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Mathematical Model for Vehicle Roll Over Detection

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Abstract: ***Lateral Dynamics plays an important role in Vehicle Dynamics which has a huge impact in Roll Over. CG Height and Wheel Track are the two parameters which decide the Static Stability Factor of the Vehicle. During Transient Condition the threshold value will differ widely from SSF. In order to detect the roll over during transient condition a Mathematical Model is created in Simulink. A Tire Model is developed in order to increase the accuracy of the results. This Mathematical Model will give an insight about the maximum Vehicle Speed and Steer Angle at which the vehicle can turn during a corner without roll over.***

Keywords: Roll Over, Simulink, Slip Angle, Brush Model, Transient, Stability Factor, Roll Damping Moment.

1. Introduction

Over the years, accidents due to roll over has created a much impact in Automobile Industry. It is very clear that Roll over fatalities is more. In order to reduce the fatality rate and to reduce the design lead time a Mathematical Model is developed in Simulink which will provide a brief analysis through the parameters which advances roll over. Based on the model it is easy to detect and rectify the errors causing roll over. This model increases effectiveness in Stability criteria. Loading condition of the vehicle is an important criterion in undergoing this simulation model, while converting from IC to EV it is necessary to change the entire parameters as the change in load will be much prone to roll over in EV Vehicles. Mathematical Model is developed for 3 DOF with help of equations of motions.

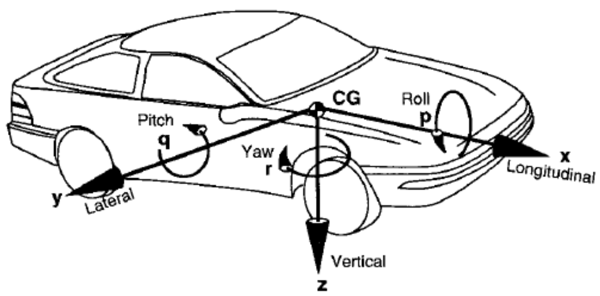

Figure 1: SAE Co-Ordinate Systems [Fundamentals of Vehicle Dynamics, Thomas D. Gillespie]

2. Roll Over

Roll over is any maneuver in which vehicle rotates at 90 degrees or more such that vehicle lifts on one side. The conditions at which roll over of a vehicle occurs are when travelling at high speed during a turn or curved road, losing control due to low co-efficient of friction of road surface (example: ice covered road), and also due to sudden steering input in a vehicle with a low roll stability. Wheels on one side will experience zero lateral Force.Lateral Acceleration at which roll over begins is defined to be roll over threshold.

$$\frac{a_y}{g} = \frac{\frac{t}{2} + \varphi h}{h}$$

This Simple Equation is the first step to measure the vehicle resistance to roll over.If the lateral Acceleration Threshold is more than the co-efficient of friction then it will lead to roll over condition. So co-efficient has to considered.

2.1 Factors affecting Vehicle Roll Over

The factors which determine the vehicle roll over are the tire and vehicle characteristics and also depend on the driver steering input and road conditions (example: Co-efficient of friction of road surface and road inclination). Roll over can happen on flat road, banked road and also off road.

2.2 Impact of Vehicle speed on Roll over

Speed of the vehicle during the turn or cornering determines the vehicle roll over. To avoid vehicle roll over, Differential brakes are used to limit the vehicle speed during cornering. The maximum speed limit of the vehicle during cornering is determined by the radius of the turn, CG height of the vehicle and vehicle wheel track.

3. Mathematical Model

Mathematical Model consists of 4 Subsystems such as Yaw Rate, Roll Model Steady State, Roll Model Transient State and Lateral Weight Transfer. Along with this 4 subsystem a Tire Model is developed.

