On Finding the Area of Triangle Using Midpoints and Eular's Phi Functions, and some Statements on Phi Function

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Abstract: ***In this paper, Author describes how to find the area of triangle ABC Using midpoints and Eulars phi function and some statements on Eulars phi function.***

Keywords: Triangle, Midpoints, Eulars phi function and etc

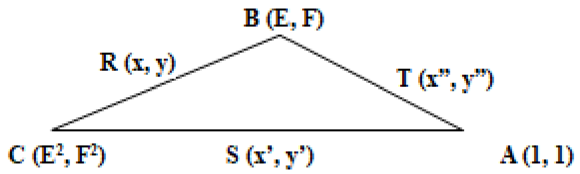
1. Introduction

In this paper there are total six parts. Author use the Concept of midpoints and Eulers phi function to find the area of triangle. Also give examples to Understand the Statement or relations easily.

[1]. Vertices of triangle ABC are of the form A(1,1), B(E,F), C(E^2,F^2), F>E.

E belongs to (2, 3) and F belongs to (3,5,7).

Note: If E is equal to 2, then F is not equal to 7.



Here, R,S,T are the midpoints of BC,CA,AB and (x,y) , (x',y'),(x'',y'') are the Coordinates of R,S,T. Midpoints R,S and T divides BC,CA and AB in 1:1.

Therefore, m_1=1=m_2
x= [m_1x_2+m_2x_1]/(m_1+m_2)
Or, x=(x_1+x_2)/2 , y=(y_1+y_2)/2
From the above triangle ABC,
x=(E+E^2)/2 , y=(F+F^2)/2 ---------- (1)
Similarly,
X'=(1+E^2)/2 , y' =(1+F^2)/2 ---------- (2)
and x"'=(E+1)/2 , y"=(F+1)/2 ----------- (3)
Let, (y.y'.y"'.x) / (x'.x"') = K_1
and phi(K_1)=K_2 , phi(K_2)=K_3 , ………..
phi(K_n)=1. (K_{n+1}=1) .

Let the total number of phi functions = n (Till 1)

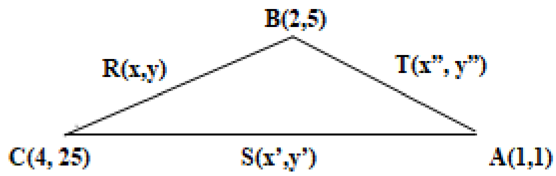
Then, the Area of triangle ABC = (ExF-n)x(a+1), Where a is the number of prime(s) between E and F.

Examples = i) Number of prime(s) between 2 and 3 is zero therefore, a=0.

ii) Number of prime(s) between 2 and 5 is 1 (i.e. 3) therefore, a=1

E and F are the Coordinates of the vertices B.

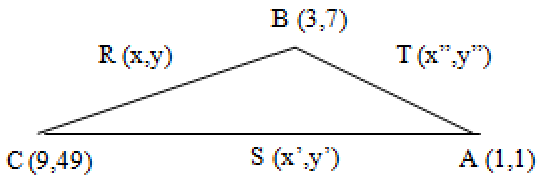
1) Example = Find the Area of triangle ABC whose vertices are A(1,1), B(2,5), C(4,25) Using midpoints and phi functions.



Let Coordinates of R are (x,y)
x = (4+2)/2 = 3
y = (25+5)/2 = 15
Let Coordinates of S are (x',y')
X' = (1+4)/2 = 5/2
Y'' = (1+25)/2 = 13.
Let Coordinates of T are (x'',y'')
X"= (2+1)/2 = 3/2
y" = (5+1)/2 = 3
then, (y.y'.y".x)/(x'.x") =
(15x13x3x3)/[(5/2)x(3/2)] = 468
Phi(468)=144,phi(144)=48,phi(48)=16,phi(16)=8,phi(8)=4,phi(4)=2 ,phi(2) = 1.
Total numbers of phi functions, n=7.
Product of Coordinates of B is ExF=2x5=10
Area of triangle ABC = (ExF-n)x(a+1)
Area of triangle ABC = (10-7)x2
= 6 sq.unit.

2) Example :Find the Area of triangle ABC whose vertices are A(1,1), B(3,7), C(9,49).

