are shown in Fig. 3.

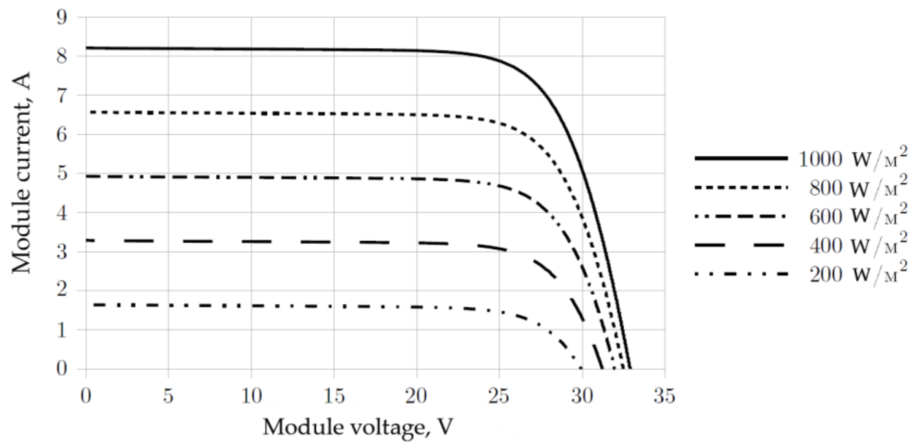

Figure 3: CVC of HS200 module under different irradiance (Newton's method)

Paper ID: SR201211161232 DOI: 10.21275/SR201211161232

Fig. 4 shows CVC calculated using the Lambert W-function by formulas (17), (18), (19). The calculations were carried out in the Maxima free computer algebra systems.

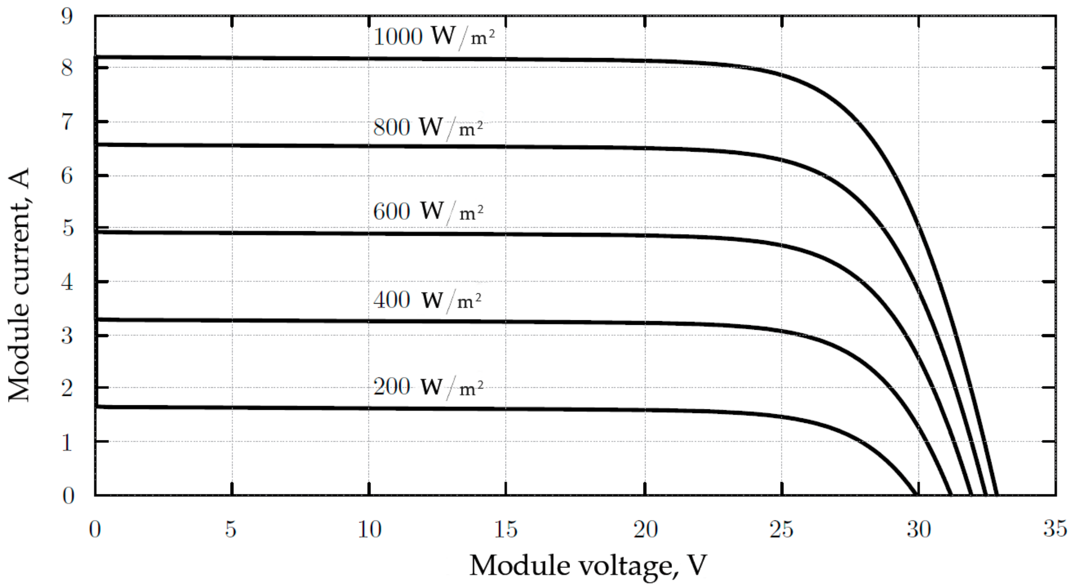

Figure 4: CVC of HS200 module under different irradiance (Lambert W-function)

7. Conclusion

In this work the parameters of a single diode equivalent circuit (IM5P) of a solar cell module were determined through the use of a Lambert W-function. We give equations for current/voltage characteristics of a solar module in two forms: $V = f(I)$ and $I = f(V)$. Also, we give equations for calculation of resistances R_s and R_{sh}. The study is made for five irradiation levels (at constant temperature). Comparison of the results obtained by the Newton method (numerical) and by the use of the Lambert function (exact) showed their coincidence with sufficient accuracy. The reported mathematical model provides the way for the end users to conduct the realistic mathematical modeling of a solar cell panel or an array.

