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# Simple Methods for Determination of Wet-Bulb Temperature and Dew-Point Temperature

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**Abstract:** ***Simple and precise methods for accurate calculation of wet –bulb temperature is presented for known values of dry-bulb temperature and relative humidity.In the present paper, the equations for determining dew point temperature for the entire pressure range (611.213 Pa to 47389.26 Pa) are also given. The equations proposed here for saturation vapour pressure for temperature range ( 0ºC to 80ºC) has an average error of 0.03% only and the same equation can be rewritten to determine the value of dew-point temperature. The deviation in calculated values of wet-bulb temperature ranged from -0.238 to 0.458ºC with mean absolute error of 0.167 for RH value of 50% while deviation in calculated values of wet-bulb temperature ranged from -0.139 to 0.481ºC with mean absolute error of 0.119 for RH value of 20%.The proposed method for wet-bulb temperature led to an average error of 0.559% for RH value of 50% and 0.358% for RH value of 20% as compared to ASHRAE approximation. The present method for calculation of wet-bulb temperature can remove a large number of iteration and provide a high degree of computational ease.***

**Keywords:** Dry-bulb temperature, Wet-bulb temperature, Dew point temperature and Relative humidity

## 1. Introduction

Any air based energy systems such as, HVAC plants, evaporative coolers, cooling towers dryers, heaters etc. require very accurate values of psychrometric properties. The knowledge of psychrometric properties is fundamental to the design of environmental control systems for plants, crops, animals and humans (Singh *et al.,* 2002). It is generally observed that psychrometric charts are most widely used because it is the quickest method to determine properties of air-water mixture. The design of very complex devices requires very accurate values of psychrometric properties with very high computational power. ASHRAE has provided very accurate values of saturation vapour pressure through exponential approximation and first order polynomial correlation for specific enthalpy with computational ease. The direct calculation of wet-bulb temperature is not possible and ASRAE provided iterative method for its determination. The value of relative humidity can be easily calculated for known values of dry-bulb temperature and wet-bulb temperatures. One can use the psychrometric equation of saturation line for this purpose for any pressure, such as standard sea level pressure (101325 Pa). However, there is no easy method to calculate wet-bulb temperature for known values of dry-bulb temperature and relative humidity. The psychrometric equation cannot be used for this purpose because it has an exponential term, one linear term and even the value of psychrometric constant in not constant**.** **T**he determination of wet-bulb temperature is really difficult. Either it is solved by trial and iterative method or the secant method (Conte, 1965) can be used, which is cumbersome and require more computational time. Computational problem may occur with this method at temperatures of 100ºC and above (Wilhelm, 1976). There is, however, no easy analytical solution to get wet –bulb temperature from dry-bulb temperature and relative humidity. Hence, there is a need to evolve a simple method for the determination of wet-bulb temperature thereby eliminating a large number of iterations.

## 2. Equations for determination of psychometric properties

### 2.1 Saturation vapour pressure

The water vapour saturation pressure is part and parcel of psychrometric calculations. Although a number of formulas for computing vapour pressure are available, only accurate relationships should be used. The calculation of vapour pressure should be as accurate as possible as it is used in determining many other psychrometric properties. Several formulas for different temperature ranges were evaluated to find the most suitable and accurate formulas for this analysis. They are very complex and lengthy. However, vapour pressure was calculated by using the equation given by Dilley (1968) and Singh *et al.*, (2002) which is simple and precise. His equation for computing saturation vapour pressure in Pascal (Pa) was found accurate for the temperature range (0ºC to 65ºC). However, it required slight modification as given below:

$$P_{wsT} = 611.213\ exp\left[\frac{17.273\ T}{237.32+T}\right] \text{------------} 1(a)$$

