Fractional Order PID Controller Using Recursive Least Square Algorithm in MIMO Systems

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Abstract: ***In this paper a new Fractional order PID controller (FOPID) using RLS algorithm is proposed to obtain precise nonlinear control model. The parameters of the FOPID can identify using Recursive Least Square (RLS) algorithm. The RLS algorithm, on the contrary of its usual application as an identification method, is used in the proposed controller to update the PID gains forcing the system to behave like a desired reference model. Unlike other techniques, the proposed fractional order PID controller has the advantage that it does not impose restrictions on the system structure such as being stable, square, minimum phase, nor almost restrict positive real. Since stability is a vital issue in the evaluation of control systems, therefore stability analysis of the proposed approach can develop using Lyapunov stability theory.***

Keywords: Fractional order PID, RLS algorithm

1. Introduction

The control processes have provided better developments in the process control industries.

During these years, the control processes have given better advances in the industry. Fractional-order proportional-integral-derivative (FOPID) controllers have received a great attention in the previous years, from both an industrial and an academic point of view. However, simple tuning rules and no effectiveness still exist for these controllers like those specified for the integer PID controllers. The PID controller, for the reason of its functional simplicity is mostly used in industrial applications. Conversely, their parameters are often adjusted using test or experiences and error methods. Unluckily, it is absolutely hard to properly adjust gains' PID, because many industrial systems are often burdened with problems such as structural complexity, uncertainties and nonlinearities.

Fractional calculus is a generalized form of classical calculus that deals with derivative and integrals of fractional order, that lead to similar tools and concept as classical calculus but has much wider applications.

Even though the concept of fractional order calculus is almost as old as traditional calculus it has only gained extensive attentions in the past few decades, fractional-order calculus has been rediscovered ,due to its extensive applications namely in systems in chemistry ,physics and especially in the field of control theory .The success of fractional calculus stems from the various efficient and effective methods of differentiation and integration of equations of non integer order formulated it the last few decades.

In recent years FOPID controller has gained much attention both from the academic and industrial point of view,as in principle they are more flexible in comparison with the standard PID controller ,were standard PID controllers have 3 controllable parameters (K_p, K_d, K_I),FOFID introduced 2 new parameters for a total of 5 such parameters($K_p, K_d, K_I, \alpha, \mu$).FOPID controller can be represented as $PI^{\alpha}D^{\mu}$.The two additional parameters α of integration and μ of derivative also made the tuning of the new FOPID controller more complex.

Many different approaches can be taken in order to get the optimum setting for the five different parameters of the fractional $PI^{\alpha}D^{\mu}$ controllers. This project proposed one such approach of utilizing "Recursive least square Algorithm" in order to update the PID gains in real time and in turn get a optimized output from the FOPID controller which is being used to control MIMO systems. The stability analysis of the proposed approach can develop using Lyapunov stability theory.

2. Literature Survey

Fractional calculus aims at further improvement of the conventional process of derivation and integration to include other than integer orders. The initial impetus to the concept of the non-integer differentiation was given by L' Hospital letter written to Leibnitzin in 1695,from then onwards numerous well-known mathematicians such as Laplace, Fourier, Abel, and Laurent efficiently working on the concept of fractional calculus. In the nineteenth century with the help of Liouville, Grünwald, Letnikov and Riemann entire theory appropriate for current mathematical development has been distinguished.

At present, fractional calculus is an unshakable theory with well framed mathematical foundation. The major cause for the dispersion of fractional calculus is that, this tool describes more exactly about physical systems[1]. It was Prof. Oustaloup who firstly introduced the fractional-order Controllers (FOC). He urbanized the three diverse version of the CRONE controller such that first, second and third generation controllers were developed[2]. I. Podlubny presented the first report on fractional order PID controller[3]. He coined a concept of fractional order $PI^{\lambda}D^{\mu}$ controller which is the extension of the classical PID controllers, where a PID controller structure with an integrator of order λ and a differentiator of order μ was introduced. The $PI^{\lambda}D^{\mu}$ controller is extra bendy and provides better prospects to alter the dynamic properties of fractional order systems in contrast with the classical PID controller[4].

In 1995 and 1996 J. Machado proposed algorithms to adopt the time domain which befits for z transform analysis and digital implementation. This study represents a first stage towards the development of motion control systems depend on the theory of FDI'S (Fractional derivatives and integrals). In 1961, Manabe introduced the frequency and transient response of the non-integer integral and its application to control systems and then after by Barbosa, Tenreiro and Ferreira in 2003. A frequency domain approach was also studied by Vinagre et al4 by using fractional-order PID controllers. Several approximation methods for continuous models and discrete models were studied and compared these methods in both time and frequency domains for implementing fractional order controllers by fractional order operators[5]. An optimization method to tune the FOPID controller has been used in such a manner that predefined design specifications are fulfilled. Further research activities led to the progress of new successful tuning techniques for fractional order controllers by conservatory of the classical control theory.

These references [6,7]provide a more bendable tuning plan using which the desired controls with respects to classical controllers can be achieved easily. Based on precise phase and gain margins with a minimum integral squared error (ISE) criterion an optimal FOPID controller is designed[8], a novel approximation method is projected which is extra accurate in the low and high frequency range compared with the well established Oustaloup's approximation method for a fractional order differentiator. In reference[9], first and second sets Ziegler- Nichols tuning rules for FOPID controllers were proposed. Another approach is achieved in reference[10] optimization method to tune the controller and auto-tuning method for FOPID controller using the relay test has been proposed.

