Profit Evaluation of an Identical Units System with Arbitrary Distribution

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Abstract: ***In this paper we do profit evaluation of parallel system of two identical units with arbitrary distributions. Every one unit has absolute failure from standard mode. The structure is measured in up-state if as a minimum of one component is functioning. Failed unit (in case of failure) is mended by server who visits the system without delay for doing repair of the unit. After giving fixed repair time to server it is replaced by new one giving some replacement time. Profit evaluation of the system is done by using arbitrary distributions to all random variables by using Semi-Markov process and RPT. The numerical study of the results obtained for a particular case has also been made.***

Keywords: Profit, Parallel-Unit System, Replacement, Maximum Repair Time, Arbitrary Distributions

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1. Introduction

In present scenario, to preserve the permanence of the services there be as many manufacturing systems such as the system of electrical transformers, in which transformers having same polarity and voltage ratio are connected in parallel in order to meet the total load requirement. To enhance the reliability of these systems, method of redundancy and making replacement of failed components by new one at the elapsed of maximum repair time are very successful techniques. Nakagawa and Osaki (1975) discussed a two-unit parallel redundant system with repair maintenance. Singh and Agarafoitis (1995) studied stochastically a two-unit cold standby system subject to maximum operation and repair time. Recently, Kumar et al. (2010) carried out cost benefit analysis of a two-unit parallel system subject to degradation after repair while chillar et al.(2013) analyze parallel system with priority to repair over Maintenance Subject to Random Shocks. Malik and Gitanjali(2012, 2014,2019) studied parallel system with two types of repairs, arrival times of server, maintenance of units with exponential distributions. In view of the above we do profit evaluation of parallel system of two identical units with arbitrary distributions. Every one unit has absolute failure from standard mode. The structure is measured in up-state if as a minimum of one component is functioning. Failed unit (in case of failure) is mended by server who visits the system without delay for doing repair of the unit. After giving fixed repair time to server it is replaced by new one giving some replacement time. Profit evaluation of the system is done by using arbitrary distributions to all random variables by using Semi-Markov process and RPT. The numerical study of the results obtained for a particular case has also been made.

2. Notations

E: Set of regenerative states
O: Unit is operative
r(t)/R(t): Constant failure rate of the unit
p(t)/ P(t): Maximum constant rate of repair time taken by the server
f(t)/F(t) : pdf / cdf of the replacement time of the unit
g(t)/G(t) : pdf / cdf of the repair time of the unit
FU_r / FU_R: Unit is failed and under repair / under repair continuously from previous state
FW_r / FW_R: Unit is failed and waiting for repair / waiting for repair continuously from previous state
FUR_p / FUR_P: Unit is failed and under replacement / under replacement continuously from previous state
m_{ij}: Contribution to mean sojourn time in state S_i∈E and non-regenerative state if occurs before transition to〖S〗_j∈E. Mathematically, it can be written as

$$m_{ij} = \int_0^\infty t\, d\left(Q_{ij}(t)\right) = -q_{ij}^{*'}(0)$$

~/*: Symbol for Laplace Stieltjes transform / Laplace transform
$\boxed{S}$ /©: Symbols for Stieltjes convolution / Laplace convolution.

The possible transitions between states along with transitions rates for the system model are shown in figure 1.The states S_0, S_1 and S_2 are regenerative while the other states are non-regenerative

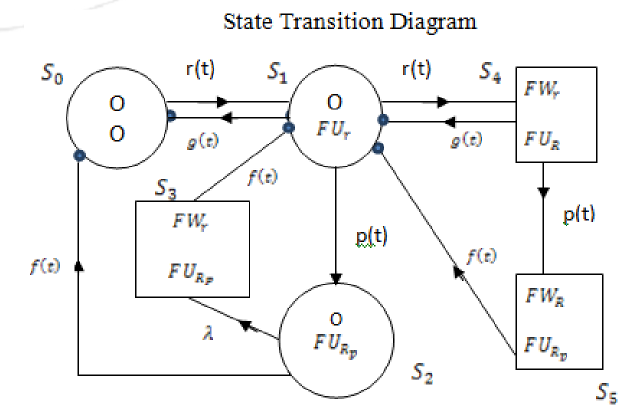

Figure1

● : Regenerative Point O: Upstate
□ : Failed State

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt$$

$p_{01} = \int_0^\infty r(t)\,dt,\ p_{10} = \int_0^\infty g(t)\,\overline{R(t)}dt,$