$(0^0C < T \leq 65.0^0C)$ and

The above equation can be rewritten for computing temperature as a function of vapour pressure as given below,

$$T_{dp} = \left[\frac{237.32\ \ln\left(\frac{P_{ws}}{611.213}\right)}{17.273-\ln\left(\frac{P_{ws}}{611.213}\right)}\right] \text{------------}\quad 1(b)$$

$(611.213\ Pa \leq PwsT\ or\ P_w \leq 25064.53\ Pa)$

For temperature above 65$^o$C, the following equation is used:

$$P_{wsT} = 611.679\ exp\left[\frac{17.2699T}{236.3435+1.01585T}\right] \text{--------} 2(a)$$

$(65^oC < T \leq 110^oC)$

The above equation can be rewritten for computing temperature as a function of vapour pressure as given below,

$$T_{dp} = \left[ \frac{236.3435 \ln\left(\frac{P_{ws}}{611.679}\right)}{17.2699 - 1.01585 \ln\left(\frac{P_{ws}}{611.679}\right)} \right] \text{----------------} 2(b)$$

The equations 1(a) and 2(a) used here are very simple and can be used with computational ease. The relationship is within 0.030 percent of the correct values of saturation vapour pressure for temperature range (0 – 80$^{o}$C). The above equations [1(a), 2(a)] and [1(b), 2(b)] calculate saturation vapour pressure as a function of temperature and temperature as a function of saturation vapour pressure, respectively. These equations should not be used outside the stated temperature or pressure ranges.

### 2.2 Dew point temperature

The dew point temperature can be calculated by using saturation vapour pressure equations 1(b) and 2(b) putting the value of actual vapour pressure ($P_w$) or saturation vapour pressure at dew point temperature, which depends on Ps Ta and RH (fraction). In this instance, the above two equations are required for entire saturation pressure ranges. The salient feature of saturation vapour pressure equations 1(a) and 2(a) is that the same equations can be rewritten as 1(b) and 2(b) for computing dew point temperatures, which are given in Table 3.

**Table 3:** Dew -point temperature ($^{0}$C)

| $T_a$ | RH (fraction) | $P_{sTa}$ | $P_w$ | $T_{dp}$ |
|---|---|---|---|---|
| 30 | 0.5 | 4246.75 | 2123.374 | 18.439 |
| 30 | 0.2 | 4246.75 | 849.3496 | 4.606 |
| 40 | 0.5 | 7382.47 | 3691.234 | 27.578 |
| 40 | 0.2 | 7382.47 | 1476.494 | 12.77 |
| 50 | 0.5 | 12349 | 6174.482 | 36.688 |
| 50 | 0.2 | 12349 | 2469.793 | 20.873 |
| 60 | 0.5 | 19954 | 9976.999 | 45.767 |
| 60 | 0.2 | 19954 | 3990.799 | 28.92 |
| 70 | 0.5 | 31199.8 | 15599.92 | 54.781 |
| 70 | 0.2 | 31199.8 | 6239.966 | 36.882 |
| 80 | 0.5 | 47389.3 | 23694.63 | 63.753 |
| 80 | 0.2 | 47389.3 | 9477.852 | 44.769 |

### 2.3 Wet-bulb temperature

The wet-bulb temperature can be estimated from the following well known psychometric relation:

$$P_w = P_{sTwb} - B\,(T_a \text{-} T_{wb}) \text{ ----------------------}(3)$$

B is given by Dr Carrier equation as,

$$B = \frac{1.8\,(P - P_{stwb})}{(2800 - 1.3\,(1.8T_a + 32))}$$

and taken as 66 Pa C$^{-1}$ for entire range of temperature
Slope of ($T_a$-$T_{wb}$) and ($T_a$-$T_{dp}$) lines are given as,

$$\frac{P_{sta} - P_w}{T_a - T_{wb1}} = \frac{(P_{sta} - P_w + 66(T_a - T_{dp}))}{(T_a - T_{dp})}$$