A fractional order controller was implemented by using PSO method[5]. Optimal problems of FOPID are solved by genetic algorithms (GA) and can be obtained for better results with PSO in contrast with conventional methods. A similar approach has been adopted for tuning of FOPID controller using integral performance index criteria with PSO. In 2010 New tuning methods for FOPID and set point weighting of FOPID are proposed [11]. Application of fractional calculus in control system is numerous and researcher's interest is increasing day by day in relevant field. Through well-known tuning methods of FOPID controllers are obtainable with numerous examples to verify the success of the methods[12]. A model reduction method and an overt FOPID controller tuning rule for high order are proposed and by simulation showing the effectively in many dynamic systems[13].

3. FOPID Controller Description

3.1 Fractional Calculus

Fractional calculus is a branch of mathematics that analyses and studies several different possibilities of defining powers of real or complex number type of differential operator D. Where,

$Df(x) = (d/dx)f(x)$ and of the integer ,

Operator J, $Jf(x) = \int_0^x f(s)ds$,

and developing a calculus for such operators generating the classic one.

3.2 Classical PID Controller

The proportional-integral-derivative (PID) controller is distinguished as the most common form of feedback. In process control today, more than 95% of the control loops are of PID type, but these controllers can be found in all areas where control is used. Despite its straightforward structure, the popularity of PID controllers lies in the simplicity of the design procedures and in the effectiveness obtained to the system performance. Those are the main reasons why PID controllers have survived many changes in technology, from mechanics and pneumatics to microprocessors via electronic tubes, transistors, integrated circuits, among others. The use of PID control consists of applying properly the combination of three types of corrective actions to the error signal, which represents how far or near is the desired output from the actual output. As widely known, these three control actions are proportional, integral and derivative.

The key aspect when tuning PID controllers is in deciding how to best combine those three terms to achieve the most efficient regulation of the process variable for the considered problem. As well known, the most obvious way is to use a simple weighted sum where each term is multiplied by a tuning constant or gain, and the results are then added together as follows:

$$u(t) = k_p\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)\right)d\,\tau + T_d\frac{de(t)}{dt}$$

Control law guarantees that the present (due to the proportional action), the past (by means of the integral action) and the future of the error (by the derivative action) are taken into account, as shown below. Two main observations can be made to the controller needs only compute the current error between the measured process variable and the desired set-point to calculate how much and how fast that difference has been changing over time, and the relative contributions of each term then can be then adjusted by choosing appropriate values of the controller parameters.

3.3 Fractional Order PID controller

The generalization to non-integer orders of (14) leads to the typical algorithm of a FOPID controller, i.e.,

$$u(t) = k_p\left(e(t) + \frac{1}{T_i}D^{-\lambda}e(t) + T_d D^{-\lambda}e(t)\right)$$

where and λ, $\mu \in$R+ are the non-integer orders of the integral and derivative terms, respectively, and D is the fractional operator definitely as Riemann–Liouville as:-

$$D^{-n}f(t) = \frac{1}{T(n)}\int_0^t f(y)(t-y^{n-1}\,dy$$

(n is a general non-integer order, and $T(n)$, the gamma function).

3.4 RLS Algorithm

The important feature of RLS filter is that its rate of convergence is typically an order of magnitude faster in that of simple LMS filter, due to the fact that the RLS filter whitens the data by using the inverse correlation matrix of the data, assumed to be of zero mean. This improvement in performance, however, is achieved at the expense of an increase in computational complexity of RLS filter.

In recursive implementations of the method of least squares, the computations with prescribed initial conditions and use the information contained in new data samples to update the old estimates therefore the length of the variable data is variable. Accordingly the cost function to be minimized as J(n), where n is the variable length of the observable data, it is customary to introduce a weighing factor in to the definition of J(n). Therefore

$$J(n) = \sum_{i=1}^{n} \beta(n,i)\,|e(i)|^2$$

where $e(i)$ is the difference between the desired response $d(i)$ and the output $y(i)$ produced by a transversal filter whose tap inputs (at time i) equal $u(i)$, $u(i-1)$,….. $u(i-M+1)$

Based on the RLS algorithms, we tune the parameters which are the FOPID gain values so that the performance index can be minimized:

3.5 Problem statement

To develop a new Fractional order PID controller (FOPID) using RLS algorithm to obtain precise nonlinear control model for MIMO systems.

3.6 Objective of research work

In this proposed work, there are three objectives:
1) To update the gain of fractional order PID controller(FOPID) using RLS agorithm
2) To obtain the optimum control model of FOPID
3) To analyse the proposed FOPID in MIMO systems.

4. Materials and Methods

1) To optimize the output of the fractional PID controller (FOPID).
2) The transfer function of FOPID controller can be represented as,

$$G(s) = K_p + \frac{K_I}{s^{\lambda}} + K_d s^{\mu}$$

K_p = Proportional gain.

K_d = Deferential gain.

K_I =Integral gain.

λ = order of integration.

μ =order of differentiation

3) The optimization is carried out by finding the optimum values for the 5 select parameter of the FOPID controller.
4) This project employed *Recursive least square* algorithm in order to find the optimum values of $K_p, K_d, K_I, \alpha, \mu$.
5) The simulink tool can be used to represent the block diagram of a closed loop system which consists of the FOPID controller and MIMO system to represent its transfer function and a unity feed back.

5. Conclusion

The desired outcome is a new fractional PID controller (FOPID) with RLS algorithm which can optimize the parameters of a FOPID controller. It is expected that the resulting approach will be possible to implement FOPID in multiple input and multiple output systems(MIMO) . Accordingly, the resulting algorithm should be robust and at the same time able to minimize the errors present in real-time while the controller is running. Lastly, it is essential that the algorithm is able to perceive platform alterations, adapt and converge against a better condition.. As a consequence, it would perform better than a standard PID controller.

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