Effects of Magnetic Parameter and Injection Velocity on Unsteady Magnetohydrodynamic Fluid Flowover a Vertical Stretching Sheet in the Presence of Induced Magnetic Field

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Abstract: ***A study of the effects of variable magnetic parameter and injection velocity on unsteady magnetohydrodynamic fluid flow, a viscous, incompressible fluid over a vertical stretching sheet medium taking into account induced magnetic field wascarried out. The magnetic induction equation governing the fluid flow was solved using implicit finite difference method.The resulting set of algebraic equation was solved using Matlab in computer program. The numerical solutions for inducedmagnetic field profiles are depicted graphically to show the effects of injection velocity (s), Prandtl number (Pr) and magnetic field parameter (M) on induced magnetic field profiles. The results showed that an increase in injection velocity leads to an increase in induced magnetic field whereas an increase in magnetic parameter leads to a decrease in induced magnetic field. Also, a decrease in Prandtl magnetic number leads to an increase in induced magnetic field.***

Keywords: Injectionvelocity, Magnetic field parameter, Magnetohydrodynamics and implicit finite difference scheme

Abbreviations: Magnetohydrodynamics (MHD); Right Hand Side (RHS)

1. Introduction

The boundary layer flow of unsteady incompressible fluid flow over a stretching sheet has gained a lot of attention by researchers owing to its wide application in industrial and engineering processes. Industrial condensation process of metallic plate in a cooling bath and extrusion of polymer sheet from a dye (Alinejad and Samarbakhshs, 2012). For instance, a number of technical processes concerning polymers involve the cooling of continuous strips extruded from a dye by drawing them through a cooling bath and in the process of drawing, these strips are subsequently stretched to achieve the desired thickness. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. The Magnetohydrodynamic (MHD) flow in electrically conducting fluid can control the rate of cooling and the desired quality of product can be achieved (Manjunatha and Gireesha, 2016). The most widely used cooling medium is water and air. However, the rate of heat exchange achievable by these fluids has been realized to be unsuitable for certain sheet materials. Hence the use of magnetic field has been preferred owing to its ease of implementation and also its intrusive nature. The rate of cooling and the desired properties of the end product can be controlled by the use of electrically conducting fluid and the application of magnetic fields (Dessie and Kishan, 2014). The study over a stretching sheet was initiated by Sakiadis (1961). Since then quite a number of researchers have been done. Crane (1970) presented an exact solution analytical solution for a two-dimensional stretching surface in a quiescent fluid while Gupta and Gupta (1977) analyzed the heat and mass transfer corresponding to the similarity solution for the boundary layer over a stretching sheet subjected to suction/blowing. Since then, quite a number of researchers have studied various aspects of the problems associated with stretching sheet involving magnetic field. Erickson, *et al* (1966) studied heat and mass transfer over a moving surface considering the effects of suction/injection. Paul (2017) considered analytical solution of one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field. The vertical plate and medium of flow are considered to be porous. The fluid is assumed to be optically thin and the magnetic Reynolds was considered small enough to neglect induced hydromagnetic effects. They found that the velocity decreased with increase of the suction parameter for both cases of cooling and heating of the porous plate whereas skin friction increased with increase of suction parameter.