It results in,

$$T_a \text{-} T_{wb1} = \frac{\frac{(P_{sta} - P_w)}{(T_a - T_{dp})}}{(P_{sta} - P_w + 66(T_a - T_{dp}))} \text{ ----------}(4)$$

$$T_{wb1} = \frac{T_a - \frac{(P_{sta} - P_w)}{(T_a - T_{dp})}}{(P_{sta} - P_w + 66(T_a - T_{dp}))} \text{ ---------------}(5)$$

Substituting the value of ($T_a$-$T_{wb1}$) in equation (3),we get saturation vapour pressure at wet-bulb temperature,

$$P_{sTwb2} = \frac{\frac{(P_w + 66\,(P_{sta} - P_w\,)}{(T_a - T_{dp}\,)}}{(P_{sta} - P_w + 66(T_a - T_{dp}))} \text{----------------}(6)$$

The value of wet-bulb temperature $T_{wb2}$ and that of dew-point temperature can be determined by putting the value of $P_{sTw2}$ and value of $P_w$in equation 1(b) or2(b),respectively as per the values of saturation vapour pressure specified therein.

The values of estimated wet-bulb temperatures, $T_{wb1}$ and $T_{wb2}$ were determined by using equations (4), (5) and (6), respectively. The values of actual vapour pressures $P_{w1}$ and $P_{w2}$ were determined by putting the values of $T_{wb1}$, $T_{wb2}$, $P_{stwb1}$ and $P_{stwb2}$ in equation (6).The values of Pstwb1 and $P_{stwb2}$ were determined by using equations 1(a) or 2(a).

**Table 4:** Estimated wet-bulb temperature

| $T_{dp}$ | $P_w$ | $P_{w1}$ | $P_{w2}$ | $T_{wb1}$ | $T_{wb2}$ |
|---|---|---|---|---|---|
| 18.439 | 2123.374 | 2172.374 | 2004.462 | 22.238 | 21.495 |
| 4.608 | 849.3496 | 1130.973 | 376.079 | 17.262 | 12.995 |
| 27.578 | 3691.234 | 3732.97 | 3536.01 | 30.468 | 29.836 |
| 12.77 | 1476.494 | 1748.818 | 838.1756 | 23.249 | 19.123 |
| 36.688 | 6174.482 | 6208.059 | 5986.862 | 38.855 | 38.346 |
| 20.874 | 2469.793 | 2712.796 | 1679.713 | 29.299 | 25.618 |
| 45.768 | 9976.999 | 10003.35 | 9760.151 | 47.391 | 46.992 |
| 28.921 | 3990.799 | 4196.332 | 3066.115 | 35.573 | 32.459 |
| 54.785 | 15599.92 | 15620.42 | 15356.61 | 56.012 | 55.702 |
| 36.881 | 6239.966 | 6408.332 | 5196.928 | 42.1 | 39.549 |
| 63.752 | 23694.63 | 23710.19 | 23433.91 | 64.692 | 64.456 |
| 44.768 | 9477.852 | 9613.712 | 8327.636 | 48.863 | 46.805 |

The value of $T_{wb3}$ is taken as,
$T_{wb3}$= ($T_{wb1}$+$T_{wb2}$)/2

Putting the value of $T_{wb3}$ into equation (7), the value of $P_{w3}$ can be determined as,

$$P_{wb3} = P_{sTwb} - B\,(T_a \text{-} T_{wb3}) \qquad \text{----------}(7)$$

Now the accurate value of $T_{wb}$ was computed by using the following equation,

$$T_{wb} = \frac{[((P_{w3} - P_w)\,T_{wb2} + T_{wb3}\,(P_w - P_{w2})]}{(P_{w3} - P_{w2})} \text{ -------}(8)$$