Assumptions are made to reduce complications
1) Roll Center is stationary.
2) Suspension Kinematics is not included
3) Un-sprung Mass Roll Dynamics are neglected
4) Roll Over Detection is possible with less complex steady state equation

3.1 Yaw Rate Subsystem

A Bi-Cycle Model is used to quantify the lateral weight transfer and yaw rate during transient condition. During turning front wheel have steer input so wheels have lateral force but rear wheels experience lateral force only after a time delay. Differential Equation to model transient yaw rate is given below:

$$\dot{r} = \frac{1}{I_z} \cdot \left[-F_{yr} \cdot b + F_{yf} \cdot a \cdot \cos(\delta)\right] \cdot [rad/s^2]$$

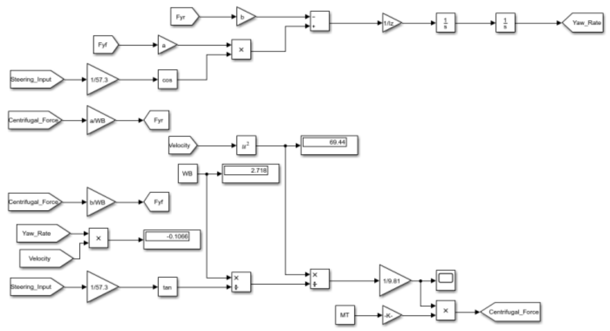

Figure 2: Yaw rate model

3.2 Roll Over Subsystem (Steady State)

The Steady Roll Over of a Vehicle can be determined by making acceleration and dynamics states of a vehicle to zero. With this simplification roll angle is analyzed as a linear function of lateral acceleration.

Equation deriving Steady State Roll Over:

$$\Phi_{ss} = \frac{W_t \cdot d_1}{k_{\Phi t} - W_t \cdot d_1} \cdot \frac{a_y}{g} [rad]$$

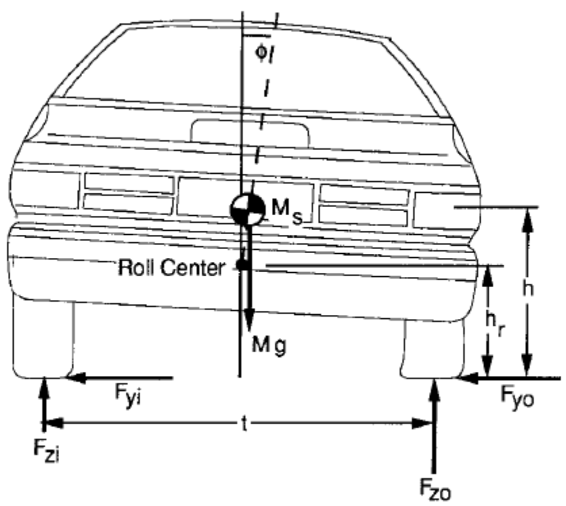

Figure 3: Vehicle roll over [Rear view]

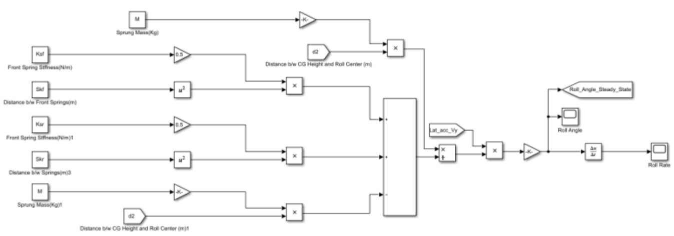

Figure 4: Roll over model

3.3 Roll Over Subsystem (Transient State)

Roll Over for transient state is complex because it is not linear. When there is sudden acceleration roll angle rises due to roll velocity it overshoots and it returns to equilibrium. Overshoot occurs when wheel lift off at lower lateral acceleration. Threshold will be lower when there is no damping. It includes roll rate, roll velocity and acceleration. Transient Roll Model is a very important equation in lateral weight transfer subsystem. Equation to model Transient Roll Model:

$$\ddot{\Phi} = \left(\frac{1}{I_x}\right) \cdot \left[-RSM - RDM + (R_z \cdot d_1 \cdot \sin\Phi) + (R_y \cdot d_1 \cdot \cos\Phi)\right] \cdot \left[\frac{rad}{s^2}\right]$$

In the above equation RSM and RDM is Roll stiffness Moment and Roll Damping Moment respectively.

