Geometric Evaluation of the Effect of Earth's Rotation on Michelson Type Interferometers

Siddhant Pinjarkar

Knowledge Centre, Priyadarshani College of Engineering, Nagpur - 440022

Abstract: ***The famous Michelson-Morley experiment used a setup of interferometer which consisted of a light source, beam splitter and mirrors, also known as Michelson interferometer. Similar Michelson-type interferometer is used to detect gravitational waves by setup used in LIGO-Hanford and LIGO-Livingston. Due to different inertial gravitational forces experienced by the two beams of light, the light in both arms should return at beam splitter at different times.This paper attempts to find the change in time of arrival and fractional fringe shift induced in Michelson-type interferometer due to the uniform rotation of the earth. It is found that the fractional fringe shift is proportional to the angular velocity of the earth, square of arm's length and inversely proportional to the wavelength of light source and speed of light.***

Keywords: Michelson interferometer, fractional fringe shift

1. Introduction

Motion relative to space, described by absolute motion in the experiment performed by Michelson-Morley in 1887. The experiment was performed to understand the absolute motion and existence of hypothetical aether [1][2] [9]. When two orthogonal coherent beams of light travel between two mirrors, Michelson interferometer compares the time taken to travel between two mirrors [2] [9]. The interferometer consists of two perpendicularly placed mirrors, light source and central half-silvered mirror. Light is emitted from a light source and splits orthogonally at the half-silvered mirror. The two light beams travel equal distance orthogonally, reflect at the end mirrors and interfere at half-silvered mirror [1] [6]. If the path of light is changed, we would obtain interference fringes at the detector. The aim of the experiment was to detect a shift in dark fringes as the apparatus moves about in space [3].

A set of second-generation interferometers, also known as Advanced LIGO detectors are installed in two sites – Hanford and Livingston. Each interferometer consists of 4 km long arms with the basic design of Michelson interferometer. The laser type being Nd: YAG with a wavelength of 1064 nm [4] [6]. Each arm of new advanced LIGO also contain Fabry-Perot resonators that used for amplification of the signal to increase the sensitivity and the interaction between the waves and the signal [11]. Figure (1) below represents the components and illustrates the working of the Michelson type interferometer used in LIGO.

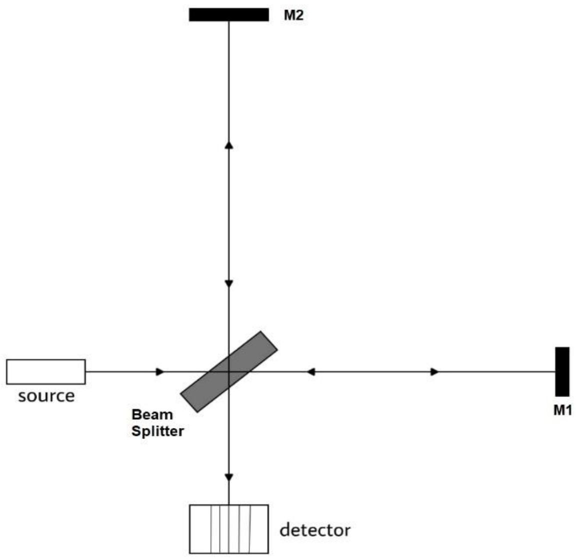

Figure 1: Setup describing Michelson interferometer with essential components used by LIGO. Where, M1 is end mirror at arm 1 and M2 is end mirror at arm 2.

LIGO uses Michelson type interferometer at kilometre scale for detection of gravitational waves to a sensitivity of 10^{-23} in metric strain. LIGO type interferometers use basic principle of calculating change in path length travelled by light in both arms. If a gravitational wave is incident on the setup, the detector at the beam splitter measures the relative distance between the two orthogonally placed mirrors [6]. Where, the path length or change in the lengths between the mirrors is given by [6]:

$$\Delta L = hL/2 \qquad (a)$$

Here h denotes the amplitude of the gravitational wave incident on the setup and L being the arm length. Further, the path length can be used to obtain fractional fringe shift induced in the setup.