Using midoints and phi functions.



Let Coordinates of R are (x,y)
x = (9+3)/2 = 6.
y = (49+7)/2 = 28.
Let Coordinates of S are (x’,y’)
X’ = (1+9)/2 = 5.
Y’ = (1+49)/2 = 25.
Let Coordinates of T are (x”,y”)
X” = (3+1)/2 = 2.
Y” = (7+1)/2 = 4.
Then, (y.y’.y”.x)/(x’.x”) =
(28x25x4x6)/(5x2) = 1680
Phi(1680)=384,phi(384)=128,phi(128)=64,phi(64)=32,phi(32)=16,phi(16)=8,phi(8)=4,phi(4)=2,phi(2)=1.
n=9 , ExF=3X7=21 , a=1.
Area of triangle ABC = (ExF-n)x(a+1) = (21-9)x2=24 sq.unit.

Triangles are:-
i) A_1(1,1), B_1(2,3), C_1(4,9)
ii) A_2(1,1), B_2(2,5), C_2(4,25)
iii) A_3(1,1), B_3(3,5), C_3(9,25)
iv) A_4(1,1), B_4(3,7), C_4(9,49) .

[2] i) Area of triangle QRS = {phi[$P^{n+1}.5^{n+2}.7^{n+2}$]}/[P^n(P-1)]
Where P is either 2 or 3 and n is a positive integer. Vertices of triangle QRS are Q($5^{n+1},7^{n+1}$), R($5^{n+2},7^{n+2}$), S($5^{n+3},7^{n+3}$).

ii) Area of triangle UVW = phi[$2^{n+1}.3^{n+2}.5^{n+2}$]/2^n.
Vertices of triangle UVW are U($3^{n+1},5^{n+1}$), V($3^{n+2},5^{n+2}$), W($3^{n+3},5^{n+3}$). Where n is 0 or any positive integer.

1) Example: Find the Area of triangle QRS whose vertices are Q (25,49), R(125,343), S(625,2401) Using phi function.
   Solution: Case(1)= When P=2,n=1 {phi[$2^2.5^3.7^3$]}/[2^1(1)]
   Area of triangle = [58800]/2
   Area of triangle = 29400 Sq.unit.
   Case(2)= When P=3,n=1.
   {phi[$3^2.5^3.7^3$]}/(3x2)=29400

2) Example: Find the Area of triangle UVW whose vertices are U(9,25), V(27,125), W(81,625) Using phi function.
   Solution :- U($3^2,5^2$), V($3^{2+1},5^{2+1}$), W($3^{2+2},5^{2+2}$).
   Area of triangle UVW = phi[$2^{n+1}.3^{n+2}.5^{n+2}$]/2^n
   n=1
   phi[$2^2.3^3.5^3$]/2 =1800.
   Area of triangle QRS = 1800sq.unit.

[3] Statement = The Area of triangle KLM whose vertices are K(p^{a-1},q^{b-1}), L(p^a,q^b), M(p^{a+1},q^{b+1}), Where a,b are the positive integers (a>1) is equal to [{LCM[phi($p^a.q^b$),pq,q^b]}x(q-p)]/(2q).

Note: There is some changes in above relation from previous relation (previous paper).Both relations are correct.

Example : Find the Area of triangle KLM whose vertices are K(3,1), L(9,11), and M(27,121) using least common multiple (LCM) and phi relation.
Solution :- K($3^{2-1},11^{1-1}$), L($3^2,11^1$), M($3^{2+1},11^{1+1}$).
Area of triangle KLM = [{LCM[phi($p^a.q^b$),pq,q^b]}x(q-p)]/(2q).
Phi(3^2.11)=60, LCM[60,33,11]=660.
Area of triangle KLM = [660x(11-3)]/(2x1)=240sq.unit.

Relation between the ratio’s of LCM’s and area of triangles.