Equation to calculate Roll Stiffness Moment:

$$RSM = \left[\left[(0.5 \cdot k_{sf} \cdot S_{kf}^2) + (0.5 \cdot k_{sr} \cdot S_{kr}^2) + k_{arbf} + k_{arbr}\right] \cdot \Phi\right] \cdot [N \cdot m]$$

Equation to calulate Roll Damping Moment:

$$RDM = \left[\left[(0.5 \cdot b_f \cdot S_{bf}^2) + (0.5 \cdot b_r \cdot S_{br}^2)\right] \cdot \Phi\right] \cdot [N \cdot m]$$

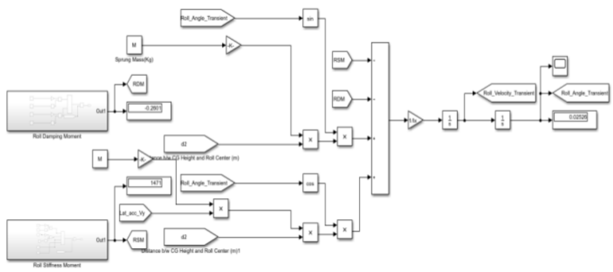

Figure 5: Roll Transient Model

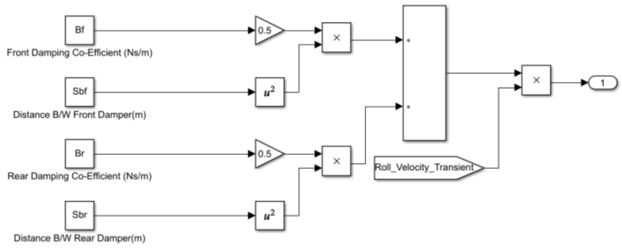

Figure 6: Roll Stiffness Moment Model

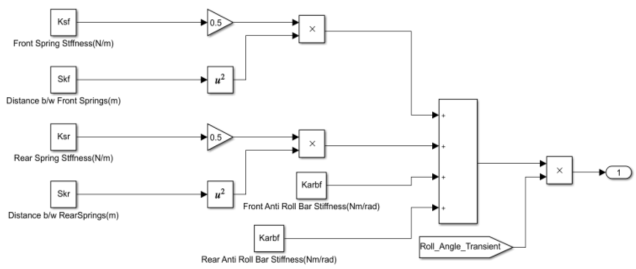

Figure 7: Roll Damping Moment Model

3.4 Lateral Weight Transfer Subsystem:

Lateral weight transfer is the difference between inner and outer tire loads.

$$dF_z = [F_{zo} - F_{zi}] \cdot [N]$$

To accurately determine the normal force on each wheel, the FBD of the un-sprungmass must be configured using both front and rear components. However, onlythe steady state roll dynamics of the un-sprung mass are modeled since the simulation isonly concerned with rollover occurrences. Once wheel lift is detected, (which is whenthe transient roll dynamics of the un-sprung mass would be critical), the vehiclesimulation is terminated and declared a rollover event. Therefore, the lateral weighttransfer of the front and rear axles are found using Equations.