$p_{12} = \int_0^\infty p(t)\,\overline{G(t)}\,\overline{R(t)}dt,\ p_{14} = \int_0^\infty r(t)\,\overline{G(t)}\,\overline{P(t)}dt,$

$p_{20} = \int_0^\infty f(t)\,\overline{R(t)}dt,\ ;\ p_{23} = \int_0^\infty r(t)\,\overline{f(t)}\,dt,$

$p_{31} = \int_0^\infty f(t)dt,\ p_{41} = \int_0^\infty g(t)\,\overline{P(t)}dt,$

$p_{45} = \int_0^\infty p(t)\,\overline{G(t)}dt,\ p_{51} = \int_0^\infty f(t)\,dt,$

$p_{11.4} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]g^*(\alpha_0),$

$p_{11.45} = p_{14}\boxed{S}p_{45}\boxed{S}p_{51},\ p_{21.3} = p_{23}\boxed{S}p_{31}$ ... (1)

It can easily be verified that

$p_{01} = p_{10} + p_{12} + p_{11.4} + p_{11.45}$ ... (2)

$= p_{20} + p_{23} = p_{20} + p_{21.3} = 1$

The mean sojourn times μ_i in state S_i is given by

$\mu_0 = \int_0^\infty P(T > t)\,dt = m_{01} = \int_0^\infty \overline{R(t)}\,dt,$

$\mu_1 = m_{10} + m_{12} + m_{14} = \int_0^\infty \overline{G(t)R(t)P(t)},$

$\mu_2 = m_{20} + m_{23} = \int_0^\infty \overline{R(t)F(t)}$ ... (3)

4. Availability Analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at $t = 0$.The recursive relation for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t)© A_1(t)$
$A_1(t) = M_1(t) + q_{10}(t)© A_0(t) + q_{12}(t)© A_2(t)$
$+ (q_{11.4}(t) + q_{11.45}(t))© A_1(t)$
$A_2(t) = M_2(t) + q_{20}(t)© A_0(t) + q_{21.3}(t)©A_1(t)$
... (6)

where $M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t)$ and $M_2(t) = e^{-\lambda t}\bar{F}(t)$

Taking $L.T.$ of relation (6) and solving for $A_0^*(s)$, we get steady-state availability as

$A_0 = \lim_{s\to 0} s\,A_0^*(s) = \frac{\mu_0(p_{10}+p_{12}(1-p_{21.3}))+M_1^*(0)+M_2^*(0)p_{12}}{\mu_1{}'+p_{12}\mu_2+\mu_0(p_{10}+p_{12}p_{20})} = \frac{N_2}{D_2}$ ...(7)

Where $N_2 = \mu_0(p_{10} + p_{12}p_{20}) + \mu_1 + p_{12}\mu_2$ and $D_2 = \mu_0(p_{10} + p_{12}p_{20}) + \mu_1{}' + p_{12}\mu_2{}'$.

