Processing Compressible 4D-CT Medical Image by using Cellular Neural Network

Thanh-Tung Cap¹, Hien-Trinh Nguyen²

¹Thai Nguyen University of Education, Hoang Van Thu, Thai Nguyen, Viet Nam
²University of Information and Communication Technology (ICTU), Quyet Thang, Thai Nguyen, Vietnam

Abstract: For 3-dimension image we use definition “voxel” (volumetric cell) instead of pixel (picture cell) of 2D. A single voxel particle consists parameter x,y,z for spatial dimensions. Data of voxel can be used to describe large objects and applied in video game, geology, astronomy, satellite image… and especially medical image processing. For surgery image collected from CT scanning picture such as brain, lung, heart, chest, hip… the object periodically change their location and size (respiration, heart rhythm, muscle and bone joint movement…) so the time element (t) must be added to parameter group of voxel. So the processing of motion image has relation with 4D-CT scan technique. The 4D-CT image processing requires IR, the virtue of which is to determine compatible cells among two consecutive image in movement. There are many methods based on various indentification criteria (on optical flow, calculation model, geometrical characteristics…). They are popularly used in processing and calculation of medical images. However there are still many constraints (such as accuracy or calculation optimization). With regards to medical image in general and lung tumour diagnosis in particular there are often many differences because of cell flexibility and disruption. In general, those methods yield good results if the strength of light is constant (Horn/ Schunk approach) but this condition is unrealistic because of lung movement based on respiratory rhythm which leads to variations of light on object (lung tissue) and causes discrepancies. We call CT image of affected tissues as compressible image. In this report we study speed calculation of optical flow of compressible CT 4D image. 4D image of large voxel volume and calculation complexity is a considerable challenge for existing PC based on sequence processing. In order to increase speed calculation we propose neuron cell parallely processing network capable of calculating 4D optical flow in real time mode.

Keywords: CNN, Cellular neural network, image registration, optical flow, medical image

1. Introduction

Since 1985, 2D optical flow processing technique has been introduced to collect speed data of each motion pixel. Nowadays, with 3D space the process has become much more complicated as object are voxel instead of pixel. 4D-CT image has popular use in healthcare, helps the diagnosis and evaluation of tumour be more accurate, especially for chest (lung tumour). 4DCT image can consist of 20 3D image volume for each phase, with size as 512x512x200 voxel for each volume. Beside very large volume data, there are possibilities of discrepancy caused by respiration, movement of body organ to be addressed. The problem has been discussed on many seminars. Sarut (2006) proposed mapping image analysis, Kessler (2006) proposed various data collecting ways, Guerrero, Gzhang, Segar, Huang, Bilton (2007) with 4D image evaluation… Many solutions are proposed on the basis of Horn&Schunk approach (for 2D image since 1981) to calculate speed of voxel in optical flow. Many conditions and assumption are added for optimization problem. Lucas Kanade proposed convolution for Gauss distribution function to eliminate disruption in 3D optical flow. But most author concentrated on 2D compressible image or 3D uncompressible image and use repeating lap for Gauss-Seidel algorithm. In this report we develop and analyze 3D compressible image and propose cell neuron network (CNN) 3 layer 3D to calculate 3D optical flow. New results are only on theoretical basis ans will be further developed in the coming time with the hope of a new methode for 4DCT motion image processing in healthcare.

2. The Problem of Calculating the Pixel Speed of Compressed Images

In analysis of motion image, it is important to determine compatibility of voxels in 2 consecutive images. Speed of voxel in optical flow is an important characteristics to the solution. In CT image, light strength of each voxel \(I(x, y, z, t)\) corresponds with the density of cell tissue \(\rho(x, y, z, t)\), in which \(x, y, z\) is coordinate in space \(\Omega\) and \(t\) is time. So the mathematic description of each 4D voxel is as follows:

\[
I(x, t) \propto \rho(x, t)
\]  \(1\)

Where \(t \in [0,1], x = (x, y) \in \Omega \) and \(\Omega \in \mathbb{R}^3\)

In all CT image processing methodes there is an assumption that cell density corresponds with light strength of each voxel is constant with time \(I(x, t) \propto \rho(x, t) = const.,\) that means:

\[
\frac{dI(x,t)}{dt} = I_t + \nabla I \cdot v = 0
\]  \(2\)