Another source that could affect the sensitivity of LIGO like interferometers or can be responsible for obtaining fringe patterns in LIGO like interferometers is uniform rotation of earth.Due to difference in inertial-gravitational forces the light in the interferometer propagating perpendicular to one

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another in the arms of the interferometershould experience difference of time dilation [5]. This can mean that the lightreturns to the beam splitter at different times, causing change in path length experienced by the light rays in both arms.This paper is an attempt to evaluate (geometrically) the difference in time taken for the light beams to reach the beam splitter and further derive the effect of rotation on fringe shift for kilometre scaled Michelson type interferometers such as LIGO.

2. Effect of earth's rotation on the interferometer

By using geometric analogies, it can be possible to calculate change in path travelled by light in the interferometer which in turn means that we can calculate fractional fringe shift for the instrument. This paper attempts to evaluate therelation betweenfringe shift observed due touniform rotation of the earth using geometric construction. The next section explains, in brief, the method used to calculate the fractional fringe shift for kilometre scaled Michelson type interferometer.

2.1 Geometric evaluation

Before, we outline the construction to understand the evaluation of change in path length, it is important to get acquainted to following facts. The surface curvature for the earth is very large as compared to the size of the setup. Thus, the two arms of the interferometer act as small tangents to the surface of the earth. Hence, it can be noted that path of the light would not change, or the position of the mirror will not change due to the small size of the setup, as compared to the earth's curvature. Also, if we consider a patch of land on earth where the interferometer is installed, the patch rotates the same way a plane rotates around a point attached to a string. Where, the rotation is uniform, hence there is no angular acceleration.

Let us outline theconstruction of a Michelson type interferometer. As discussed above, the light source splits at the half-silvered mirror (beam splitter) to meet at the end mirrors. Let A (in figure 2) be the point where the light splits, the light rays travel orthogonal to each other towards M_1 and M_2. Both mirrors are aligned perpendicular to the propagation of light. Fringe shift is observed when there is a change of path length. If rotation of the earth is not taken into consideration, the light rays should retrace the path travelled along the axis of propagation. As for LIGO type detectors, the arm lengths are equal. Hence, the change in path length for the light rays should equal to zero. Thus, there should be no fractional fringe shift observed.

Now, if we consider earth's rotation, the light rays start from point A towards mirrors M_1 and $M_2$and reach in time, let's say δt. By the time light rays reach both mirrors, due to rotation of earth, the position of mirrors has changed as the setup itself has shifted its position by δθ. Which means,by the time the rays reach mirrors M_1 and M_2, both mirrors have undergone rotation of $\delta\theta = \Omega\delta t$ (Ω being angular velocity of earth) and as the rotation is uniform the rotation in both mirrors should be the same causing similar inclination in both mirrors, where Ω is the angular velocity of earth's rotation.

Now, the light traces a new path that is affected by the rotation of the earth instead of following the length of the arm's 'L'. By using geometry of the setup, the new length traced by the light (up to M_1 and M_2) can be found to be as '$L/\cos\delta\theta$'. Therefore, in figure 2, AM_1 and AM_2 illustrate the new path traced by light given by: '$L/\cos\delta\theta$'. Here the value of δθ is very small, so the value of cosine is close to 1. The light rays reach M_1 and M_2, which have already changed their position by δθ due to rotation of the earth. This is represented in figure 2 by black solid line near M_1 and M_2. The inclination observed in the mirrors causes the light rays to change its angle of incidence by δθ and by using laws of reflection the rays are reflected back to position A'. The angle of reflection should be same as angle of incidence, therefore the total angle made between AM_1 and M_1A' is '2δθ'. It also worth noting that, according to the laws of reflection the light ray should lie in the same plane. Similar analogy can be used for the other arm.