5. Busy Period Analysis Due To Repair

Let $B_i^1(t)$ be the probability that the server is busy in repairing the unit at an instant 't'given that the system entered regenerative state i at $t = 0$. The recursive relations for $B_i^1(t)$ are given as

$B_0^1(t) = q_{01}(t)© B_1^1(t),$
$B_1^1(t) = W_1(t) + q_{10}(t)© B_0^1(t)+q_{12}(t)© B_2^1(t) + (q_{11.4}(t) + q_{11.45}(t))© B_1^1(t),$
$B_2^1(t) = q_{20}(t)© B_0^1(t) + q_{21.3}(t)© B_1^1(t)$ ... (8)

where

$W_1(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t) + (\lambda e^{-\lambda t}©\mathbf{1}©e^{-\alpha_0 t})\bar{G}(t)$ ...(9)

Taking $L.T.$ of relation (8) and solving for $B_0^{1*}(s)$, we get in the long run the time for which the system is under repair is given by

$B_0^1 = \lim_{s\to 0} s\,B_0^{1*}(s) = \frac{N_3}{D_2}$ ...(10)

where $N_3 = P_{01}w_1^*(0)$ and D_2 is already specified.

6. Busy Period Analysis due to Replacement

Let $B_i^2(t)$ be the probability that the sever is busy in replacing the unit at an instant 't'given that the system entered regenerative state i at $t = 0$. The recursive relation for $B_i^2(t)$ are given by:

$B_0^2(t) = q_{01}(t)©B_1^2(t)$
$B_1^2(t) = q_{10}(t)©B_0^2(t)+q_{12}(t)© B_2^2(t) + (q_{11.4}(t) + q_{11.45}(t))© B_1^2(t)$
$B_2^2(t) = W_2(t) + q_{20}(t)©B_0^2(t) + q_{21.3}(t)© B_1^2(t)$ ... (11)

where

$W_2(t) = e^{-\lambda t}\bar{F}(t) + (\lambda e^{-\lambda t}©1)\bar{F}(t)$ ...(12)

Taking $L.T.$ of relation (11) and solving for $B_0^{2*}(s)$, we get the time for which the system is under replacement is given by

$B_0^2 = \lim_{s\to 0} s\,B_0^{2*}(s) = \frac{N_4}{D_2}$ ...(13)

where $N_4 = P_{12}w_2^*(0)$ and D_2 is already specified.

7. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relation for $N_i(t)$ are given by

$N_0(t) = Q_{01}(t)\boxed{S}[1 + N_1(t)]$
$N_1(t) = Q_{10}(t)\boxed{S}N_0(t)+Q_{12}(t)\boxed{S}N_2(t) + (Q_{11.4}(t) + Q_{11.45}(t))\boxed{S}N_1(t)$
$N_2(t) = Q_{20}(t)\boxed{S}N_0(t) + Q_{21.3}(t)\boxed{S}N_1(t)$ ... (14)

Taking $L.S.T.$ of relation (14) and solving for $\widetilde{N_0}(s)$, we get the expected number of visits per unit time as

$N_0 = \lim_{s\to 0} s\,\widetilde{N_0}(s) = \frac{N_5}{D_2}$ ...(15)

Where $N_5 = P_{10} + P_{12}P_{20}$ and D_2 is already specified.

8. Expected Number of Replacements of the Unit

Let $R_i(t)$ be the expected number of replacements by the unit in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relation for $R_i(t)$ are given by:

$R_0(t) = Q_{01}(t)\boxed{S}R_1(t)$

$R_1(t) = Q_{10}(t)\boxed{S} R_0(t)+Q_{12}(t)\boxed{S} R_2(t) + \left(Q_{11.4}(t) + Q_{11.45}(t)\right)\boxed{S} R_1(t)$

$R_2(t) = Q_{20}(t)\boxed{S} [1 + R_0(t)] + Q_{21.3}(t)\boxed{S} [1 + R_1(t)]$ (16)

Taking $L.S.T.$ of relation (16) and solving for $\widetilde{R_0}(s)$, we get the expected number of replacements per unit time as

$$R_0 = \lim_{s\to 0} s\,\widetilde{R_0}(s) = \frac{N_6}{D_2} \quad \ldots(17)$$

where $N_6 = P_{11.45} + P_{12}$ and D_2 is already specified.

