

Spectrum Sensing Time Optimization Using Kullback-Leibler Divergence (KLD OSS)

Manimegalai Munisamy¹, Janani Munisamy²

Department of ECE, TPGIT, Vellore, India
Email: manimega.samy[at]gmail.com

Department of ECE, TPGIT, Vellore, India
Email: jananigms[at]gmail.com

Abstract: Effective and quick spectrum sensing forms the major deciding criterion in any cognitive radio environment. A spectrum sensing scheme, using Kullback-Leibler Divergence with optimized spectrum sensing (KLD OSS) time in order to avail and identify the underutilized spectrum effectively is presented in this paper. Simulation results highlighting the competitive edge of this scheme, with higher probability of detection over various Signal to noise ratio (SNR) is also shown. This optimized scheme utilizes Log-Likelihood ratio (LLR) of all Secondary Users (SU) during each sensing instance, with Fusion Centre (FC) acquiring and assimilating these information to estimate the likelihood of spectrum availability.

Keywords: Spectrum Sensing Time Optimization, Kullback-Leibler Divergence, KLD OSS

1. Introduction

Cognitive Radio (CR) is an innovative concept in wireless communication which paved the way for building efficient intelligence Radios. Prime motive of CRs is to achieve effective utilization of underutilized bandwidth [1] & [2]. With the age-old practice of licensed spectrum usage, in a non-homogenous urban infrastructure and land topography, overcrowding and underutilization of spectrum is inevitable. A recent research study made by Signals Research Group [3], highlights that around 88 percent of licensed spectrum was underutilized in licensed frequency spectrum in indoor environment. Thus the cognitive radio aims to achieve effective and efficient spectrum management, allowing other potential Secondary Users (SU) to temporally gain access to the spectrum that is not dynamically utilized by licensed users [4].

Many spectrum detection techniques such as Energy based detection [5], matched filter detection [1] and cyclo-stationary feature detection [2] have already been explored for spectrum sensing with their own pros and cons. While sensing the spectrum, to tackle the impact of fading phenomenon in the wireless environment, cooperative spectrum sensing is utilized to take the advantage of spatial diversity in wireless networks. Cooperative sensing depends on multiple SU nodes to sense the presence or absence of PU [7]. Each SU communicates the probability of spectrum availability with the FC to estimate the likelihood of spectrum availability for the given probability of false alarm. The Fusion Centre then combines the results of the individual SUs to make the decision on the presence or absence of a PU [2] & [6]. When the Fusion Centre arrives at an estimation, time taken towards Spectrum sensing by each SU has to be minimal enough to make the Cognitive radio network effective. Basic tradeoff in sensing time of each SU is that it shall be low enough so that the cognitive node has sufficient time to transmit its own data and high enough to ensure that the interference caused to the primary user is minimized. Conventionally, the Spectrum sensing time allocated to all SU is assumed to be identical and fixed.

This paper presents the performance of the CR network with proposed Optimized spectrum sensing method with Kullback Leibler Divergence (KLD OSS). This method significantly reduces the number of samples required to sense the licensed frequency band which in turn reduces the spectrum sensing time.

The rest of the paper is organized as follows: In section II, System model is presented. In section III, Spectrum sensing method using KLD is presented. The Sensing time optimization is discussed in section IV. Simulation results are presented in section V. Finally Conclusions are drawn in section VI.

2. System Model

Consider the cognitive radio environment with N Secondary users (SU). Each SU is capable of sensing the spectrum band of interest with anyone of the spectrum sensing methods as discussed in [5], [1] and [2]. Further each SU has the computational capability to identify the Primary Users presence with LLR calculation and consequently forward their decision to the Fusion Centre as represented in fig. 1. The Fusion Centre then accumulates and assimilates these received decisions from all SUs to arrive at an estimate between the two binary hypotheses H_0 and H_1 which are defined as follows

$$H_0: \text{PU is absent}, H_1: \text{PU is present}$$

The acquired signal at the n^{th} ($n=1, 2, 3, 4, \dots, N$) SU is given by

$$\begin{aligned} H_0: Y_n(s) &= W_n(s), \\ H_1: Y_n(s) &= h_n(s) * X_n(s) + W_n(s), s=1,2,3 \dots \end{aligned} \quad (1)$$

