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Subsidizing Black Scholes Inefficiency Using ANN in Nifty Index

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Abstract: ***Purpose**: The aim of this paper is to build a model using the artificial neural network (ANN) to eliminate or subsidize the incongruence in Black Scholes fair value and actual market value. Also, this paper aims to provide a review of the diffuse and scattered literature on this particular theme. **Research gap**: Although many linear and non- linear, stochastic models including jump and diffusion has been used to capture volatility and measure the efficiency of Black-Scholes Model (BSM), but none of them have been substantially correct and consistent. This study uses ANN to reduce the inaccuracy of the given model. **Research methodology**: To examine the aforesaid purpose CNX NIFTY 50 call and put options data have been collected over a period of 6 months beginning from 1st Jan 2019 to 30th June 2019. Also, for capturing volatility closing price of the index is collected for a period of 1 year (1st July 2018 to 30th June 2019). GARCH volatility is computed to bring out the volatility parameter of the model. Later, a feed-forward algorithm/ back propagation algorithm will be used to process the ANN model. For testing the accuracy of the models error metrics will be computed (mean error, the total mean squared error. Root mean squared error). Excel and R are the used workspace. **Findings**: The result shows that ANN model outperforms the classic Black Scholes Model. For comparing the performance of the two model, error metrics (MSE, RMSE) are used which reflects that ANN reports minimum error. **Originality**: Neural network in options pricing in India is at blooming phase. This paper will be an addition in this genre. These neural network techniques do not presumes any relationship among the variables, but most of the work in this area either follows linear or nonlinear relationship among the variables. **Practical implications**: This paper will try to educate the options trader about the uses of neural networks to reduce the systematic biases in the most celebrated option pricing model. Also, the volatility trader will get a know-how of the sphere of volatility in the index chosen.*

Keywords: volatility, Black-Scholes, GARCH, ANN, error metrics, NIFTY index options

1. Introduction

1.1 Black Scholes Model

Ever since the derivation of the classicBlack-Scholes-Merton (BSM) model in 1973, it has been criticized and challenged on various grounds like constant volatility, constant risk-free rate, assumption offrictionless market, etc. To address these unrealistic assumptions various model has been framed in recent past like stochastic volatility model by Heston (1993), the jump-diffusion model by Bates (1996), and deterministic volatility by Dumas et al (1998). However, the BSM model still remains the most celebrated model probably because of its relative simplicity and speed when looked into other recently developed sophisticated models.

The model primarily works on the following assumption which is discussed below:

1) Log normal distribution- themodel believes stock prices are log-normally distributed as the stock prices can never be below zero.
2) Efficient market- it believes that the market is efficient and cannot be consistently predicted by traders. In other sense, stock prices follow a random walk.
3) Constant volatility and constant risk-free rate- this model works on the principle of known volatility and default risk rate and believes them to be constant. Which is the most debated assumption as it does not fit in the real market. **(Hull 2008)**
4) No dividend- the traditional BSM makes other unrealistic assumptions that the stocks pay no dividend. Although, a modification has been made in the model later to relax this assumption. **(John. C. Hull)**
5) No transaction cost- this model assumes that traders do not incur any cost while purchasing or selling the securities which are again not true.
6) European options- this models apply to options that can be exercised only on expiry which excludes more flexible options like American options.
7) Liquidity- BSM believes that securities can be purchased and sold infractions and at any time.
8) Arbitrage opportunity- risk-free arbitrage opportunity is not available, which implies that the put-call parity holds for the market.

The classic BSM formula is quoted below:

$$C = S_0\, e^{-qt} * N(d_1) - X\, e^{-rt} * N(d_2)$$

$$P = X\, e^{-rt} * N(-d_2) - S_0\, e^{-qt} * N(-d_1)$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

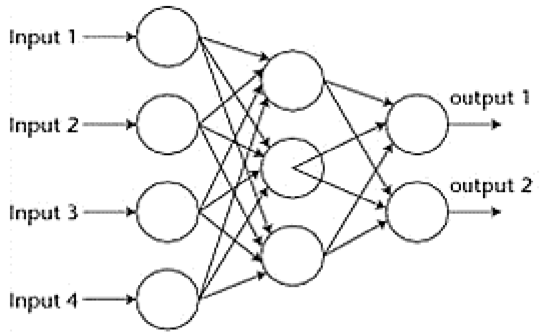
Where, C/P = f (S, K, t, r, σ)
C=Value of the Call option price, P= Value of Put option, S=Current stock (or other underlying) price, X=Strike price, r=Risk-free interest rate, t=Time to maturity, σ = volatility

There has been a rigorous study to calculate the σ (volatility) as other determinants of the model are available from the market itself. Initially,the historical volatility (HSD) which used log-returns of stock prices to compute volatility (standard deviation) came into practice, but soon other measures also evolved. Literature shows that moving average measure, weighted implied volatility measure, implied volatility measure (Latane and Rendleman, 1976), GARCH family models are also exhaustively used. Mixed results are found throughout the literature and no particular measure can be crowned as the best measure. This paper uses the GARCH (1,1) model to capture the heteroskedastic variance of the data and to come up with an unconditional variance (historical volatility).