9. Cost-Benefit Analysis

Profit incurred to the system model in steady state is given by:

$P = K_1 A_0 - K_2 B_0^1 - K_3 B_0^2 - K_4 R_0 - K_5 N_0$

Where

K_1 = Revenue per unit uptime of the system

K_2 = Cost per unit time for which server is busy due to repair

K_3 = Cost per unit time for which server is busy due to replacement

K_4 = Cost per unit time replacement of the unit

K_5 = Cost per unit visits by the server

10. Particular Case

Let us consider

$g(t) = \theta t^{p-1}\exp[-\theta t^p]$,

$p(t) = \alpha_0 p t^{p-1}\exp[-\alpha_0 t^p]$, $r(t) = \lambda p t^{p-1}\exp[-\lambda t^p]$

$f(t) = \beta p t^{p-1}\exp[-\beta t^p]$

Where $t \geq 0;\ \theta, \alpha_0, \beta, \lambda, > 0$.

By using the non-zero element p_{ij}, we obtain the following results:

$MTSF\,(T_0) = \frac{N_1}{D_1}$, Availability $(A_0) = \frac{N_2}{D_2}$,

Busy Period for repair $(B_0^1) = \frac{N_3}{D_2}$,

Busy period for replacement $(B_0^2) = \frac{N_4}{D_2}$

Expected number of visits $(N_0) = \frac{N_5}{D_2}$,

Expected number of replacement $(R_0) = \frac{N_6}{D_2}$

where $N_1 = \frac{1}{2\lambda} + \frac{\alpha_0+\beta+\lambda}{(\beta+\lambda)(\theta+\lambda+\alpha_0)}$, $D_1 = \frac{\lambda(\alpha_0+\beta+\lambda)}{(\beta+\lambda)(\theta+\lambda+\alpha_0)}$

$N_2 = \frac{1}{\theta+\lambda+\alpha_0}\left[1+\frac{\theta}{2\lambda}+\frac{\alpha_0}{2\lambda}\left(2-\frac{\beta}{\beta+\lambda}\right)\right]$, $N_3 = \frac{1}{\theta+\lambda+\alpha_0}\left[1+\lambda-\frac{\alpha_0\lambda}{\beta+\lambda}\right]$

$N_4 = \frac{1}{\theta+\lambda+\alpha_0}\left[\frac{\alpha_0(1+\lambda)}{\beta+\lambda}\right]$, $N_5 = \frac{\alpha_0\beta+\theta(\beta+\lambda)}{\theta+\lambda+\alpha_0(\beta+\lambda)}$

$N_6 = \frac{\alpha_0}{\theta+\lambda+\alpha_0}$,

$D_2 = \frac{1}{\theta+\lambda+\alpha_0}\left[1+\frac{\lambda\alpha_0}{\theta+\alpha_0}\left(\frac{1}{\alpha_0}+\frac{1}{\beta}\right)+\frac{\alpha_0}{2\lambda}\left(\frac{\beta+2\lambda}{\beta+\lambda}\right)+\frac{\theta}{2\lambda}\right]$

11. Conclusion

For a more concrete study of system behavior, we plot the curves for mean time to system failure (MTSF) and Profit function as shown in figures 2 and 3 respectively for different values of repair rate (θ), maximum repair time (α_0) taken by the server, failure rate (λ) and the costs K_2 and K_4 against the replacement rate β keeping the values of other parameters fixed as: K_1 = 5000, K_3 = 700 and K_5=50. These figures indicate that MTSF and profit increases with the increase of replacement rate (β) and repair rate (θ) while their values decrease as failure rate (λ) increases. The values of MTSF becomes more by the increase of repair time α_0 for β>7.5(approx). However, the profit declines with the increase of maximum repair time(α_0) and also by interchanging the values of K_2 = 600 and K_4 = 450. Thus on the basis of the results obtained for a particular case, it is concluded that a parallel system of two-identical units in which maximum time is given to the server for repair of the unit can be made more profitable by making replacement of the unit with high rates giving less replacement cost.

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