where $h_n(s)$ represents the fading channel coefficients, $X_n(s)$ represents the independent and identically distributed signal samples of the PU signal acquired by the SU with mean 0 and variance $\sigma_{x,n}^2$ i.e., $X_n(s) \sim N(0, \sigma_{x,n}^2)$ and $W_n(s)$ represents the independent and identically distributed additive Gaussian white noise samples with mean 0 and

variance $\sigma_{w,n}^2$ i.e., $W_n(s) \sim N(0, \sigma_{w,n}^2)$. Under the binary hypotheses H_0 and H_1 , the acquired signal distribution at the n^{th} SU is described by the probability density function $P_{0,n}(Y_n(s))$ and $P_{1,n}(Y_n(s))$ respectively.

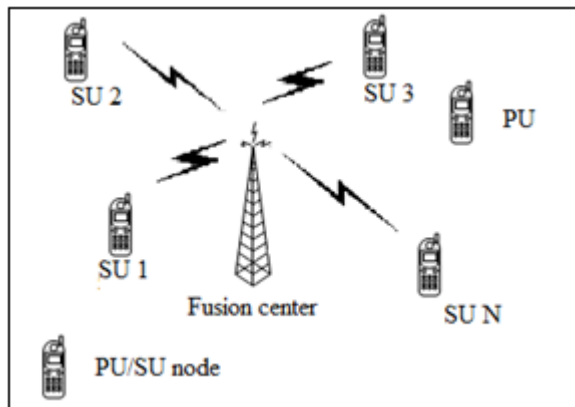


Figure 1: Deployment scenario

The performance of the spectrum sensing method is evaluated and compared in terms of spectrum sensing time and Receiver Operating Characteristic (ROC) curve achieved based on different Signal to noise ratio (SNR).

Probability of detection (P_d), Probability of false alarm (P_f) and Probability of misdetection (P_m) are the various performance metrics used to evaluate the performance of the detection method. For a given P_d , the P_f and P_m are represented by

$$P_f = P(\text{deciding on } H_1 \text{ when } H_0 \text{ is true}) = P(\text{decision} = H_1 | H_0)$$

$$P_m = P(\text{deciding on } H_0 \text{ when } H_1 \text{ is true}) = P(\text{decision} = H_0 | H_1)$$

3. Spectrum Sensing using KLD

Conventionally, the number of sensing samples acquired by the SU to detect the frequency spectrum of interest is fixed. With fixed sample size method, all SU senses the spectrum of interest using any conventional sensing methods and posts this information to Fusion centre. The Fusion Centre accumulates these information and then compute the LLR for a given fixed sample size (S_{fix}). The mathematical expression for LLR calculation is given below in (2).

$$LLR = \sum_{s=1}^{S_{fix}} \sum_{n=1}^N \ln \left(\frac{P_{1,n}(Y_n(s))}{P_{0,n}(Y_n(s))} \right) \quad (2)$$

With Conventional method, the number of samples required to detect the PU signal is fixed as in the Neyman-Pearson approach [8]. To detect the presence or absence of a PU signal, the Likelihood ratio test is performed based on

$$\begin{aligned} \text{If } LLR > \lambda, & \text{Decide as } H_1 \\ \text{If } LLR \leq \lambda, & \text{Decide as } H_0 \end{aligned} \quad (3)$$

The threshold value λ and the sample size S_{fix} are selected based on the Probability of false alarm (P_f) and Probability of misdetection (P_m) bounded to the pre-assigned values α and β such that $0 < \alpha, \beta < 1$ i.e.,

$$P_f \leq \alpha \text{ and } P_m \leq \beta \quad (4)$$

Based on equation (4), the expressions for S_{fix} and λ are given by

$$S_{fix} \approx 2 \left(\frac{\sqrt{\sum_{n=1}^N \left(1 - \frac{\sigma_{0,n}^2}{\sigma_{1,n}^2}\right)^2} Q^{-1}(\alpha) + \sqrt{\sum_{n=1}^N \left(\frac{\sigma_{1,n}^2}{\sigma_{0,n}^2} - 1\right)^2} Q^{-1}(\beta)}{\sum_{n=1}^N \frac{\sigma_{1,n}^2}{\sigma_{0,n}^2} + \sum_{n=1}^N \frac{\sigma_{0,n}^2}{\sigma_{1,n}^2} - 2N} \right)^2 \quad (5)$$

