Influence Analysis of Physical Quantity of Weight, Buoyant Force, and the Density of Floating Object in the Fluid

Fakhruddin

Lecturer in Physics Education Study Programme, The Faculty of Teacher Training and Educational Sciences, Nusa Cendana University

Abstract: ***A study has been done to analyze the influence of physical quantity of weight w_B, buoyant force F_A, and density ρ of the floating object in the fluid. If $F_{A_D} > w$ in dynamic equilibrium state, $F_{A_{SD}} > w$ or $F_{A_{SS}} = w$ in static equilibrium state, with $F_{A_D} > F_{A_{SD}} > F_{A_{SS}}$ and the density of the fluid is higher than the density of an object or $\rho_f > \rho_B$. Generally, floating object in the liquid happens if the buoyant force is higher or equal to the weight, with the general formulation $F_A \geq w_B$ and the density $\rho_f > \rho_B$.***

Keywords: Floating Object, Buoyant Force, Weight, Density

1. Introduction

Physics is the knowledge about natural phenomena of matter and wave in the scope of space and time. The physicists study the behaviour and the characteristic of matter and wave in various fields, starting from the sub-microscopic particle that makes every matter (particle physics) to the behaviour of matter of the universe as the unity of the cosmic. In learning physics, there is a tendency to form a systematical mindset which is more careful, to stimulate the growth of logical thinking, and to sharpen make the accuracy of thinking skill. In learning physics, there is a need of mathematical analysis skill to clarify or to prove a physical phenomenon. In learning mathematics, one will always analyse and simplify the problems to gain an accurate and sufficient solution.

One problem in physics that is investigated by a mathematician is how an object floats in a fluid. It is Archimedes (287-212 SM) who states that if an object submerged fully or partially in a fluid, the fluid will give the buoyant force to that object, in which the upward force *(buoyant force)* is equal to the weight of the fluid that is exerted from the container. This is well known as Archimedes' principal.

Based on Archimedes' principal, the physicists can explain the object's condition when floating, drifting, or sinking in the fluid. Specifically, in explaining a floating object in a fluid, there are a lot of investigations done by the physicists in some books in which there are some differences in the way they analyse it.

For example, a study by Halliday and Resnick (2006) in their book entitled *Physics 6 third edition* explains the floating condition using physics only without mathematical explanation. Mean while Giancoli (2001) in the book *Physics* (*seventh edition*) explains using both mathematics and physics but not in a detail explanation why the buoyant force is equal to the weight *w* or in mathematical formulation $F_A = w$. Tripler (2001) in his book *Physics for science and engineers* (*third edition*) states that based on Archimedes' principal, an object will float if the object density ρ_B is lower than the density of the fluid ρ_f in which the buoyant force F_A is greater than the weight of the object w_B. Eventually, the object will be accelerated to the surface of the fluid unless it is detained. The study by Tripler is only explain physically without any derivation of mathematical formulation in a detail way. Wilson, Buffa, and Lou (2007) in their book *College Physics* (*third edition*) state that an object floats because there is upward buoyant force F_A which is greater than the weight w_B in which these forces are equivalent if the object is equilibrium. However, this also not explain in a detail way why $F_A > w_B$ and $F_A = w_B$.

Similarly, Kanginan (2008) in his book *Physics for Senior High School Grade XI* explains the floating condition using Archimedes' principal and Newton's First Law in static equilibrium which at the end gets the formulation similar to Giancoli (1996) and Wilson, Buffa, and Lou (2007) that $F_A = w_B$. However, Kanginan does not illuminate what happens in dynamic equilibrium. Moreover, Indrajit (2008), Pauliza (2008), and Mikrajuddin et al (2008) elucidate that the floating condition happens when $F_A > w_B$ without any further explanation.

The discrepancy in the explanation about floating object in the fluid by the experts could create misconception. Therefore, there is a need to investigate how an object floats using both physics and mathematics explanation in order to gain a more detail and comprehensive understanding and to avoid the misconception.

2. Material and Methods

Achimedes' Principal

ISSN: 2319-7064
ResearchGate Impact Factor (2018): 0.28 | SJIF (2018): 7.426

Archimedes (287-212 SM) is an ancient Greek mathematician who found the way to formulate how to measure the volume of irregular objects. He discovered his notion when he was taking a bath and found that the water inside the bath is exerted out of the bath because of the buoyant force of the water. When measured, it was found that the displaced water is equal to his weight.

An object takes a shape of a cylinder with the height h has the wide of longitudinal section A in which all of the body is submerged in the fluid with the density of the object ρ_B as shown in Figure 1.

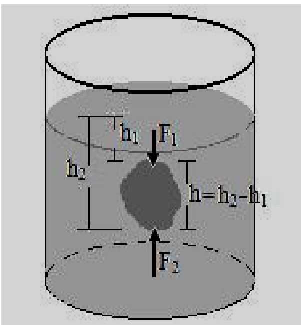

Figure 1: The forces work in an object in the fluid.