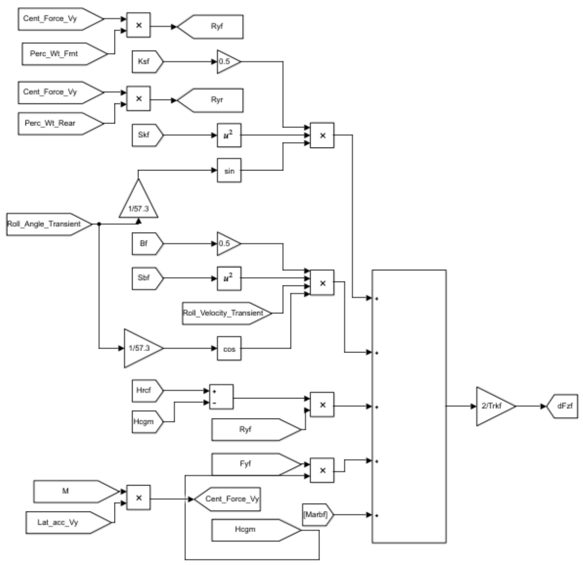

Figure 8: Lateral Weight Trasnfer Front

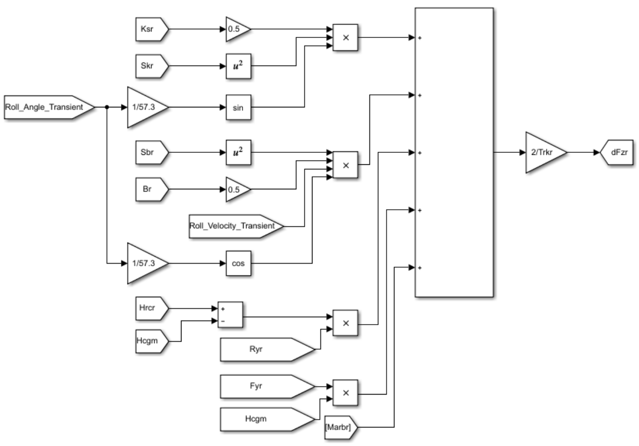

Figure 9: Lateral Weight Transfer Rear

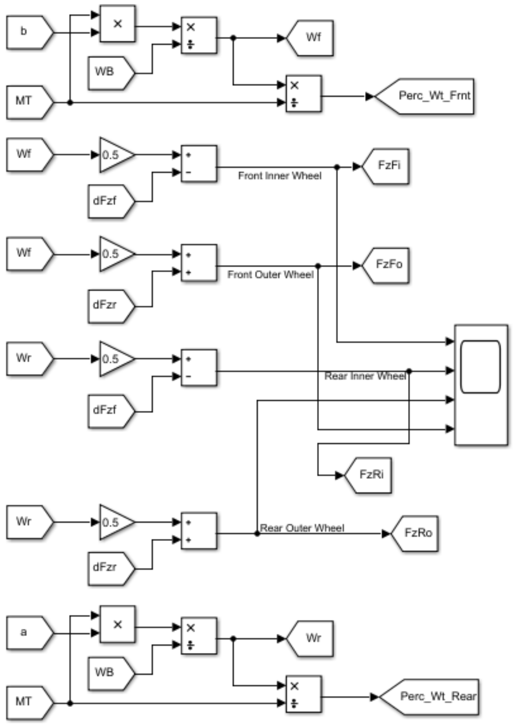

Figure 10: Normal Loads on each wheels.

3.5 Tire Model

Brush Tire Model is developed to measure the lateral force of the tire for various load conditions.

Fig 11 is the Mathematical Model behind the Brush Tire Model

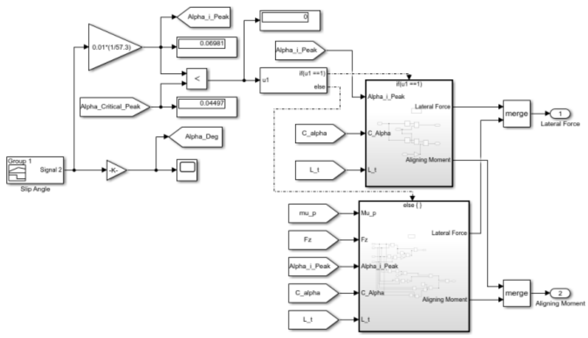

Figure 11: Tire Model

Based on the Brush Tire Mathematical Model it is possible to co relate the Tire Slip Values with Lateral forces for various Vertical Loads. Tire Slip is calculated based on Vehicle Lateral Force and Yaw Rate . Result of the model is shown in graph Fig 12.

$$\alpha_f = \left(\tan^{-1}\left(\frac{V_y + r \cdot a}{V_x}\right) - \delta\right) \cdot [rad]$$