Where \(I_t = \frac{\partial I}{\partial t}, \quad \nabla I = \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial z} \end{bmatrix}, \quad v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \quad \nabla I = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}, \quad J_t = \begin{bmatrix} I_{tt} \\ I_{tx} \\ I_{ty} \\ I_{tz} \end{bmatrix} \)

Equation (2) is coverson of mass equation expressing continuity or continuity equation of image, which is constant with time or incompressible image. In CT image, chest, cell density changes from image to image due to elasticity of lung during respiration. Although partial density of cell are
uneven in lung, we assume that total mass of lung is unchanged during respiration:

$$\int_\Omega \rho(x,0) \, d\Omega = \int_\Omega \rho(x,t) \, d\Omega$$

(3)

Now, the equation for preserving the mass looks like:

$$I_t + \nabla^T \cdot \mathbf{v} + \nabla \cdot \mathbf{I} = 0 \quad \text{div}(\mathbf{v}) = 0 \quad (4)$$

Where $\text{div}(\mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ is divergence of pixel speed vector (optical flow). Equation (4) is also known as the equation of the speed of the Compressible Image.

To calculate the image flow velocity:

$$\mathbf{v} = \begin{bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{bmatrix}$$

with 3 unknowns, the speed equation (2) or (4) alone is not eligible. Therefore, we need to find more constraints to be able to calculate the image flow rate vector consistently. Applying the idea of Horn - Schunck [9], we combine equation (4) with the assumption that the velocity of neighboring voxels is little changed into a function (5) and find the minimum functional value this:

$$\min_{\mathbf{v}} \left\{ \frac{1}{2} \int \left| I(x) + \nabla^T \cdot \mathbf{v} + \nabla \cdot \mathbf{I} \right|^2 + \alpha \sum \left| \mathbf{v} \right| \right\} \, d\Omega$$

(5)

Where $\alpha > 0$ is smoothing coefficient. We can find $\mathbf{V}$ to minimize function (5) by Variational Calculus. As (5) is a convex square functional so the necessary and sufficient condition for optima $\mathbf{V}$ is to satisfy corresponding Euler-Lagrange equation:

$$\nabla \cdot \mathbf{v} = 0 \quad (6)$$

The boundary condition is:

$$\nabla \mathbf{v} \cdot \mathbf{n} = 0 \quad i = 1, 2, 3 \quad (7)$$

In (6) and (7) we use symbols:

$$f = h^2 + \alpha^2 \sum \left| \nabla \mathbf{v} \right|^2$$

$$h = I + \nabla^T \cdot \mathbf{v} + \nabla \cdot \mathbf{I}$$

(8)

Where $\mathbf{e}_i$ is the unit vector of the axis $I_n$ is the normal vector to the boundary surface. After calculating the derivatives of (6) and reducing, the Euler-Lagrange equation has the form:

$$-\nabla^2 \mathbf{v} + \alpha \mathbf{v} = \mathbf{0} \quad (9)$$

Continuing to implement the Euler - Lagrange equation (9), we get the following system of equations:

$$\begin{cases}
I(\mathbf{u}_x + \mathbf{u}_y + \mathbf{v}_z) + \alpha \nabla^2 \mathbf{v} = 0 \\
I(\mathbf{v}_x + \mathbf{v}_y + \mathbf{w}_z) + \alpha \nabla^2 \mathbf{v} = 0 \\
I(\mathbf{w}_x + \mathbf{w}_y + \mathbf{w}_z) + \alpha \nabla^2 \mathbf{v} = 0
\end{cases} \quad (10)$$

Where:

$$\nabla^2 \mathbf{v} = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \quad (11)$$

In [9], Horn - Shunk presented the approximate calculation for image derivatives as follows (with $h_i = h_0 = h_1 = I$):

$$\begin{align*}
I_x &= (I_{i+1,j,k} - I_{i-1,j,k})/2 \\
I_y &= (I_{i,j+1,k} - I_{i,j-1,k})/2 \\
I_z &= (I_{i,j,k+1} - I_{i,j,k-1})/2
\end{align*}$$

The Laplace operator is discrete:

$$\nabla^2 \mathbf{v} = 6u_{i,j,k} - u_{i+1,j,k} - u_{i-1,j,k} - u_{i,j+1,k} - u_{i,j-1,k} - u_{i,j,k+1} - u_{i,j,k-1}$$