CRediT authorship contribution statement
Mamadsho Ilolov: Conceptualization, Methodology, Writing - original draft, Writing - review & editing. Khakim Akhmedov: Writing - review & editing. Ahmadsho M. Ilolov: Writing - review & editing. Anvar S. Qodirov: Methodology, Resources, Writing - review & editing. Jamshed Rahmatov: Writing - review & editing. Narimon Sh. Yusufbekov: Software.

Declaration of competing interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] IEA (2019). World Energy Outlook 2019, IEA, Paris http://www.iea.org/reports/world-energy-outlook-2019.
[2] M.Ilolov, Sh. Kabirov, A.M. Ilolov, M.Q. Subhoni, T. Yamamoto, On the Production of Aluminum and Silicon by Electrolysis, Conference on Sustainable Development of Energy, Water and Environment Systems, Palermo, 30.9-4.10.2018, Book of Abstracts (2018) 293.
[3] M. Ilolov, Sh. Kabirov, A. M. Ilolov, On the electrolytic method of producing Solar-grade Silicon: innovations and new technologies, AIP Conference proceedings 1999, 130008 (2018). Published online: 10 August 2018.
[4] E. Becquerel, Memoire sur les effects electrigues products sondesragonssolaries, Comptus Rendus 9 (1839) 561-567.
[5] A. Einstein, On a heuristic point of view concerning the production and transformation of light, Annalen der Physic 17 (1905) 132-148.
[6] Zh.I. Alferov, V.M. Andreev, V.D. Rumantsev, Trends and Perspectives of Solar Photovoltaics, Physics and Techniques of Semiconductors, 38(8) (2004) 937-948.
[7] D.M. Chapin, C.S. Fuller, G.L. Pearson, A new silicon p-n junction photocell for converting solar radiation into electrical power, Journal of Applied Physics 8 (1954) 676.
[8] J. Lindmayer, J. Allison, The violet cell: an improved silicon solar cell, COMSAT Technical Review 3 (1973) 1–22.
[9] K.A. Emery, C.R. Osterwald, Solar cell efficiency measurements, Solar Cells 17 (1986) 253–274.
[10] Martin A. Green, The Path to 25% Silicon Solar CellEfficiency: History of Silicon Cell Evolution, Prog. Photovolt: Res. Appl. 17 (2009) 183–189.
[11] K. Roberts, J.R. Valluri, On calculating the Current-Voltage Characteristics of Multi-Diode Models for Organic Solar Cells, Arxiv: 1601.02079vI [physics.comp._ph] (2015).
[12] S.A. Afgan, H. Almusavi, H.Geza, Simulating the electrical characteristics of a photovoltaic cell based on a single-diode equivalent circuit model, MATEC Web of Conferences. 126, 03002, 2017, Annual Session of Scientific Papers IMTORADEA (2017).

[13] M.G. Vilalva, J.R.Gazoli, E.R. Filho, Comprehensive approach to modelling and simulation of photovoltaic arrays, IEEE Transactions on Power Electronics 24 (5) (2009) 1198-1208
[14] G.R. Walker, Evaluating MPPT topologies using a MATLAB PV Model, Journal of Electrical and Electronics Engineering 21 (1) (2001) 49-56.
[15] J. Cubas, S. Pindado, C.D. Manuel, Explicit Expressions for Solar Panel Equivalent Circuit Parameters based analytical Formulation and the Lambert W-function Energies 7 (7) (2014) 4098-4115.
[16] M. Wolf, H. Raushenbach, Advance Energy Conversions 3 (1963) 455-479.
[17] A.E. Dubinov, I. D. Dubinova, S. K. Saikov, Lambert W-function and its applications in mathematical problems of physics, Sarov Russian Federal Nuclear Center(2006) 106.
[18] R.M. Corless*et al.* On the Lambert W-function. Advances in Computational Mathematics 5 (1996) 329-359.
[19] K. Roberts, A Robust approximation to a Lambert-Type Function, arXiv:1504.01964v1 [math. NA] (2015).