In the above mentioned literature, the magnetic field is considered uniform and the induced magnetic field effects are neglected. Induced magnetic field play an important role, especially when the magnetic Prandtl number is large. Jha and Aina (2017) did a theoretical investigation of an MHD mixed convection flow in a vertical microchannel formed two electrically non-conducting infinite vertical plates. The influence of an induced magnetic field arising due to motion of an electrically conducting fluid is taken into consideration. In their results, they found that the effect of the Hartmann number and magnetic Prandtl number on the induced current tend to decrease at the central region of the microchannel. Ghosh *et al* (2011) considered steady, fully developed magnetohydrodynamic flow of a viscous, incompressible, electrically-conducting Newtonian fluid between parallel plates under the action of an inclined magnetic field and a constant pressure gradient along the longitudinal axis of the channel. Their results showed that an elevation of the Grashof number enhances the secondary induced magnetic field component; however, there was a decrease in magnitude of the primary induced magnetic field component with increasing Grashof number. El-Aziz and Afify (2008) did a numerical investigation of the steady MHD boundary layer flow near the stagnation point over a stretching surface in the presence of induced magnetic field, viscous dissipation, slip velocity phenomenon, and heat absorption/generation effects. The Casson fluid model was used to characterize the non-Newtonian fluid behaviour. Results predicted that the magnetic parameter with $\alpha < 1$has the tendency to enhance the heat transfer rate, whereas the reverse trend is seen with $\alpha > 1$. It is also noticed that the rate of heat transfer is a decreasing function of the reciprocal of a magnetic Prandtl number, whereas the opposite phenomenon occurs with the magnitude of the friction factor. Rwanda et al (2018) studied MHD boundary layer flow of a viscous incompressible fluid over a porous exponentially stretching sheet with an inclined magnetic field in the presence of thermal radiation and injection. They observed that fluid velocity was suppressed by increasing the strength of magnetic field, angle of inclination and permeability property of the material, whereas it was boosted when injection and stretching on the material were increased. It was also observed that increasing the magnetic field, angle of inclination and permeability of the material on the path of flow of the fluid lowered the skin friction, but increased when the material was stretched exponentially with injection. Kamran *et al* (2018) did a mathematical analysis of single and multi-walled carbon nanotubes along with two different base fluids, water and kerosene oil, subjected to a strong magnetic field with induced magnetic effects. They observed that the heat transfer rate was greater in kerosene oil based nanofluid as compared with the water based nanofluid. Uwanta and Hamza (2014) investigated the effect of suction/injection on unsteady hydromagnetic natural convection flow of viscous reactive fluid between two vertical porous plates in the presence of thermal diffusion. Ndungu *et al* (2015) studied unsteady hydromagnetic couette flow of a viscous incompressible electrically conducting fluid in a rotating system with injection through the lower plate in the presence of a variable transverse magnetic field. The plates were considered porous and fluid flowing past the plates was induced by the movement of lower plate with the upper plate set stationary. They noted that rotation retarded flow whereas injection accelerated the flow. Also, induced magnetic field increased with increase in magnetic Reynolds number. Nyariki *et al* (2017) considered unsteady MHD couette flow of an incompressible electrically conducting fluid between two parallel infinite plates in the presence of an inclined variable magnetic field fixed relative to the moving porous plate with suction. In their results, they noted that an increase in suction lead to a decrease in the induced magnetic field, the velocity and temperature profiles. Also, as the angle of inclination and magnetic parameter increased, the velocity and the induced magnetic field increased. The current paper focuses on the effects of variable magnetic parameter, suction velocity and Prandtl magnetic number on induced magnetic field. The magnetic induction equation governing the fluid flow was solved using implicit finite difference method

2. The Induction Equation

From the electromagnetic equations we established the relationship between electric field $\boldsymbol{E}$, the magnetic field, $\boldsymbol{B}$, and the induction current density, $\boldsymbol{J}$. Ohm's law for a medium moving along a velocity field **q,** the conductor current density $\boldsymbol{J}$is related to the magnetic field **B** as:

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{q} \times \boldsymbol{B}) \tag{1}$$

From Faraday's law which states that the electromotive force generated around a closed loop equals minus the rate of change of magnetic flux through the loop, that is:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{2}$$

Combining equations (1) and (2) we obtain:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = -\nabla \times \left(\frac{\boldsymbol{J}}{\sigma} - \boldsymbol{q} \times \boldsymbol{B}\right) = -\frac{1}{\sigma}\nabla \times \boldsymbol{J} + \nabla \times (\boldsymbol{q} \times \boldsymbol{B}) \tag{3}$$