$$\alpha_r = \tan^{-1}\left(\frac{V_y + r \cdot b}{V_x}\right) \cdot [rad]$$

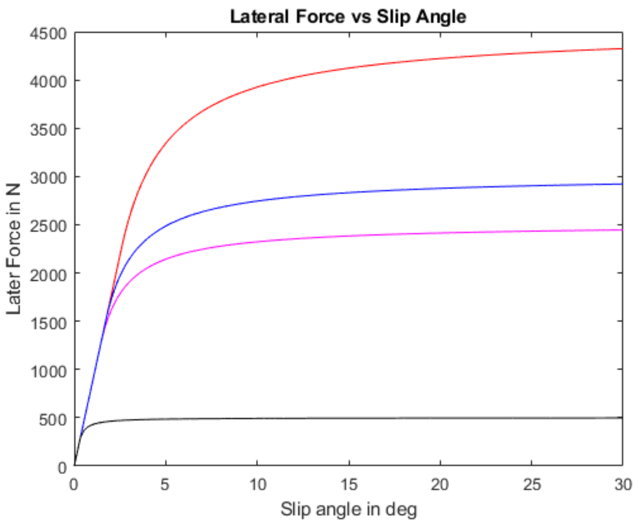

Figure 12: Slip Angle Vs Lateral Force

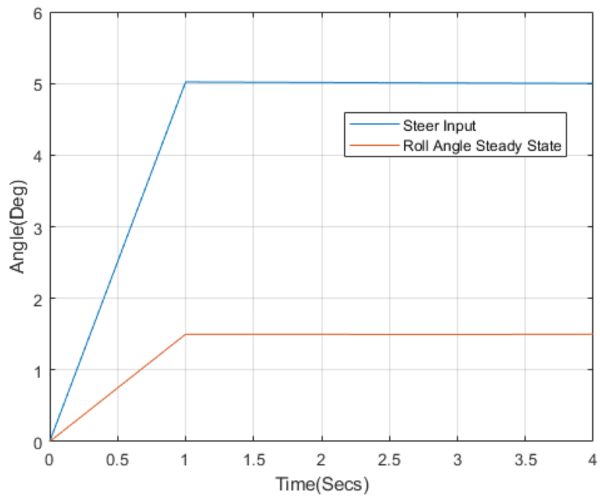

Figure 13: Steer Angle and Steady State Roll Over

4. Input Parameters

Table 1: Input parameters for Simulink model

Input	*Units*	*Values*
Vehicle mass	Kg	1907
Sprung mass	Kg	1525
Wheel base	M	2.718
Vehicle Yaw inertia	Nsm^2	3833.31
Vehicle Roll inertia	Nsm^2	734.04
Distance of CG from Front axle	m	1.216
Distance of CG from Rear axle	m	1.502
CG height Sprung mass	m	0.669
CG height UnSprung mass	m	0.35
Distance between Springs Front	m	0.7747
Distance between Springs Rear	m	0.9906
Distance between Dampers Front	m	0.7747
Distance between Dampers Rear	m	0.7620
Anti Roll bar Stiffness Front	Nm/rad	700
Anti Roll bar Stiffness Rear	Nm/rad	400
Spring Stiffness Front	N/m	75000
Spring Stiffness Rear	N/m	70000
Damping Co-Efficient Front	Ns/m	5000
Damping Co-Efficient Rear	Ns/m	4000
Roll center height Front	m	-0.1
Roll center height Rear	m	0.35
Test speed	Km/h	30
Wheel track Front	m	1.445
Wheel track Rear	m	1.405
Tire Normal force	N	5000
Peak Co-efficient of Friction	-	0.9
Peak Co-efficient of friction Asphalt	-	0.5
Peak Co-efficient of Friction Gravel	-	0.6
Peak Co-efficient of Friction Ice	-	0.1
Length of contact patch	m	0.2
Tire Cornering Stiffness	N/rad	50000

These parameters are used as inputs in Simulink model. These input parameters are the values of selected vehicle for testing.

5. Results

Validation of steady state steady roll over with respect to steer angle is done using mathematical model to measure the threshold value which is shown in Fig 13.

The reason behind comparing steady state Roll model with transient Roll model is to check the accuracy and it is well maintained in the threshold value. In this Transient Roll Model a Slope Steer is used as an input and the result is shown in Fig 14. Transient Roll Model is calculated based on Roll Angle, Roll Velocity and Roll Acceleration,

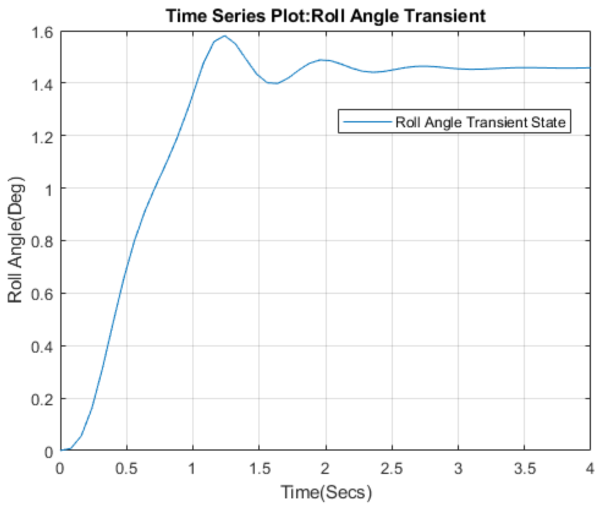

Figure 14: Transient Roll Angle

Lateral Weight Transfer is function of roll dynamics in transient condition ehicl includes roll damping moment and roll stfifness moment.Simulation is done for 4 seconds assuming the vehicle speed to be around 30Kmph with steer angle of 5 Deg . Lateral Weight transfer during this condition is measured in form of graph Fig 15.