$$\begin{aligned} \lambda \approx & \frac{S_{fix}}{2} \sum_{n=1}^N \left(1 - \frac{\sigma_{0,n}^2}{\sigma_{1,n}^2}\right) + \frac{S_{fix}}{2} \sum_{n=1}^N \ln \left(\frac{\sigma_{0,n}^2}{\sigma_{1,n}^2}\right) + \\ & \sqrt{\frac{S_{fix}}{2} \sum_{n=1}^N \left(1 - \frac{\sigma_{0,n}^2}{\sigma_{1,n}^2}\right)^2} Q^{-1}(\alpha) \end{aligned} \quad (6)$$

In order to minimize the spectrum sensing time, following enhancements were introduced into the spectrum sensing algorithm such that better optimization is achieved. Here, each SU performs the Log-Likelihood Ratio (LLR) for each and every spectrum sensed sample in a sequential manner as stated in Wald's equation [8]. The sensing test is performed after acquiring a sample at each CR and then FC accumulates these values, in which the LLR value is compared with two threshold levels η_m and η_f . If the accumulated LLR value lies between the two thresholds levels, then one more sample is taken and further the same routine is repeated until LLR satisfies the following condition $\eta_f \leq LLR \leq \eta_m$, otherwise decide as H_1 and terminate if $LLR \geq \eta_m$ or else decide as H_0 and terminate if $LLR \leq \eta_f$, as described in Algorithm below, the Log likelihood ratio is given by

$$LLR = \sum_{s=1}^S \sum_{n=1}^N \ln \left(\frac{P_{1,n}(Y_n(s))}{P_{0,n}(Y_n(s))} \right) \quad (7)$$

Threshold levels η_m and η_f can be tuned to get the desired detection performance, i.e., depending on the tolerance level of probability of false alarm P_f and miss-detection P_m . It is shown in [9] that if the probability of the false alarm P_f and the probability of the miss-detection P_m are sufficiently small, then for LLR based test expressions, η_f and η_m can be given as

$$\begin{aligned} \eta_f &= \ln \left(\frac{1 - \beta}{\alpha} \right) \frac{1}{\eta} \\ \eta_m &= \ln \left(\frac{\alpha}{1 - \beta} \right) \frac{1}{\eta} \end{aligned} \quad (8)$$

Algorithm: Optimized Spectrum sensing using Kullback-Leibler Divergence (KLDOS)

Step1: Initially set $S=0$ & $LLR_S=0$ at the Fusion Centre.

Step2: **do loop**

Step 3: $S=S+1$.

Step 4: All the $n(n=1,2,3,\dots,N)$ SU radios compute $\ln \left(\frac{P_{1,n}(Y_n(s))}{P_{0,n}(Y_n(s))} \right)$ from the acquired sample $Y_n(s)$.

Step 5: Each SU then sends the computed $\ln \left(\frac{P_{1,n}(Y_n(s))}{P_{0,n}(Y_n(s))} \right)$ to the Fusion centre

Step 6: The Fusion centre then updates the LLR value according to

$$LLR_S = LLR_{S-1} + \sum_{n=1}^N \ln \left(\frac{P_{1,n}(Y_n(s))}{P_{0,n}(Y_n(s))} \right)$$

Step 7: **while loop** when $LLR_S \leq \eta_f$ or $LLR_S \geq \eta_m$

Step 8: If $LLR_S \leq \eta_m$, it is decided that the 'PU is absent: H_0 ' else if $LLR_S \geq \eta_f$ it is decided that the 'PU is present: H_1 '.