1.2 Artificial Neural Network (ANN)

The increased complexities in the financial market make it difficult for a naïve model to stand as efficiently as they did before. The use of ANN is not a new touch in financial literature. Financial studies show how the world has now inclined to use ANN for better modeling and forecasting. **Dutta and Shekar** (1988) and **Surkan and Xingren** (1991) used ANN for the valuation of Bonds. Similarly, literature evidence that ANN was in use to forecast bank failure, to forecast stock prices, to forecast stock volatility, etc. There is satisfactory literature worldwide that shows the use of ANN in forecasting option pricing.

ANN is an advanced machine learning program that tries to imitate the functioning of a human brain. It tries to study the pattern of association between provided inputs and outputs. The relationship of inputs and outputs is not known, rather it study and observe the data set to grasp the relationship between the two. As a human brain works with 10^{11} neurons and 10^{15} interconnections, ANN also works with neurons which are structured in machine recognized pattern. These neurons are the basic computational unit also called a node. These nodes as dendrites of a biological neural network (BNN) receive inputs from other nodes or any external source. Each input node is then assigned a weight (w) based on their relative importance to other inputs. Mostly these inputs are passed through a single/ multiple hidden layers. Then an activation function is assigned to those weighted sums of inputs to get an output.



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Input Nodes (input layer): No computation is done here within this layer, they just pass the information to the next layer (hidden layer most of the time). A block of nodes is also called a **layer**.

Hidden nodes (hidden layer): In Hidden layers is where intermediate processing or computation is done, they perform computations and then transfer the weights (signals or information) from the input layer to the following layer (another hidden layer or to the output layer).

Output Nodes (output layer): Here we finally use an activation function that maps to the desired output format

There are various types of ANN available like feedforward neural network, Single-layer Perceptron, multi-layer perceptron, convolutional neural network. Also, there aremany activation functions available namely, sigmoid, Tanh, ReLU, Logistics. In this paper, we process a simple Multi-layer perceptron in R software which has a default Logistic activation function.

2. Literature Review

TamalDatta Chaudhuri and Indranil Ghosh (June 2015), developed an ANN using backpropagation to predict volatility in the Indian stock market through the volatility of NIFTY returns and Gold returns. They used multiple inputs multiple output structure using two different neural architecture and nine learning algorithms. For experiments, only one hidden layer was used while the number of hidden neurons has been varied at three levels (20, 30 & 40 respectively). Hence the total number of trials was fifty-four (2*9*3). They concluded that their framework could satisfactorily forecast volatility for 2015 using training data for 2013-14.

Marry Malliars and Linda Salchenbeger (Sept 1993) developed a neural network model that processed financial input data to estimate the market price of options at closing. Later the network's ability to estimate closing prices is compared with the Black Sholes model. They conclude that the neural network outperforms the Black Sholes model significantly. Joachim Tobias Hahn (November 2013), worked on the improvement of option pricing measures with the help of ANN. He primarily focused on computing volatility using ANN and then used it in the model to be verified and paralleled with the direction option pricing approach. Finally, he concluded that ANN has certain usefulness in forecasting volatility. Henrik Amilon also while comparing option prices computed using BSM and MLP concluded that BSM is superior to the former. In their study, they used historical and implied volatility with BSM as the benchmark.

S. K.Mitra also attempted to improve option pricing models as the BSM has known systematic biases. An attempt was made to use ANN to reduce those biases as ANN takes suitable parameters to adjust the given inputs. NIFTY call option for 3 years was the chosen data. The study finally concluded that results from the ANN model are superior to the original BSM. Zeynepøltüzer Samur and Gül Tekin Temur (2009) aimed to analyze the success of using ANN in S&P 100 index data both (European and American) option contracts were undertaken for the study. The study reveals that although ANN has a better performance for the call option it has an inferior performance for the put options.

Kai Lio and Xiao Wang (2013) tried to provide a pragmatic option pricing model by combining skewness and kurtosis adjusted Black Sholes, time series with ANN. The models were tested on the FTSE 100 index. The results showed that the adjusted BS still had greater deviation but deviation in time series with ANN was much less.