Replacing the above approximation into equation (10), we have the Euler-Lagrange equation in discrete form as follows (Equation 11):

$$\begin{align*}
&I_{i,j,k} + I_{i+1,j,k} + I_{i-1,j,k} + I_{i,j+1,k} + I_{i,j-1,k} + I_{i,j,k+1} + I_{i,j,k-1} + 6u_{i,j,k} - u_{i+1,j,k} - u_{i-1,j,k} - u_{i,j+1,k} - u_{i,j-1,k} - u_{i,j,k+1} - u_{i,j,k-1} \\
&+ I_{i,j,k} + I_{i,j+1,k} + I_{i,j-1,k} + I_{i+1,j,k} + I_{i-1,j,k} + I_{i,j,k+1} + I_{i,j,k-1} + 6u_{i,j,k} - u_{i,j+1,k} - u_{i,j-1,k} - u_{i+1,j,k} - u_{i-1,j,k} - u_{i,j,k+1} - u_{i,j,k-1} \\
&+ I_{i,j,k} + I_{i,j-1,k} + I_{i,j+1,k} + I_{i-1,j,k} + I_{i+1,j,k} + I_{i,j,k+1} + I_{i,j,k-1} + 6u_{i,j,k} - u_{i,j-1,k} - u_{i,j+1,k} - u_{i-1,j,k} - u_{i+1,j,k} - u_{i,j,k+1} - u_{i,j,k-1} \\
&+ I_{i,j,k} + I_{i-1,j,k} + I_{i+1,j,k} + I_{i,j+1,k} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k+1} + I_{i,j,k-1} + 6u_{i,j,k} - u_{i-1,j,k} - u_{i+1,j,k} - u_{i,j+1,k} - u_{i,j-1,k} - u_{i,j,k+1} - u_{i,j,k-1} \\
&+ I_{i,j,k} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k+1} + I_{i,j,k-1} + 6u_{i,j,k} - u_{i,j,k+1} - u_{i,j,k-1} - u_{i,j,k+1} - u_{i,j,k-1} - u_{i,j,k+1} - u_{i,j,k-1} \\
&+ I_{i,j,k} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k+1} + I_{i,j,k-1} + I_{i,j,k+1} + I_{i,j,k-1} + 6u_{i,j,k} - u_{i,j,k+1} - u_{i,j,k-1} - u_{i,j,k+1} - u_{i,j,k-1} - u_{i,j,k+1} - u_{i,j,k-1} \\
&+ I_{i,j,k} + I_{i,j,k} + I_{i,j,k} + I_{i,j,k} + I_{i,j,k} + I_{i,j,k} + I_{i,j,k} + I_{i,j,k} + 6u_{i,j,k} - u_{i,j,k} - u_{i,j,k} - u_{i,j,k} - u_{i,j,k} - u_{i,j,k} - u_{i,j,k} - u_{i,j,k}
\end{align*}$$

The problem of speed of 4D voxels has become Euler-Lagranger equation system to find speed of discrete voxels $u_{i,j,k}, v_{i,j,k}, w_{i,j,k}$ in space $\Omega$. This is a very large and complicated problem. In case of 4D-CT image consisting of 20 3D volume with size $512x512x200$ of each volume, the total number of voxel for calculation is 100 mln voxels. All sequence calculating PC can not give quick answer, especially motion image processing in real time. In this article we propose CNN paralley processing method to increase calculation in optical flow for compressible 4D image.

3. Methods of determining 4D compressible image flow rate using cellular neural networks

CNN is a paralley processing system to meet demand for the fastest processing system in image processing. Basic
volume of CNN is cell. It consists of linear and nonlinear circuit. A single cell connects only with neighboring cells. Adjacent cells can directly affect each other. Cells which do not directly connect with each other can only affect other cells indirectly by the spread of CNN. Each cell $C_{ij}$ has neighboring cells $C_{ik}$. Basic equation describing movement of each cell of CNN one layer 2D is as follows:

**Status equation:**

$$
\dot{x}_i = -x_i + z_i + \sum_{C_{ijk}} A(i,j,k,l) y_{ij} + \sum_{C_{ijk}} B(i,j,k,l) u_{ij} \quad (12)
$$