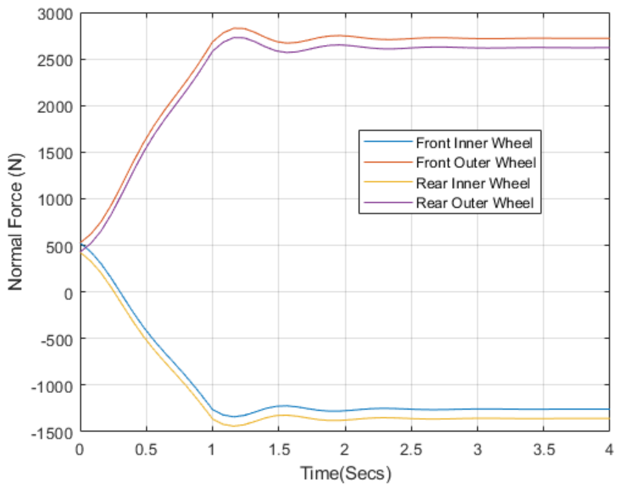

Figure 15: Normal Force on each Wheel

From the graph it is clear that inner wheels (Front and Rear) tends to achieve normal forc less than zero which means inner wheels are not in contact with the ground, It is clear that at a particular speed and steering agle vehicle with specifications mentioned in Table 1 can sustain in ground for lesser seconds. This graph gives an insights of vehicle roll over behavior bu using this Mathematical Model.

6. Summary / Conclusion

This mathematical model gives the steady state roll angle and transient state roll angle of the selected vehicle for a given test speed of the vehicle. From the result graph of Normal force on each wheel, it is inferred that when the Normal forces of the Front inner wheel and Rear inner wheel value becomes less than zero or negative, the roll-over of the vehicle starts. This mathematical model reduces the design lead time and makes it easier to optimize and analyze the roll over parameters of a vehicle when it is converted from IC to EV. This Mathematical Model gives the maximum Vehicle Speed and Steer Angle at which the vehicle can turn during a corner without roll over.

7. Future Developments

This mathematical model can be developed into a driver assist feature in the upcoming vehicles, using wheel speed sensor and steering angle sensor to warning the driver of maximum speed the vehicle can maneuver with the maximum steering angle input. This driver assist feature will help in reducing the number of accidents occurring due to vehicle roll over. This feature will be effective in reducing roll over of load carrying commercial vehicles, which are more prone to roll over.

8. Nomenclature

I_z – Vehicle yaw inertia
I_x – Vehicle roll inertia
a – Distance of CG from front wheel
b – Distance of CG from rear wheel
K_{sf}, K_{sr} – Spring stiffness front, rear
S_{kf}, S_{kr} – Distance between springs front, rear
b_f, b_r – Damping co-efficient front, rear
S_{bf}, S_{br}– Distance between dampers front, rear
K_{arbf},K_{arbr} – Anti roll bar suspension stiffness front, rear
d_1 – Difference of CG height and Roll center height
T_{rkf}, T_{rkr}– Wheel track front, rear
F_{yf}, F_{yr}– Tire lateral force front, rear
V_x, V_y– Vehicle velocity in x-axis, y-axis
α_f, α_r – Tire slip angle front, rear
Φ – Roll angle
Φ_{ss} – Steady state roll angle
$\ddot{\Phi}$ – Transient roll angle
δ – Steer angle
W_t – Total sprung weight
$K_{\Phi t}$ – Total roll stiffness
a_y – Lateral acceleration
g – Acceleration due to gravity
φ – slope of the road surface
t–Wheel track
h – CG height
r – Yaw rate
F_{zo},F_{zi} – Load on outer wheels, inner wheels
dF_z – Lateral weight transfer

References

[1] Thomas D Gillespie, “Fundamentals of Vehicle Dynamics” Society of Automotive Engineers, Inc.
[2] C.Smith, “Tune to Win”, AERO publishers Inc.
[3] Rill,Georg, “Road Vehicle Dynamics – Fundamentals and Modeling”
[4] Rajamani R, “Vehicle Dynamics and Control”
[5] Dean Karnopp, “Vehicle Dynamics, Stability, and Control”