The fluid gives the pressure on the top of the surface $p_1 = \rho_f g\ h_1$ and in the bottom $p_2 = \rho_f g\ h_2$. The force given by the pressure on the top of the cylinder is $F_1 = p_1\ A = \rho_f g\ h_1$ meanwhile the force given by the bottom of the cylinder is $F_2 = p_2\ A = \rho_f g\ h_2$. Thus, the net force is the result of the fluid's pressure which is called buoyant force F_A. The buoyant force has the upward direction and the magnitude of

$$\begin{aligned} F_A &= F_2 - F_1 \\ &= \rho_f g\ A(h_2 - h_1) \\ &= \rho_f g\ A\ h \\ &= \rho_f g\ V_B \end{aligned} \qquad (1)$$

V_B is the volume of the cylinder while $\rho_f g\ V_B$ is the weight of the exerted fluid w_f that has the volume that is equal to cylinder's volume. Therefore, the buoyant force F_A is equal to the weight of the exerted fluid w_f that is displaced by the cylinder w_f. This result is valid to all object with any form. This discover is well known as Archimedes' principal that states that the buoyant force in a submerged object, whether it is fully or partially immersed in a fluid, is equal to the weight of the fluid exerted by the object (Tipler; 2001, Giancoli;2001, and Wilson, Buffa, and Lou; 2007).

Newton's Law

Newton in his study about motion in his book Principia in 1687 (Giancoli, 2001) states that:

An object will remain at rest or in uniform motion in a straight line unless acted upon it the total force that change these states. This is called Newton's First Law or mathematically written

$$\sum \vec{F} = \vec{0} \qquad (2)$$

The acceleration of a body is directly proportional to the net unbalanced force and inversely proportional to the body's mass. The direction of the acceleration is in the same direction as the total of given forces. This is called Newton's second law or mathematically written

$$\vec{a} = \frac{\sum \vec{F}}{m} \qquad (3)$$

For every action, there is an equal and opposite reaction. This is called Newton's third law or mathematically written

$$\vec{F}_{aqksi} = -\vec{F}_{reaksi} \qquad (4)$$

Newton's three laws can explain an object's motion whether it moves or in the rest..

Therefore, a study about floating object in the fluid can be explained using theoretical framework taken by some references that use mathematics and physics to analyse.

Weight, Buoyant Force and Density

An object will float in the fluid if the buoyant force F_A is equal to the object's weight w in a static equilibrium, or mathematically written

$$\begin{gathered} F_A = w \\ \rho_f\ V_f^{'}\ g = \rho_B\ V_B\ g \end{gathered} \qquad (5)$$

With V_B as the object's volume and $V_f^{'}$ as the volume of the exerted fluid. From the formulation (5) it is known that an object will float in the fluid if the density of the object ρ_B is lower than the density of the fluid ρ_f or $\rho_B < \rho_f$ (Tipler; 2001, Giancoli; 2001, and Wilson, Buffa, and Lou; 2007).

If the density of the object ρ_B is lower than the density of the fluid ρ_f or $\rho_B < \rho_f$ then the buoyant force F_A is greater than object's weight w or $F_A > w$. As the result, the object will be accelerated upward to the surface of the fluid (Tipler; 2001 and Wilson, Buffa, and Lou; 2007).

3. Discussion

The buoyant force F_A occurs because the fluid's weight w_f gives the force around the object that is submerged in the fluid. This makes the object get the buoyant force (upward force) F_A that is equal to the exerted fluid $w_f^{'}$. Three states can be used to investigate how an object floats.

3.1 Dynamic State

If a floated object in the fluid is in dynamic state then the object moves upward the surface of the fluid with the acceleration $\vec{a}$ in which Newton's second law applies.

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

$$\vec{a} = \frac{\vec{F}_{AD} - \vec{w}}{m} \quad (6)$$

Based on the formulation (6), it is known that in the dynamic state, the object moves with the acceleration $\vec{a}$ upward the surface of the fluid. The magnitude of the buoyant force in the dynamic state F_{A_D} is greater than the object's weight w or mathematically written

$$F_{A_D} > w \quad (7)$$

If density is the only factor to be reviewed, then the formulation (7) can be written

$$F_A > w$$

$$\rho_f g V_f^{'} > \rho_B g V_B \quad (8)$$

Because the object's volume is fully submerged in the fluid, then $V'_B = V_B$ with the magnitude that is equal to the volume of exerted fluid $V_f^{'}$ or $V_B^{'} = V_B = V_f^{'}$. Therefore, the density of the object ρ_B is lower than the density of the fluid ρ_f or mathematically written $\rho_B < \rho_f$.