**Output equation:**

$$
y_{ij} = f(x_{ij}) = \frac{1}{2} |x_{ij}| + \frac{1}{2} |x_{ij} - l| \quad (13)
$$

![Diagram of a cell](image1)

**Figure 1:** Diagram of a cell

(12), (13) basic equation system of CNN is continuous in time and discrete in space. Because structure of cell are identical so it is convenient for networking on chip. CNN processing programme is implemented through statements describing partial connections between cells in A, B, Z network (template). Each statement corresponds with a template. In library of CNN there are basic templates which are added to be richer and richer. Programmer can find suitable template for problems to be addressed. To determine speed of voxel we must solve Euler-Lagrange discrete equation system (11). We can have variable of (11) by finding stable variable of differential equation system on virtual time $\tau$ as follows:

$$
\begin{align*}
\frac{d^2 v_{ij}}{dt^2} & = -4x_{ij} + 6v_{ij} + 2v_{ij}^2 - v_{ij}^3 \\
\frac{d v_{ij}}{dt} & = f(x_{ij})
\end{align*}
$$

**Figure 2:** Three-layer CNN model 3D calculating speed voxel

By analyzing relations between elements of voxels in equation system (14) we can determine weighted average matrixes between cells as follows:

$$
A_u = \frac{A_{u+1}}{A_{u+1}} \quad \text{where} \quad A_{u+1} = \begin{bmatrix} 0 & 0 & 0 \\
0 & -a^2 & 0 \\
0 & 0 & 0 \end{bmatrix}
$$

$$
D_{uv} = \frac{D_{uv+1}}{D_{uv+1}} \quad \text{where} \quad D_{uv+1} = \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \end{bmatrix}
$$

$$
A = \begin{bmatrix} 0 & -I_{11} & l^4 - a^2 \\
-I_{11} & 1 & 4a \\
I_{11} & 4a & -a^2 \end{bmatrix}
$$

$$
D_{uv} = \begin{bmatrix} \frac{l^4}{4} & -\frac{l^2}{2} & -\frac{l^2}{2} \\
-\frac{l^2}{2} & 1 & -\frac{l^2}{2} \\
-\frac{l^2}{2} & -\frac{l^2}{2} & \frac{l^4}{4} \end{bmatrix}
$$

We build CNN structure to solve differential equation system (14) as shown in Figure 1. This is an CNN 3 layer 3D. The upper layer to calculate speed element $u_{i,j,k}$ of voxel $i,j,k$ in 3D space, the middle to calculate speed element $v_{i,j,k}$ and the lower to calculate element $w_{i,j,k}$. The status of network is voxel speed $v(x,y,z)$. Two 4D image $I(x,y,z,t), I(x,y,z,t+\Delta t)$ are used to calculate elements of nonlinear template of network. The primary value of summation in network is 0. Network is calculated on condition that margin cell is 0. Output of network is values of speed elements $v(x(t), y(t), z(t))$. We can see the relationship between nonlinear layer with light strength.
Motion image processing for 4DCT chest image is urgent because of flexibility of lung cell during respiration. Assumptions that light strength in CT voxel is constant in nowadays method can cause discrepancies. This report studies and develops optical flow for 4D compressible image in chets 4D image as well as healthcare image. The concerning problems are optical flow equation, function minimization on Horn-Schunk model, Euler-Lagranger discrete equation which all are applied and developed for 4DCT compressible image. In order to increase calculation speed we propose CNN 3 layer 3D to solve Euler-Lagranger discrete equation system for 4DCT image to calculate speed of voxels in 2 consecutive images. This is a new parallellly processing method based on nature of CNN and network solidity on FPGA chip. By this way, we can integrate more methods and techniques to reduce disruption, increase quality for 4DCT image. This problem shall be dealt in another article. We hope to improve processing technique for motion compressible image by using CNN technology.

### References


2. R. Adrian, "Twenty year of particle image velocimetry", Experiments in Fluid, 2005, 39(2)


08.2000.


Author Profile

Thanh-Tung Caphas been born in Vietnam. He obtained his master degree in University of Information and Communication technology. His research interests include computer science, data mining and machine learning, CNN technology, software engineering, images processing.

Hien-TrinhNguyenschas been born in Vietnam. She obtained his master degree in University of Information and Communication technology. Her research interests include computer science, fuzzy constraint network, data mining and machine learning.