Area of triangle KLM (A_1) = [{LCM[phi($p^a.q^b$),pq,q^b]}x(q-p)]/(2q) ------------(1)
From previous paper,
Area of triangle I J S(A_2) = [{LCM[phi(p^a.q),pq,q^2]}x(q-p)/(2q^2) -------------(2)
From (1) and (2), we get

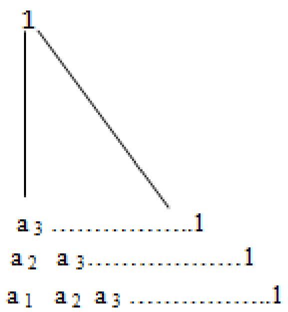
[LCM[phi($p^a.q^b$),pq,q^b]]/[LCM[phi(p^a.q),pq,q^2]]=
(A_1/A_2)x(1/q).

Vertices of triangle A_1 and A_2 are K(p^{a-1},q^{b-1}), L(p^a,q^b), M(p^{a+1},q^{b+1}) and I(p^{a-1},1), J(p^a,q), S(p^{a+1},q^2).
Where a,b are the positive integers (a>1).

Example: Find the ratio of the Area of triangles Whose vertices are K(5,121), L(25,1331),

M(125,14641) and I(5,1), J(25,11), S(125,121) using LCM and phi relation.
Solution:- p=5, q=11, a=2, b=3, (a>1).
Phi($5^2.11^3$)= 24200
LCM[phi($p^a.q^b$),pq,q^b] = LCM[24200,55,1331] = 266200
Now, phi(5^2.11) = 200.
LCM[phi(p^a.q),pq,q^2] = LCM[200,55,121] = 24200

Acoording to relation,
(A_1/A_2) = [266200/24200]x11 {q=11}
(A_1/A_2)= 121
[4]. Statement :- For first 14 odd primes.



Where a_2 = phi(a_1), a_3 = phi(a_2), ……., 1= phi(a_{n-1}). {a_n=1}.
a_1 is an odd prime, a_1 belongs to (3,5,7,…….47).
Let, nx1=n
(n-1)xa_{n-1}=K_1
(n-2)xa_{n-2}=K_2
:
:
1xa_1=a_1
Then, [phi(n)+phi(K_1)+phi(K_2)+…….+phi(a_1)]-a_1 is equal to P^m. Where P is an odd prime and m belongs to (1,2,4).

Example = Let a_1=11.

1
2   1
4   2   1
10   4   2   1
11   10   4   2   1
5x1=5
4x2=8
3x4=12
2x10=20
1x11=11
[phi(5)+phi(8)+phi(12)+phi(20)+phi(11)]-11
[4+4+4+8+10]-11= 19.
$P^m=19^1$, P=19, m=1.
Where 19 is an odd prime.
Example: When a_1=19 , then P^m =25
Or, $P^m=5^2$. P=5 and m=2.

[5]. Bi-phi disintegration number, Equally Bi-phi disintegration number, Neither Bi-phi nor Equally Bi-phi disintegration number.

i) **Bi-phi disintegration numbers** :- 2,4,5,6,9,11,13,14,16,18,21,……, let K is a positive integer K,K_1,K_2,………1.

K_1= phi(K),K_2=phi(K_1),……..1=phi(K_{n-1}) {K_n=1}.
Let, K+K_1=a_1
K+K_1+K_2=a_2
.
.
.
K+K_1+K_2+……...+1=a_n.
Now, phi(a_1)=b_1, phi(a_2)=b_2,…….phi(a_n)=b_n.
Again,phi(b_1)= C_1, phi(b_2)= C_2,……..phi(b_n)=C_n.
If phi [C_1+C_2+…….+C_n]= phi(K).
Then, the number K is Bi-phi disintegration number.
Example = K=5.
5,4,2,1.
4=phi(5), 2=phi(4), 1=phi(2). 5+4=9, 5+4+2=11, 5+4+2+1=12.

Now, phi(9)=6, phi(11)=10, phi(12)=4. again, phi(6)=2, phi(10)=4, phi(4)=2.
phi[2+4+2]=4=phi(5) Or, phi(8)=4=phi(5).
Thus, 5 is Bi-phi disintegration number.
Note:- 2 is the only number which have C_1 only.

ii) Equally Bi-phi distintegration numbers :-
3,8,12,………
If C_1+C_2+………+C_n=K, then the number K is Equally Bi-phi disintegration number.
Example :- K=12.
12,4,2,1.
4=phi(12), 2=phi(4), 1=phi(2). 12+4=16, 12+4+2=18, 12+4+2+1=19.
Now, phi(16)=8, phi(18)=6, phi(19)=18. again, phi(8)=4, phi(6)=2, phi(18)=6.
phi[4+2+6]=4=phi(12)
Here, 4+2+6=12.
Therefore, 12 is Equally Bi-phi disintegration number.

