Contribution of the Duality Theory of ε - And π - Tensor Products of Baire Spaces

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Abstract: ***The basic study of the tensor product of the duality theory is a locally convex space in terms of its dual is the central part of the modern theory of topological vector spaces, for it provides the setting for the deepest and most beautiful results of the subject. Various authors have mentioned In this paper we have proved that the dual space of the ε – tensor product of the two given metrizable locally convex spaces is equal to the ε – tensor product of their topological dual spaces . Duality theory of ε – tensor product of two metrizable locally convex spaces considered about topological dual space of tensor products. objective of this paper is For all our purposes, topological vector spaces are locally convex, in the sense of having a basis at consisting of convex opens. We prove below that a separating family of semi norms produces a locally convex topology. Conversely, every locally convex topology is given by separating families of semi-norms: the semi norms are functionals associated to a local basis of balanced, convex opens. Giving the topology on a locally convex V by a family of semi norms exhibits V as a dense subspace of a projective limit of Banach spaces, with the subspace topology. This chapter presents the most basic results on topological vector spaces. With the exception of the last section, the scalar field over which vector spaces are defined can be an arbitrary.***

Keywords: ε – tensor product, Metrizable, Locally, convex spaces, Topological dual spaces

1. Introduction

This chapter presents the most basic results on the tensor product of the duality theory, topological vector spaces. With the exception of the scalar field over which vector spaces are defined can be an arbitrary with the uniformity derived from its absolute value. The purpose of this generality is to clearly identify those properties of the commonly used real and complex number that are essential for these basic results. The description of vector space topologies in terms of neighborhood bases of a the uniformity associated with such a topology. Constructing new topological vector spaces from given ones.The standard tools used in working with spaces of finite dimension are collected, which is followed by a brief discussion of affine sub spaces the extremely important notion of boundedness. Metrizability is treated although not overly important for the general theory, deserves special attention for several reasons among them are its connection with category, role in applications in analysis, and its role in the history of the subject. By the Herms [1] , Kelly [2] , Komura [3] , Kaplan [4] , Loventz [5] and Nakaro [6] duaity theory of dual space of the ε – tensor product of the given metrizable locally convex spaces is equal to the ε – tensor product of their topological dual spaces. We have proved about duality theory of ε – tensor product of two metrizable locally convex spaces. In this Connection we have considered about topological dual space of tensor products, polar of sets, field of scalars, subsets of tensor product spaces with topological dual spaces.

2. Notation

We denote by β(E×F) the space of all continuous bilinear forms on E×F for two locally convex spaces E and F. By E⊗ ε F We denote the ε -tensor product of two spaces E and F. We denote by E⊗π F the π -tensor product of two spaces E and F. By f(ExF) we denote the space of continuous bilinear forms of finite rank in E × F and at the same time by . We denote the space of integral bilinear forms on E × F. By $\sqrt{G}$ we denote the absolutely convex and closed subset of the space E. We denote by α_b (E, F)the space of all continous linear forms form E into F' such that the topology on α β(E,F) is the strong topology. By E⊗ ε F we dentoe the complete - tensor product of E and F' By(E⊗ ε F) we denote the strong topological dual of (E⊗ ε F)

Definition I

Let E and F be locally convex spaces. Then the dual of E⊗π F is identified with β(E×F) which is the space of all continuous bilinear forms on E×F' the π -equicontinuous subsets of (E×F') are the equicontinuous subsets of β(E×F)

Definition II

Let E and F be locally convex spaces which have a fundamental system of zero neighbourahoods consisting of convex sets. T_e is weaker on E⊗ F than T π every continuous linear functional on (E⊗ F)' is represented by a uniquely defined of *f*(E×F) B(E×F) element of *f*(E×F) ¥ are becomes a subspace of said to be integral bilinear forms on E×F' such that E'⊗ F'⊂ *f*(E×F) ⊂ β(E×F)

Definition III

Let E and F be locally convex spaces. Then *f*(E×F) = (E⊗ ε F)' is the union of all sets *f* (H1⊗ H2), Where H_1 and H_2 are equicontiuous subset of E' and F' respectively . The closure of *f* (H1⊗ H2) in taken is β(E×F) for the Γs(E⊗π F) topology. Every set Γ(H1⊗ H2) is equicontinuous. Every equicontinuous subset of *f*(E×F) is contained in some Γ(H1⊗ H2).