Consider the following figure (2), here, as the mirrors is inclined due to the earth's rotation, the angle of incidence of light rays 'changes to δθ with respect to the normal of the mirror and reflect with the same angle of δθ from the normal of mirrors. Thus, the reflected light rays should meet approximately at A'. Therefore, to evaluate change in path length, from $\Delta A'M_1M_2$;

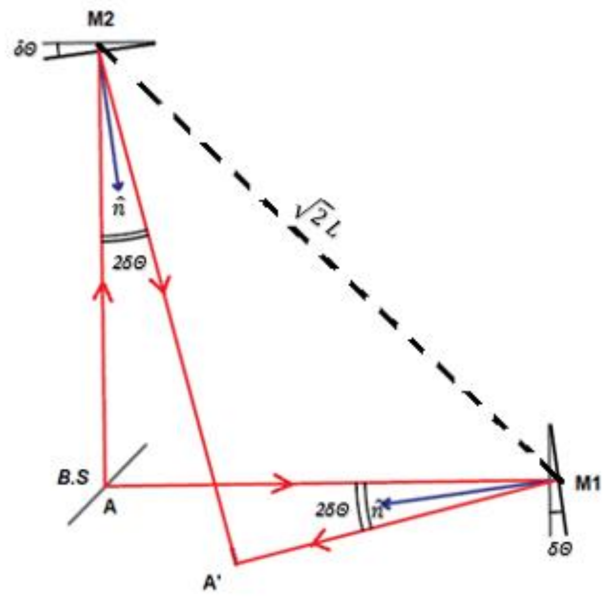

Figure 2: Path of light after leaving beam splitter and reflecting from mirrors which meet at A'. Grey coloured mirrors represent initial position of interferometer i.e. before the rotation of the earth and when the light rays leave the beam splitter (B.S). Whereas, Black coloured mirrors represent the approximate position of mirrors affected by the rotation of the earth. Blue solid lines represent the direction of the normal after the mirrors have tilted and the red solid line represents the path of light rays.

Therefore;

$$Sin\left(\frac{\pi}{4} + 2\delta\theta\right) = \frac{M_1A'}{\sqrt{2}L} \quad (1)$$

$$Cos\left(\frac{\pi}{4} + 2\delta\theta\right) = \frac{M_2A'}{\sqrt{2}L} \quad (2)$$

The path difference travelled by the light is given by:

$$\Delta L = M_1A' - M_2A' \quad (3)$$

Expanding Sine and Cosine terms and substituting for M_1A' and M_2A', we get the value for path difference. Substituting the terms, the value of ΔL is given by:

$$\Delta L = 4L\delta\theta \quad (4)$$

Where, Cos 2δθ ~ 1 and Sin 2δθ ~ 2δθ, as the value of δθ is very small.

It is known that the difference in time for light rays to travel from both mirrors to beam splitter is given by:

$$\Delta T = \frac{\Delta L}{c} \quad (5)$$

Where c is the speed of light.

It is also known that fractional fringe shift is given by:

$$\Delta N = \frac{\Delta v}{v} = \frac{\Delta T c}{\lambda} \quad (6)$$

Substituting the value of ΔT into the equation for the fractional fringe shift, we get:

$$\Delta N = \frac{4L\delta\theta}{\lambda} \quad (7)$$

Where δθ is equal to the product of Ω and δt. Here, δt is the interval of time it takes for the light rays to travel arm length of the interferometer.

Therefore, the fractional fringe shift is given by the equation:

$$\Delta N = \frac{4L^2\Omega}{\lambda c}$$

3. Conclusion

From the equation of fractional fringe shift obtained in the above expression, it can be seen clearly that the fractional fringe shift is dependent on the square of arm's length, the angular velocity of the earth's rotation, speed of light and wavelength of the light source. The inclusion of 'Ω' in the equation proves that fractional shift is obtained due to uniform rotation of the earth. It is also evident that through a simple geometrical consideration, for uniformly rotating earth,the change in path length for the two light beams and the equation for a fractional fringe shift can be obtained. This in turn means that due to uniform rotation of earth the light beams experience a change in their path which induces a change in path length, finally resulting in fractional fringe shift for the setup. A similar expression was obtained in the famous Michelson-Gale experiment [8] and by Silberstein [10] for a different setup of interferometry.

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Author Profile

Siddhant Pinjarkar, received his bachelor's degree in Mechanical engineering from Pune University at Sinhgad college of engineering. Further, he has a post graduate degree in Astrophysics from University of Glasgow. Currently he is working on publications with Astrophysics field and wants pursue PhD in the future to get into research.