Using Ampere's law (Okello and Mutuku, 2020) $\nabla \times \boldsymbol{B} = \mu_e \boldsymbol{J}$ in equation (3) we obtain:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{q} \times \boldsymbol{B}) - \frac{1}{\sigma \mu_e}\nabla \times (\nabla \times \boldsymbol{B}) \tag{4}$$

Using the vector identity $\nabla \times (\nabla \times \boldsymbol{B}) = \nabla(\nabla.\boldsymbol{B}) - \nabla^2 \boldsymbol{B}$and the magnetic Gauss law $\nabla.\boldsymbol{B} = 0$, we can formulate the induction equation as:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{q} \times \boldsymbol{B}) + \frac{1}{\sigma \mu_e}\nabla^2 \boldsymbol{B} \tag{5}$$

Where, μ_e, is the magnetic permeability. The first term on the RHS of equation (5) is magnetic tension force which is restoring force that acts to straighten bent magnetic field lines and the second term is the magnetic pressure force which is energy density associated with magnetic field.

2.1Discretization of induction equation

The non-dimensionalised magnetic induction equation is given as:

$$\frac{\partial H}{\partial t} + S\frac{\partial H}{\partial y} = M\frac{\partial u}{\partial y} + \frac{1}{Pr_M}\frac{\partial^2 H}{\partial y^2} \tag{6}$$

Equation (1) can be discretized Using Crank-Nicolson methodas follows

$$\frac{H_{i,j+1} - H_{i,j}}{\Delta t} + S\frac{H_{i+1,j+1} - H_{i-1,j+1} + H_{i+1,j} - H_{i-1,j}}{4\Delta y} = Mu_y + \frac{1}{Pr_M}\frac{H_{i+1,j} - 2H_{i,j} + H_{i-1,j} + H_{i+1,j+1} - 2H_{i,j+1} + H_{i-1,j+1}}{2(\Delta y)^2} \tag{7}$$

Multiplying equation (7) by $4Pr_M\Delta t$ and letting $\frac{\Delta t}{\Delta y} = \emptyset, \frac{\Delta t}{(\Delta y)^2} = \mu$ we get the equation

$$4Pr_M H_{i,j+1} - 4Pr_M H_{i,j} + SPr_M\emptyset H_{i+1,j+1} - SPr_M\emptyset H_{i-1,j+1} + SPr_M\emptyset H_{i+1,j} - SPr_M\emptyset H_{i-1,j} = 4Pr_M\Delta t M u_y + 2\mu H_{i+1,j} - 4\mu H_{i,j} + 2\mu H_{i-1,j} + 2\mu H_{i+1,j+1} - 4\mu H_{i,j+1} + 2\mu H_{i-1,j+1} \tag{8}$$

Also, letting $\Delta t = 0.01,\ \Delta y = 0.5$, equation (8) becomes:

$$(4Pr_M + 0.16)H_{i,j+1} + (0.02SPr_M - 0.08)H_{i+1,j+1} + (0.02SPr_M - 0.08)H_{i-1,j+1} = (4Pr_M - 0.16)H_{i,j} + (0.08 - 0.02SPr_M)H_{i+1,j} + (0.02SPr_M + 0.08)H_{i-1,j} \tag{9}$$

Taking $i = 1,2,3,4,5\ \ and\ \ j = 0$ into equation (9),we get the following set of systems of linear algebraic equations

$$\begin{aligned} i = 1: (4Pr_M + 0.16)H_{1,1} &+ (0.02SPr_M - 0.08)H_{2,1} \\ &+ (0.02SPr_M - 0.08)H_{0,1} \\ &= (4Pr_M - 0.16)H_{1,0} \\ &+ \ (0.08 - 0.02SPr_M)H_{2,0} \\ &+ (0.02SPr_M + 0.08)H_{0,0} \end{aligned}$$