According to Wald’s equation, the average number of samples required for detection is a random variable and it depends on Kullback-Leibler divergence between the two probability distributions. Thus the average number of samples required for detection depends on the signal conditions rather than being fixed as in conventional method. The average number of samples required for spectrum sensing in the absence of the PU is given by

$$E_{H_0}\{S_{Stop}\} = \frac{-\left(\eta_f - \eta_m - \eta_f e^{\eta_m} + \eta_m e^{\eta_f}\right)}{\left(e^{-\eta_f} - e^{-\eta_m}\right)\left[\sum_{n=1}^N D\left(P_{0,n}\left(Y_n(s)\right) \parallel P_{1,n}\left(Y_n(s)\right)\right)]} \quad (9)$$

The average number of samples required for spectrum detection in the presence of PU is

$$E_{H_1}\{S_{Stop}\} = \frac{\eta_f - \eta_m - \eta_f e^{-\eta_m} + \eta_m e^{-\eta_f}}{\left(e^{-\eta_f} - e^{-\eta_m}\right)\left[\sum_{n=1}^N D\left(P_{1,n}\left(Y_n(s)\right) \parallel P_{0,n}\left(Y_n(s)\right)\right)]} \quad (10)$$

Where $D(f||g)$ represents the Kullback-Leibler divergence between the probability distributions. Kullback-Leibler divergence or relative entropy is a measure of the distance between two probability distributions. This distance however is not symmetric in general, so it is not a distance in the Euclidean sense. The Kullback-Leibler divergence between the two continuous probability density functions $f(x)$ and $g(x)$ is defined as

$$D(f||g) = E\left[\log \frac{f(x)}{g(x)}\right] \quad (11)$$

Where the expectation is taken with respect to $f(x)$. $D(f||g)$ is only finite if the support set of $f(x)$ is contained in the support set of $g(x)$ [8]. Another important property of the Kullback-Leibler divergence is that it is non-negative, i.e., $D(f||g) \geq 0$ and is non-symmetric i.e., $D(f||g) \neq D(g||f)$.

The number of required samples on an average is dependent on the KL divergence as seen in equations (9), (10). Intuitively, the larger the KL divergence, the more the two hypotheses differ from each other, which in turn requires lesser number of samples to detect the spectrum and thus the sensing time is reduced. Thus if the KL divergence calculated at each SU is increased, the time required at the SU to sense the spectrum is reduced.

4. Sensing Time Optimization

The cognitive radio’s frame structure is shown in Fig. 2, which consists of consecutive frames, where T_f is the frame duration and T_s is the sensing time required to sense the PU signal and the remaining time $T_d = T_f - T_s$ is used for transmitting the data of the Secondary user(SU). The spectrum sensing time is chosen such that more time is allocated for transmission of SU data.

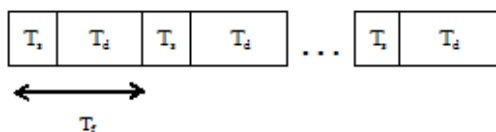


Figure 2: Frame structure of the Cognitive radio network

The Probability of detection (P_d) is defined as the probability of identification of the PU (Licensed user) presence

correctly [10]. Probability of false alarm (P_f) is defined as the probability of detecting the presence of PU when it is actually inactive. Probability of misdetection (P_m) refers to the probability of accepting the absence of the PU signal when it is actually present. The lower the probability of false alarm, the more the channel can be reused when it is available. For a good detection method, the P_d should be as high as possible and the P_f should be as low as possible.

The expressions for P_d and P_f are given by

$$P_d = Q\left(\frac{\lambda - SN\sigma_{1,n}^2}{\sqrt{2SN\sigma_{1,n}^2}}\right) \quad (12)$$

$$P_f = Q\left(\frac{\lambda - SN\sigma_{0,n}^2}{\sqrt{2SN\sigma_{0,n}^2}}\right) \quad (13)$$

As seen in expressions (12)&(13), the Probability of detection and false alarm are dependent on the threshold λ , the number of samples required for sensing S and the number of Secondary Users N .

5. Simulation Results

The optimized spectrum sensing scheme using Kullback-Leibler divergence (KLD OSS) is evaluated and compared with the conventional fixed sample size method based on simulation. The simulated cognitive radio network consist of five SU nodes (i.e., $N=5$) which are efficiently utilized in cooperative spectrum sensing. For simulation purposes, the targeted PU signal and the noise are considered as Gaussian distributed with zero mean and variance $\sigma_{x,n}^2$ & $\sigma_{w,n}^2$ respectively. Conventional detection method with fixed sample size uses S_{fix} and λ as specified by Neyman – Pearson detector, while the detection method using KLD OSS uses η_f & η_m as determined in equation (8). The system parameters set up for simulation is shown in Table 1.