**Table 5:** Determination of accurate wet-bulb temperature

| $T_a$ | RH | $T_{wb}$ | $ASHRAET_{wb}$ | Deviation C | Error (%) |
|---|---|---|---|---|---|
| 30 | 0.5 | 22.021 | 22.234 | 0.213 | 0.952 |
| 30 | 0.2 | 15.67 | 15.701 | 0.031 | 0.198 |
| 40 | 0.5 | 30.334 | 30.506 | 0.172 | 0.566 |
| 40 | 0.2 | 22.015 | 22.025 | 0.01 | 0.046 |
| 50 | 0.5 | 38.778 | 38.889 | 0.111 | 0.288 |
| 50 | 0.2 | 28.433 | 28.399 | -0.034 | 0.12 |
| 60 | 0.5 | 47.348 | 47.386 | 0.038 | 0.084 |
| 60 | 0.2 | 35.007 | 34.911 | -0.096 | 0.272 |
| 70 | 0.5 | 55.988 | 55.975 | -0.013 | 0.024 |
| 70 | 0.2 | 41.745 | 41.591 | -0.154 | 0.371 |
| 80 | 0.5 | 64.679 | 64.626 | -0.053 | 0.082 |
| 80 | 0.2 | 48.646 | 48.438 | -0.208 | 0.429 |
|  |  |  |  | 0.095 (mean Absolute deviation C) | 0.286 Av error (% ) |

## 3. Accuracy and Limitations

The accuracy of the calculated values of dew point temperature depends on the limits of temperature and vapour pressure. Within the limits specified the equations 1(b) and 2(b) compute the accurate values of dew point temperature, but if calculations are to be made outside the stated limits, the above equations need to be reformulated. The equation 1(a) and 2(a) have the accuracy of 0.030 per cent of the correct values of saturation vapour pressure for the temperature range (0 ˚C to 8 ˚C). The errors go on accumulating, if we go for higher temperatures i.e. 0.042 per cent for 110˚C. There are other formulas available in the literature for calculating saturation vapour pressure (Perry, IAPWS, ASHRAE W & G, Fernandez and Chargoy, 1990, Keenan and Keyes, 1936 and Wilhelm,1976).But the formula given by Fernandez and Chargoy is not accurate for large range of temperature and those given by Keenan and Keyes and Wilhelm, IAPWS, ASHRAE W & G, are lengthy and complex. The equations proposed here 1(a), 2(a), 1(b) and 2(b) give very accurate values of both saturation vapour pressure and dew-point temperature.

The values of $T_{wb}$ are given in Table 5 along with that of $T_{wb}$ values computed by using ASRAE approximation. The values of $T_{wb}$ determined using present method were found in close proximity with that of ASHRAE values. The deviation in calculated values of wet-bulb temperature ranged from -0.238 to 0.458ºC with mean absolute error of 0.167 for RH value of 50% while deviation in calculated values of wet-bulb temperature ranged from -0.139 to 0.481ºC with mean absolute error of 0.119 for RH value of 20%.The proposed method for wet-bulb temperature led to an average error of 0.559% for RH value of 50% and 0.358% for RH value of 20% as compared to ASHRAE approximation. The present method for calculation of wet-bulb temperature can remove a large number of iteration and provide a high degree of computational ease.

## 4. Summary

The paper presents two equations for calculation of saturation vapour pressure in two temperature ranges (0˚C to 65˚C) and (65˚C to 110˚C).The equations to calculate dew point temperature as a function of saturation vapour pressure are also given. The relationships proposed here are valid from 0˚C to 110˚ C. Outside the stated limits, the equations need to be reformulated. This paper also provides an easy method to calculate accurate wet-bulb temperature, one of the most important thermodynamic properties of air-water mixture, by direct calculations.

**Appendix A**

List of symbols

| | |
|---|---|
| Temperature, °C | T |
| Relative humidity,fraction | $\gamma$ |
| Partial pressure,Pa | P |
| Psychrometric constant | B |

**Appendix B**

Subscript

| | |
|---|---|
| Dry air | db |
| Wet-bulb | wb |
| Dew point | dp |
| Condition for actual water vapour | w |
| Condition at saturation | s |

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