$$i = 2: (4Pr_M + 0.16)H_{2,1} + (0.02SPr_M - 0.08)H_{3,1} + (0.02SPr_M - 0.08)H_{1,1} = (4Pr_M - 0.16)H_{2,0} + {} - 0.02SPr_M)H_{3,0} + (0.02SPr_M + 0.08)H_{1,0}$$

$$i = 3: (4Pr_M + 0.16)H_{3,1} + (0.02SPr_M - 0.08)H_{4,1} + (0.02SPr_M - 0.08)H_{2,1} = (4Pr_M - 0.16)H_{3,0} + {} - 0.02SPr_M)H_{4,0} + (0.02SPr_M + 0.08)H_{2,0}$$

$$i = 4: (4Pr_M + 0.16)H_{4,1} + (0.02SPr_M - 0.08)H_{5,1} + (0.02SPr_M - 0.08)H_{3,1} = (4Pr_M - 0.16)H_{4,0} + (0.08 - 0.02SPr_M)H_{5,0} + (0.02SPr_M + 0.08)H_{3,0}$$

(0.08

$$i = 5: (4Pr_M + 0.16)H_{5,1} + (0.02SPr_M - 0.08)H_{6,1} + (0.02SPr_M - 0.08)H_{4,1} = (4Pr_M - 0.16)H_{5,0} + (0.08 - 0.02SPr_M)H_{6,0} + (0.02SPr_M + 0.08)H_{4,0}$$

(0.08 (10)

Taking the initial conditions $H(y,0) = 1$ and boundary conditions $H(0,t) = 0$ into the set of systems of linear algebraic equations in (10), we get the matrix

$$\begin{pmatrix} (4\Pr_m - 0.16) & (0.02s\Pr_m - 0.08) & 0 & 0 & 0 \\ (0.02s\Pr_m - 0.08) & (4\Pr_m - 0.16) & (0.02s\Pr_m - 0.08) & 0 & 0 \\ 0 & (0.02s\Pr_m - 0.08) & (4\Pr_m - 0.16) & (0.02s\Pr_m - 0.08) & 0 \\ 0 & 0 & (0.02s\Pr_m - 0.08) & (4\Pr_m - 0.16) & 0 \\ 0 & 0 & 0 & (0.02s\Pr_m - 0.08) & (4\Pr_m - 0.16) \end{pmatrix} \begin{pmatrix} H_{1,1} \\ H_{2,1} \\ H_{3,1} \\ H_{4,1} \\ H_{5,1} \end{pmatrix} = \begin{pmatrix} \Pr(4 - 0.04s) \\ 4\Pr - 0.16 \\ 4\Pr - 0.16 \\ 4\Pr - 0.16 \\ 4\Pr - 0.16 \end{pmatrix} \quad (11)$$

3. Results and Discussions

The effects of suction velocity, magnetic parameter and magnetic Prandtl number on induced magnetic field are discussed. Various values of S, M and Pr are selected and substituted into equation (11).The results are obtained as shown in table 1, 2 and 3 and presented in Fig 1, 2, 3for injection velocity, magnetic parameter and magnetic Prandtl number respectively.

3.1 Effects of Injection velocity

The other fluid properties of Pr_M *and* $M$are maintained at their default values, the Injection velocity wasvaried as $S = 20, 22$ *and* 24. The constants were taken as$Pr_M = 0.1, M = 1$ while varying $S = 20,\ 22,\ 24$in equation (5) ,we obtain the results as shown in table 1 and presented in figure 1

Distance along x	$S = 20$	$S = 22$	$S = 24$
1	0.6076942	0.5891623	0.5707914
2	0.5077189	0.4980797	0.488849
3	0.5003701	0.4920797	0.4840655
4	0.4974628	0.4897800	0.4822969
5	0.4641048	0.4600573	0.4561312