Table 1: System Parameters Set Up For Simulation

S. No.	System Parameters	Values
1	SNR at the PU (γ_p)	-12 to 3 dB
2	$\alpha = \beta$	0.01
3	No. of SUs (N)	5
4	SNR at the SU (γ_s)	20 dB
5	Frame Duration (T_f)	2 sec

In Fig.3, it is observed that the Sensing time is substantially reduced with Optimized sensing method. It can also be observed that for lower SNR levels of PU signal, lesser the average samples required compared to conventional fixed sample size method, thus proving the method is substantially adaptive for lower SNR levels of primary user.

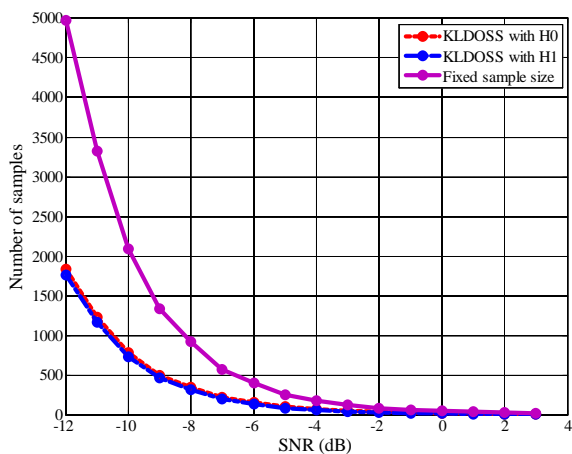


Figure 3: Average number of samples required for fixed sample size and the detection method using KLD OSS under the binary hypotheses H_0 & H_1 for different values of SNR.

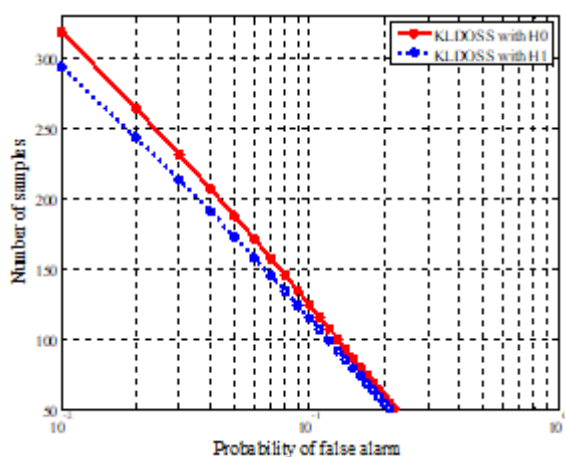


Figure 4: Average number of samples required using KLD OSS for the binary hypotheses H_0 & H_1

In Fig. 4, it can be seen that the average number of samples required is reduced with the increase in Probability of false alarm. It is also observed that the number of samples required for sensing under the hypotheses H_0 is comparatively higher than the number of samples required under H_1 .

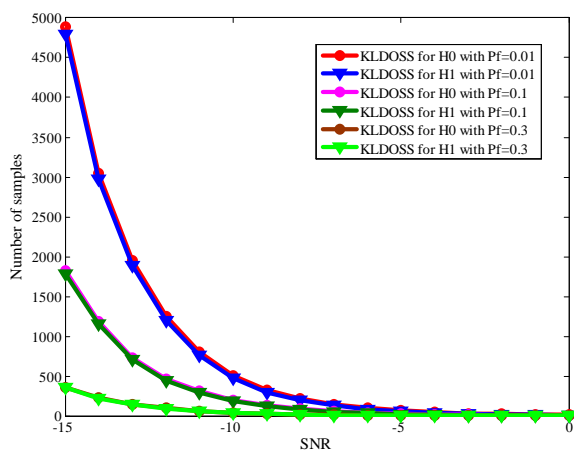


Figure 5: Average number of samples required with KLD OSS for binary hypotheses H_0 & H_1 for different values of SNR in dB.

In Fig. 5, it can be seen that the average number of samples required for sensing decreases with the increase in Probability of false alarm.