3. Data Set and Research Methodology

In this section, the research framework of this paper has been discussed. Options prices are first computed with Black Scholes Model and then an ANN has been built. As traditional Black Scholes appliesto European options, NIFTY 50 INDEX has been chosen as all the indices traded at NSE are European and are settled in cash. Also, NIFTY 50 INDEX is among the most traded index options among 7 available domestic indices at NSE. For this study, 6 months NIFTY 50 option data (both call and put) from the period 1st January 2019 to 30th June 2019 has been collected from NSE's official website. Only near option contracts are entertained. The classic Black Scholes formula requires 5 inputs – the strike price (k), price of underlying (s), volatility (σ), risk-free rate (r), time to expiry (t). The strike price and underlying value of the asset are readily available from the option data available from the site. Volatility has been computed using GARCH (1, 1) from daily closing prices of the index taken from the period of 1st July 2018 to 30th June 2019.Average of one year (1st July 2018 to 30th June 2019) overnight MIBOR rate is computed to figure out the risk-free rate. And To make the data more precise, options contracts are filtered based on illiquidity, moneyness, and maturity days. Following is the table showing the filtration:

Table 1: Filter statistics of NIFTY index

Basis	Call	Put
Total no. of contracts	10693	10693
No. of contracts >= 50	6212	6041
Maturity > 3 days	785	790
Moneyness ±0.15	339	593
Remaining data	3352	3274

Unconditional variance (volatility) is computed with long term variance from GARCH (1,1). The obtained daily volatility is then annualized by multiplying 252 (assumed number of trading days).Once the volatility is computed, and other determinants are structured, the Black Scholes option valuation formula has been computed in R using f Options. After options prices have been synthesized, the computed prices are compared to the actual prices obtained from NSE. Lastly, to check the efficiency of the model error metric has been functionalized.

4. Data Analysis and Discussion

4.1 Volatility Clustering

For the computation of volatility, NIFTY index log differenced return computed for a year is checked for volatility clustering by ARCH test (LM test) and L-jung box test. The test shows that the data has clustering,

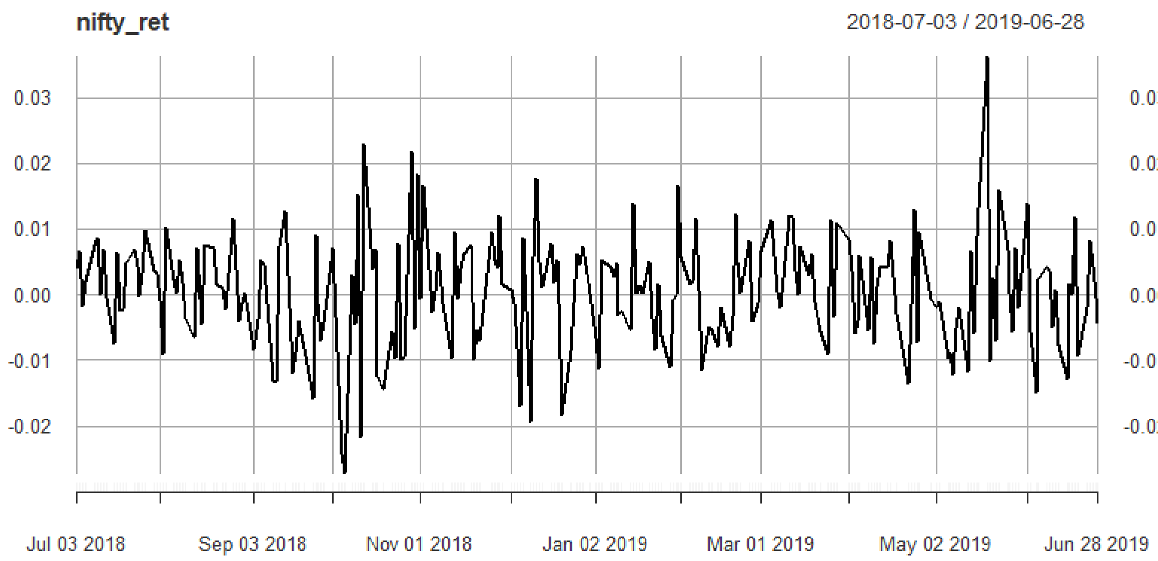

Figure 1: Volatility clustering in NIFTY returns

Table 2: Results of ARCH test

TEST	p-value	Hypothesis
ARCH (LM test)	0.02882	Not accepted
L-jung box test	0.0007826	Not accepted

4.2 Volatility Computation

Further, the GARCH (1,1) is tested to compute the unconditional variance (volatility) which results as under:

Optimal parameters

	Estimate	Std. Error	t-value	Pr(>|t|)
Mu	0.000864	0.000539	1.6035	0.108827
ar1	0.089023	0.067305	1.3227	0.185943
omega	0.000005	0.000002	2.9727	0.002952
alpha1	0.128815	0.031724	4.0605	0.000049

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

Where γ V_L is generally replaced by Omega (w).Which in other words mean given the value of omega(w) V_L can be computed as :
w/γ or,
w/1-(α+β)

The resultant daily long-term variance (unconditional variance/ historical volatility) amounts 0.084% which when annualized stands as 21.68%.