The results in table 1 are presented graphically as shown in Fig 1

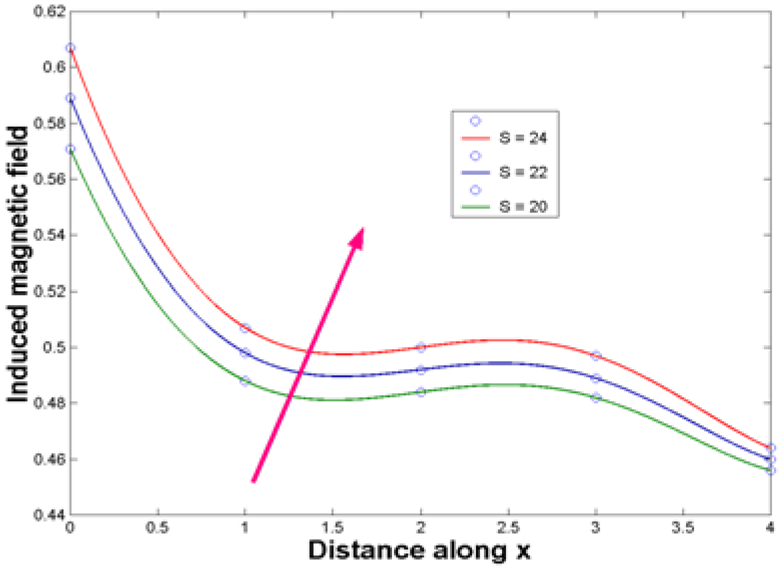

Figure 1: Induced magnetic field against distance along x with varying suction velocity

From the figure 1 it is observed that an increase in suction velocity leads to an increasein induced magnetic field. Injection (blowing) through the wall results to an increase in induced magnetic field. Increase in induced magnetic field is due to pushing of the heated fluid away from the wall, resulting to reduction in boundary layer thickness, that is, less viscosity on the wall.

3.2 Effects of Magnetic Prandtl Number

The other fluid properties of magnetic parameter M and suction velocity $S$are maintained at their default values, the Magnetic Prandtlnumber was varied as Pr_M= 0.3, 0.5, *and* 0.7. The constants were taken as$M = 0.1, S = 20$ while varying $Pr_M = 0.3,\ 0.5,\ 0.7$ in equation (5) ,we obtain the results as shown in table 2 and presented in fig 2

Distance along x	$Pr_M = 0.3$	$Pr_M = 0.5$	$Pr_M = 0.7$
1	0.8310073	0.8406543	0.84992911
2	0.7525931	0.7646771	0.78211292
3	0.75656907	0.7720792	0.7855803
4	0.75692054	0.774627	0.79121199
5	0.7511087	0.765485	0.77932045

The results in table 2 are presented graphically as shown in Fig 2

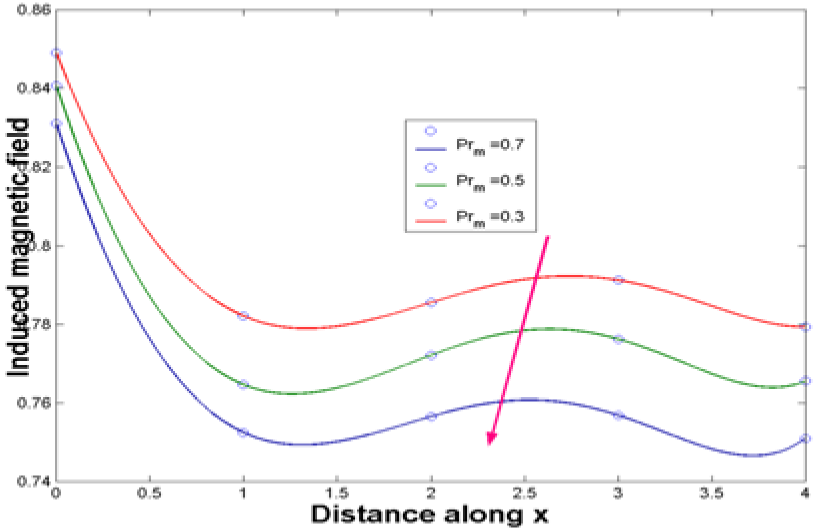

Figure 2: Induced magnetic field against distance along x with varying magnetic Prandtl number

From the figure 2 it is observed that an increase in magnetic Prandtl number leads to a decrease in induced magnetic field. This can be attributed to the fact that increasing the value of magnetic Prandtl number is equivalent to decreasing magnetic diffusivity and consequently the strength of magnetic field is reduced.