iii) Neither Bi-phi nor equally Bi-phi disintegration numbers are 7,10,15,17,……

Example= K= 17.
17,16,8,4,2,1.
16=phi(17), 8=phi(16), 4=phi(8), 2=phi(4), 1=phi(2).
17+16=33, 17+16+8=41, 17+16+8+4=45, 17+16+8+4+2=47, 17+16+8+4+2+1=48.
Now, phi(33)=20, phi(41)=40, phi(45)=24, phi(47)=46, phi(48)=16. again, phi(20)=8, phi(40)=16, phi(24)=8, phi(46)=22, phi(16)=8.
phi[8+16+8+22+8]= 30, which is not equal to phi(17).
Therefore, 17 is Neither Bi-phi nor Equally Bi-phi disintegration number.

[6]. Questions based on previous paper's formula [Relation between the Area of Triangle (Coordinate Geometry) and Eular's phi Function and Some Results].

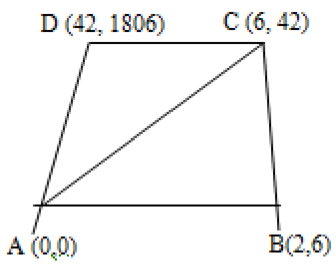
According to author's previous paper.
Area of triangle PQR = [phi(n)x(P_r-P_{r-1})]/[2X$P_1^{e_1-1}$.$P_2^{e_2-1}$……..$P_r^{e_r-1}$x(P_1-1).(P_2-1)………(P_{r-2}-1)].
P_{r-2} is the second last prime (excluding last two primes).

(1) Question : Using phi function (Above formula) find the value of K if the vertices of triangle PQR are P(0,0), Q(4,K), R(20,42), where K+1 is a prime and Area of triangle PQR is 24.

Solution :- 24=[phi{$(4+1)^2$x(K+1)} x(K+1-5)]/{2x(4+1)},
24x10=5^2x(K-1)x(4/5)x{K/(K+1)}x(K-4)
240=5x4x(K^2-4K)
12=K^2-4K , K^2-4K-12=0.
K=6,-2.
K+1=6+1=7, which is a prime Therefore, K=6.
Note = We can take any power of (4+1) and (K+1).
[In above example] and also we can take any primes(s) which comes before (4+1).

(2) Question : Find the Area of the quadrilateral ABCD whose vertices are A(0,0), B(2,6), C(6,42) and D(42,1806) Using phi function.

Solution:-



Area of quadrilateral ABCD = Area of triangle ABC+Area of triangle ACD.
A (0,0), B(3-1,7-1), C[3(3-1),7(7-1)], D[7(7-1),43(43-1)]

Now, Area of triangle ABC = [phi{$(2+1)^2$x(6+1)}x(7-3)]/(2x3^{2-1})
= (36x4)/6=24.

Therefore, Area of triangle ABC = 24 sq.unit.

Now, Area of triangle ACD = [phi{2^2x3x(6+1)x(42+1)} x(43-7)]/[2x2^{2-1}x(2-1)x(3-1)]
= 4536 sq.unit.

Therefore, Area of quadrilateral = 24+4536=4560 sq.unit.

Note: We can apply the phi function to find the area of quadrilateral only when:-

1) Vertices are in general form.
2) Join the point A to C or AC (not B to D or BD).
3) Take the area of (triangle) ABC not ACB.
   Similarly, Take the Area of ACD not ADC.

2. Benefits

It should encourage students to think more about this topic.

3. Acknowledgement

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References

[1] Elementary number theory by David. M. Burton
[2] First year and second year BSC mathematics textbook

Author Profile

Mr. Chirag Gupta is studying in S.Y.BSC SICES Degree College of Arts, Science, Commerce (Ambernath). He has published his three papers in International Journal of Science and Research in month of february, march and august. (2020) . Recently published paper of Chirag Gupta: Relation between the Area of Triangle (Coordinate Geometry) and Eular's phi Function and Some Results.