3.2 Dynamic Equilibrium State

If an object floats in the fluid in a dynamic equilibrium moves upward to the surface of the fluid with constant velocity v or the object does not have any acceleration $\vec{a} = \vec{0}$, it means that Newton's first law applies so that the formulation (8) becomes

$$\sum \vec{F} = \vec{0}$$

$$\vec{F}_{A_{SD}} - \vec{w} - \vec{f}_s = \vec{0} \quad (9)$$

with $\vec{f}_s$ as the friction towards the object or stoke force that against the magnitude of the object's motion. Because the force only happens in one straight line, then the only factor that can be investigated is the value of the force hence the formulation (9) can also be written

$$F_{A_{SD}} = w + f_s \quad (9.a)$$

Based on the formulation (9.a), it is known that the buoyant force in dynamic equilibrium $F_{A_{SD}}$ is greater than the object's weight w or mathematically written

$$F_{A_{SD}} > w \quad (10)$$

If the only factor that is reviewed is the density, then the formulation (10) can be written

$$F_{A_{SD}} > w$$

$$\rho_f g V_f^{'} > \rho_B g V_B \quad (11)$$

Because the object's volume is fully submerged in the fluid, then $V'_B = V_B$ with the magnitude of the exerted volume of the fluid $V_f^{'}$ or $V_B^{'} = V_B = V_f^{'}$, therefore the density of the object ρ_B is lower than the density of the fluid ρ_f or mathematically written $\rho_B < \rho_f$.

3.3 Static Equilibrium State

If an object floats in the fluid and in a rest or in static equilibrium or when an object is in a rest in the surface of the fluid as shown in Figure 4, the weight of the object w gets the buoyant force in the static equilibrium with upward direction $F_{A_{SS}}$ and the magnitude that is equal to the weight of the object w or in accordance with Newton's First Law.

$$\sum \vec{F} = \vec{0}$$

$$\vec{w} - \vec{F}_{A_{SS}} = \vec{0} \quad (12)$$

Because the force applies in a straight line though with the opposite direction, then the only factor that can be investigated is the magnitude of the force. Thus, the formulation (12) becomes

$$w - F_{A_{SS}} = 0$$

$$w = F_{A_{SS}} \quad (13)$$

The formulation (13) fits with the statement of Tipler (2001), Giancolli (201), Wilson, Buffa, and Lou (2007), and Kanginan (2008).

Meanwhile if the review is only the effect of the density, then the formulation (13) can be writte

$$\rho_B g V_B = \rho g V_f^{'} \quad (14)$$

Because the volume of the submerged object V'_B is equal to the volume of the exerted fluid $V_f^{'}$ or $V_B^{'} = V_f^{'}$, then the volume of the object V_B is greater than $V_f^{'}$ or $V_B > V_f^{'}$ thus the density of the object ρ_B is lower than the density of the fluid ρ_f or mathematically written $\rho_B < \rho_f$.

The result of the investigation of $F_A > w$ in a dynamic state is in accordance with the notions of Tipler (2001) and Wilson, Buffa, and Lou (2007). It is not only be applied when $\rho_B < \rho_f$ but also can be applied in static equilibrium $F_A = w$. Until now, there is no study about the floating object in the fluid in dynamic equilibrium with $F_A > w$.

Based on the analysis, it is known that an object can float in the fluid if the buoyant force F_A and the object's weight w fulfil $F_A > w$ in the dynamic equilibrium state or in the dynamic state with the formulation $F_A = w$ in the static state.

In conclusion, an object can float in the fluid if its initial state is dynamic state with $F_{A_D} > w$ and then the state will be dynamic equilibrium if $F_{A_{SD}} > w$ and finally the object is in static equilibrium state or in the rest in the surface of the fluid with $F_{A_{SS}} = w$ and $F_{A_D} > F_{A_{SD}} > F_{A_{SS}}$. Therefore, an object will float in the fluid if $F_A \geq w$ with the density of the fluid ρ_f is greater than the density of the object ρ_B or $\rho_B < \rho_f$.

4. Conclusion

1) The floating object in the fluid happens due to the buoyant force $\vec{F}_A$ of the fluid that is equal to the weight of exerted fluid $w_f^{'}$ or $F_A = w_f^{'}$. This makes the fluid surrounding the object gives the upward force towards the object's weight w_B that happens in dynamic state if $F_{A_D} > w$, in dynamic equilibrium state if $F_{A_{SD}} > w$ or in static equilibrium state if $F_{A_{SS}} = w$ with $F_{A_D} > F_{A_{SD}} > F_{A_{SS}}$.
2) A floating object in the fluid can occur in dynamic state, dynamic equilibrium state and static state with the buoyant force F_A is greater than or equal to the object's weight w_B or $F_A \geq w_B$ and the density of the fluid ρ_f is greater than the density of the object ρ_B or $\rho_f > \rho_B$.

References

[1] Giancoli. Douglas C..Physics (seventh edition). Prantice Hall Inc, New Jesley, 2001.
[2] Halliday & Resnick. Physics (6rd Edition). John Wiley & Sons, Inc., New York, 2006.
[3] Kanginan, M.. 2008. Fisika untuk SMA Kelas XI. Erlangga. Jakarta.
[4] Mikrajuddin, Saktiyono, dan Lutfi . IPA terpadu SMP dan MTs untuk kelas VIII semester II. Esis. Jakarta, 2008.
[5] Pauliza, Osa.. Fisika kelompok teknologi dan kesehatan untuk sekolah menengah kejuruan kelas XI, Grafindo Media Pratama. Bandung, 2008.
[6] Tipler Paul A Physics for scientists and engineers (third edition). Worth Publisher, Inc. New York, 2001.
[7] Wilson, Buffa, and Lou. College Physics (sixth edition). Pearson Education, Inc. New Jesley, 2007.