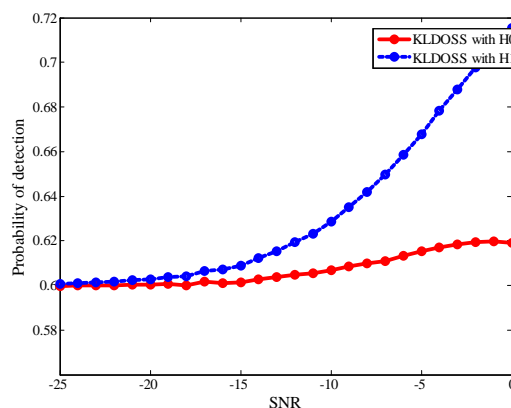


Figure 6: ROC of spectrum sensing method using KLD for different values of SNR in dB under H_0 & H_1

In Fig.6, it is shown that the performance of sensing varies with the Signal to Noise Ratio and it can be seen that the performance of detection increases with increase in SNR.

6. Conclusion

Thus, in contrast to the conventional Neyman-Pearson based fixed sensing time detector, the optimized spectrum sensing method using KLD significantly reduces the average sensing time that is required to achieve the same Cognitive Radio network objective. Performance metrics such as P_d , P_f and SNR are used for the analysis of the proposed KLD OSS. Thus with the proposed sensing model, sensing time is optimized. Simulation results also demonstrate that the detection method using KLD is robust and outperforms the existing conventional signal detectors.

References

- [1] Mitola, III and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Personal Communications.*, vol. 6, no. 4, pp. 13–18, 1999.
- [2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Communications.*, vol. 23, no. 2, pp. 201–220, Feb.2005.
- [3] SpiderCloud Wireless, "Poor In-building Coverage Leaves Valuable Licensed Spectrum Under-utilized", Press release 25th February 2013.
- [4] Federal Communications Commission, "Spectrum Policy Task Force Report," FCC Document ET Docket No. 02-155, Nov. 2002.
- [5] António Furtado, LuísIriio, Rodolfo Oliveira, Luís Bernardo, RuiDinis "Spectrum Sensing Performance in Cognitive Radio Networks With Multiple Primary Users" *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, VOL. 65, NO. 3, MARCH 2016
- [6] J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE J. Sel. Topics Signal Processing*, vol. 2, no. 1, pp. 18–27, Feb. 2008.
- [7] M. Matsui, H. Shiba, K. Akabane, and K. Uehara, "A novel cooperative sensing technique for cognitive radio," in *Proc. 18th IEEE Int. Symp. Personal, Indoor,*

Mobile Radio Communications (PIMRC), Sep. 2007, pp. 1–5

- [8] Wald, *Sequential Analysis*. New York: Wiley, 1947.
- [9] Q. Zou, S. Zheng, and A. H. Sayed, "Cooperative Sensing via Sequential Detection," *IEEE Transaction on Signal Processing (TSP)*, vol. 58, no. 12, pp. 6266-6283, December 2010.
- [10] Fadel F. Digham, Member, IEEE, Mohamed-Slim Alouini, Senior Member, IEEE, and Marvin K. Simon, Fellow, IEEE. "On the Energy Detection of Unknown Signals Over Fading Channels", *IEEE TRANSACTIONS ON COMMUNICATIONS*, vol. 55, NO. 1, JANUARY 2007, pp. 21-24.

Author Profile



Manimegalai Munisamy received her B.E degree in Electronics and communication engineering from Thanthai Periyar Government Institute of technology, Vellore and M.E degree in Communication systems from College of Engineering, Guindy, India. She is currently working as Assistant Professor in Thanthai Periyar Government Institute of Technology, Vellore, India. Her field of interest includes Wireless communication and Wireless Sensor networks. She is currently pursuing her Ph.D in the field of Wireless communication.



Janani Munisamy received her B.E degree in Electronics and communication engineering from Thanthai Periyar Government Institute of technology, Vellore and M.E degree in Applied Electronics from College of Engineering, Guindy, India. She is currently working as Assistant Professor in Thanthai Periyar Government Institute of Technology, Vellore, India. Her field of interest includes VLSI, Wireless communication, Image Processing and Wireless Sensor networks. She is currently pursuing her Ph.D in the field of VLSI.