3.3Effects of Magnetic parameter

The otherfluid properties of Pr_M *and* S are maintained at their default values, the magnetic parameter isvaried as $M =$ 4,6 *and* 8. The constants were taken as$Pr_M = 0.7, S = 20$ while varying $M = 4,\ 6,\ 8$ in equation (5) ,we obtain the results as shown in table 3 and presented in figure 3

Distance along x	$M = 4$	$M = 6$	$M = 8$
1	3.938719	4.459206	5.979693
2	0.8319965	1.103756	1.400547
3	1.1418196	1.375202	1.532207
4	1.243736	1.413261	1.522785
5	1.506504	1.506536	1.556569

The results in table 3 are presented graphically as shown in Fig 3

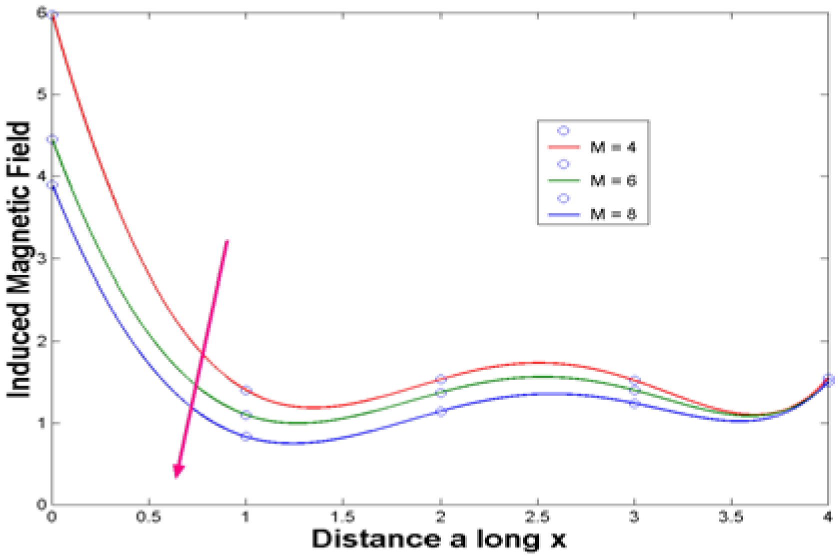

Figure 3: Induced magnetic field against distance along x with varying magnetic parameter

It is noticed that there is a decrease in induced magnetic field as magnetic parameter increases as in figure 3. The physical explanation is that increase in magnetic field strength makes the Lorentz force large thus reducing the fluid motion. This increases the no slip effect, that is, boundary layer thickness reduces. Furthermore, there exists points of intersection on the vertical stretching sheet where the induced magnetic field is independent of the magnetic parameter at$x = 4$. The results show that a magnetic parameter can be used to destabilize the induced magnetic boundary layer away from the fixed end of the stretching sheet. Thus, the magnetic parameter destabilizes the growth of the induced magnetic boundary layer.

Conclusion

From the present numerical investigation, following conclusions have been drawn:

- It was observed that an increase in magnetic parameter leads to a decrease in induced magnetic field.
- It was alsoobserved that increasing magnetic Prandtl number leads to a decrease in the inducedmagnetic field near the plate, while this trend is reversed away from the plate. This also reduces the induced magnetic field.
- Suction velocity increases with increase in induced magnetic field.

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