Definition - IV

Let E and F be normed spaces. Then the norm of (E⊗ ε F

)'b is said to be the integral norm || ||, || ||$_1$ on f f(E×F) which is given by||w ||l = sup. ||z || ε≤l[< W,Z>] Where W ∈ f(E×F),Z∈ (E⊗ ε F), where we can set E' = E'b for the normed space E. Let E and F be (B) spaces and let W ∈ E'x F' then there exists a relation. || W ||, || W|| π

We utilize the above notations and definitions along with theorems, propositions, Corollaries, hypothesis etc. from books under the references to prove the following theorems which stands as problem and which have not been considered as yet by others.

Theroem :-
Let P be a locally convex space such that f(P x P) =(P⊗ ε P)' then P is a Baire space.

Proof:
Let each Hn be an equicontinuous subset of the topological dual P' of the locally convex space P. Then by hypothesis we

have $f(P \times P) = (P \otimes \varepsilon P)' \equiv \bigcup_{r=1}^{n} (\Gamma H r \otimes H r)$ 1

We consider that corresponding to each Hn there exists each absolutely convex, closed, balanced absorbing subset

Gn of P such that $(P \otimes \varepsilon P)' = \bigcup_{r=1}^{n} (G r \otimes G r)$ 2

From the concept of (2) we have $P = \bigcup_{r=1}^{n} Mr$ 3

Where We set $\bigcup_{r=1}^{n} Mr \otimes \bigcup_{r=1}^{n} {}^{Mr}_{r=1} = \bigcup_{r=1}^{n} (G\Gamma\ r \otimes G\Gamma\ r)$ 4

Obviously each Gn can be assumed to be a barrel in P such that in particular, each Mn is also a barrel in P...... 5

Since P is a locally convex space. Hence the system in(P)of zero neighbourhood Dn in P consists of convex sets Cn.

We consider in a special case that these convex set Cn are closed, balanced absorbing and absolutly convex
.....6

Form the concept of (6) it follows that U(P) = {Dn} with Cn C

Dn Such that $P = \bigcup_{r=1}^{n} Dr$ 7

On the basis of the concepts of (3) and (7) it is obvious that

$$\bigcup_{r=1}^{n} Mr = \bigcup_{r=1}^{n} Mr \quad .... 8$$

from the concept of (8) it is clear that each barrel Mn is a zero neighbourhood Dn in P 9

From the concept at 9 it follows that P is a barrelled space. 10

We have known that a barrelled space has been proved to be a baire space. 11

From the concepts of (10) and (11) it is clear that

P is a Baire space. 12

Thus the theorem is proved

Corollary
For every locally convex space E and n ∈ N, LI (nE) := (⊗ nE, ε) 0 β is a complemented subspace of the strong dual PI (nEn) := (⊗n s En , εs) 0 β . The elements in LI (nE) (resp. PI (nEn)) are called integral n-linear mappings (resp. integral n -homogeneous polynomials) and appear in several papers and books, among others

Concluding remarks
The Baire category theorem (BCT) is an important tool in general topology and functional analysis. The theorem has two forms, each of which gives sufficient conditions for a topological space to be a Baire space. The theorem was proved by René-Louis Baire in his 1899 doctoral thesis A Baire space is a topological space with the following property: for each countable collection of open dense sets their intersection is dense.

(**BCT1**) Every complete metric space is a Baire space. More generally, every topological space which is homeomorphic to an open subset of a complete pseudometric space is a Baire space. Thus every completely metrizable topological space is a Baire space.

(**BCT2**) Every locally compact Hausdorff space is a Baire space. The proof is similar to the preceding statement; the finite intersection property takes the role played by completeness.

Note that neither of these statements implies the other, since there are complete metric spaces which are not locally compact (the irrational numbers with the metric defined below; also, any Banach space of infinite dimension), and there are locally compact Hausdorff spaces which are not metrizable (for instance, any uncountable product of non-trivial compact Hausdorff spaces is such; also, several function spaces used in Functional Analysis; the uncountable Fort space). See Steen and Seebach in the references below.

(**BCT3**) A non-empty complete metric space, or any of its subsets with nonempty interior, is not the countable union of nowhere-dense sets.

This formulation is equivalent to BCT1 and is sometimes more useful in applications. Also: if a non-empty complete metric space is the countable union of closed sets, then one of these closed sets has *non-empty* interior.

In our work we propose to consider problems of following types

1) The equality of ε– tensor product of two metrizable locally convex spaces can be equal to π – tensor product of said two dual metric locally convex spaces.
2) Two given locally convex spaces can be normable if the duality of ε – tensor products of the given first dual space and the second given locally convex space is equal to the π – tensor products of the first given bidual space and the second given dual locally convex space.
3) The duality of ε – tensor product of two given dual nuclear dual metric spaces can be equal to the π – tensor product of the first given dual space and second given bidual space

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