4.3 Black Scholes Model

Then the Black Sholes option pricing has been computed for 6 months call and 6 months put with the respective strike, underlying value, maturity of each day along with the computed volatility and risk-free rate which is a constant for the entire data set. After the computation, the calculated option prices of BSM is compared to the market prices extracted from NSE. Lastly, the RMSE is computed to check the efficiency of the model.

Table 3: RMSE of BSM

Month	Jan	Feb.	March	Apr.	May	June
RMSE (call)	38.19219	40.52436	38.15259	36.52036	62.23461	48.90554
RMSE (put)	77.96447	52.668	54.67059	69.26814	52.18773	59.51727

4.4 The ANN Process

Supervised learning consists of using multi-layered algorithms for finding the optimal relationship between inputs predictors and output targets. In this paper regression supervised deep learning is worked upon. R software by default uses logistic sigmoid activation function. The same data set of NIFTY call and put are used for building an MLP (Multilayer perceptron) for 6 months call and 6 months put. For this purpose first datasets are normalized using min-max normalization. Then the dataset isdivided in the ratio of 70:30 as training and test set respectively. The model is developed on the framed training data with 3 given inputs and one output. Settle price (Sp) is the target output and underlying value (UV), the strike price (k), expiration (exp), are the inputs given. Volatility (sigma), risk-free rate (rfr) are constant they will not impact the association of the model and hence they are excluded.

Figure 2 envisages the obtained neural network model. The model has 2 neurons in its hidden layer. Weights assigned are shown with the black lines. Backpropagation algorithm is used to calculate the weights. Bias term is displayed from the blue line.

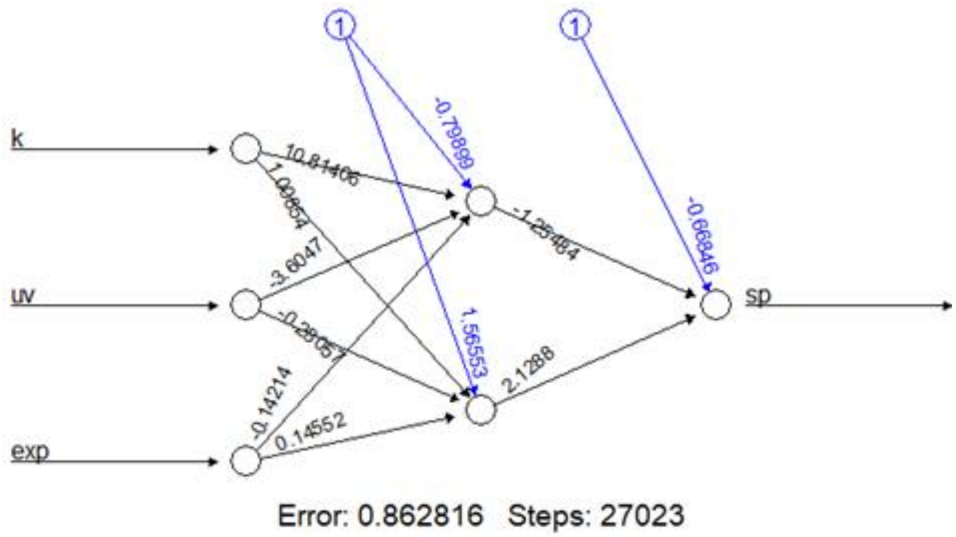

Figure 2: ANN model computed

Table 4: RMSE of ANN

MLP	MSE	RMSE
(3,2,1)	0.0007581938	0.027533

5. Concluding Remark

Although BSM model is still the most popular and exhaustively used option pricing model, the study shows that it has a very high RMSE for the selected data. It has an incongruence with the actual market price. While the MLP used with supervised learning is a better predictor of the option prices. The study clearly reflects that the developed ANN model has lesser error and hence it closer to actual market price of the options.

The work is in line with previous literature. Further study can be taken on using ANN for valuing American options and other exotic options.

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