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$\tilde{\mu}_{(m,n)}$ −Open and Closed Sets in Bigeneralized Topological Spaces

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Abstract: ***In this paper, the authors introduced the concept of $\tilde{\mu}_{(m,n)}$ −open (respectively $\tilde{\mu}_{(m,n)}$ −closed)sets in bigeneralized topological spaces (briefly BGTS). Properties and characterization of these sets are also determined.***

Keywords: ***$\tilde{\mu}_{(m,n)}$ −open sets, $\tilde{\mu}_{(m,n)}$ −closed sets, bigeneralized topological spaces***

1. Introduction

The concepts of generalized topological spaces were first introduced by $\acute{A}. Csasz\acute{a}r$[3]. In [2], he obtained and investigated the notions of μ-semi-open sets, μ -pre-open sets, μ -α-open sets, and μ -β-open sets in generalized topological spaces. Moreover, the concepts of BGTS were introduced by C. Boonpok in [1] and studied (m, n)-closed sets and (m, n)-open sets.

The concepts of $\tilde{\mu}$-open sets, which is analogous to μ-semi-open sets, were introduced by D. Saravanakumar, N. Kalaivani, and G. SaiSundra Krishnan [4]. They also introduced the class of all $\tilde{\mu}$-open sets denoted by $\tilde{\mu}$O(x), $\tilde{\mu}$-interior, $\tilde{\mu}$-closure, $\tilde{\mu}$-boundary and $\tilde{\mu}$-exterior operators and studied their fundamental properties.

In this paper, the concepts of $\tilde{\mu}_{(m,n)}$ −open (respectively $\tilde{\mu}_{(m,n)}$ −closed) sets in bigeneralized topological spaces are introduced and characterized. Also, their properties and relationships are investigated.

2. Preliminaries

Definition 2.1 [3] Let X be a set. A collection μ of subsets of X is a generalized topology (briefly GT) on X if it satisfies the following: (O1) $\emptyset \in \mu$; and (O2) If $\{M_i : i \in I\} \subseteq \mu$, then $\bigcup_{i\in I} M_i \in \mu$.

If μ is a GT on X, then (X, μ) is called a generalized topological space (briefly GTS), and the elements of μ are called μ -open sets. The complement X\A of a μ -open set A is called μ -closed set. A generalized topology μ is said to be a strong GT if X∈ μ. If μ is a strong GT then (X, μ) is said to be a strong GTS.

Definition 2.2 [2, 4] Let (X, μ) be a GTS and A ⊆ X. Then
(i) μ-semi-closure of A denoted by c_{s_u}(A), is the intersection of all μ-semi-closed sets containing A. That is, the smallest μ-semi-closed set containing A.
(ii) μ-semi-interior of A denoted by isi_{s_u}(A), is the union of all μ-semi-open sets contained in A. That is, the largest μ-semi-open set contained in A.

Definition 2.3 [4] Let (X, μ) be a GTS. A subset A of X is said to be a $\tilde{\mu}$-open set if there exists a μ-open set U such that U ⊆A⊆ c_{s_u}(U). The family of all $\tilde{\mu}$-open sets is denoted as $\tilde{\mu}$O(X). A subset A of X is $\tilde{\mu}$-closed if its complement X\A is $\tilde{\mu}$-open.

Definition 2.4 [1] Let X be a nonempty set and let μ_1 and μ_2 be generalized topologies on X. The triple (X, μ_1, μ_2) is said to be a bigeneralized topological space (briefly BGTS). If both $\mu_1$and μ_2 are strong generalized topologies, then the BGTS (X, μ_1, μ_2) is said to be a strong bigeneralized topological space (briefly SBGTS). Let (X, μ_1, μ_2) be a BGTS and A, a subset of X. The μ-closure of A and the μ-interior of A with respect to μ_m are denoted by c_{μ_m}(A) and i_{μ_m}(A) respectively.

3. On $\tilde{\mu}_{(m,n)}$ Open Sets

In [4], Saravanakumar, Kalaivani and SaiSundara Krishnan introduced and defined $\tilde{\mu}$-open sets in GTS. In this section, $\tilde{\mu}$-open (resp. $\tilde{\mu}$-closed) sets are defined in a BGTS. Properties of these sets are introduced and studied. All throughout this section, (X, μ_1, μ_2) is a BGTS and m and n are elements of {1,2} where m≠n.

Definition 3.1 Let (X, μ_m, μ_n) be a BGTS. A subset A of X is said to be $\tilde{\mu}_{(m,n)}$ −open if there exists a μ_m-open set U of X such that U⊆A⊆ $c_{s_{\mu_n}}$(U), The complement X\A of $\tilde{\mu}_{(m,n)}$ −open is said to be $\tilde{\mu}_{(m,n)}$ −closed set.

To illustrate Definition 3.1, consider the next example.

Example 3.2 Let X = {a,b,c} with two GTs μ_1 = {∅, {a}, {c}, {a,c}} and μ_2 ={∅, X, {a} {a,b}}. Then the $\tilde{\mu}_{(1,2)}$-open sets are ∅, X, {a}, {a,b}, {a,c} and {c}. And the $\tilde{\mu}_{(2,1)}$-open sets are ∅, X, {a} and {a,b}. Also the $\tilde{\mu}_{(1,2)}$-closed sets are ∅, X, {b,c}, {c} and {b} and the $\tilde{\mu}_{(2,1)}$-closed sets are ∅, X, {b,c}, {c}.

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The following remark follows from Definition 3.1.

Remark 3.3 Let (X, μ_1, μ_2) be a BGTS.

(i) If a μ_m-open set U is equal to $c_{s_{\mu_n}}$(U), then U is $\tilde{\mu}_{(m,n)}$-open.
(ii) The empty set is $\tilde{\mu}_{(m,n)}$ open in (X, μ_1, μ_2).
(iii) X is $\tilde{\mu}_{(m,n)}$- closed.

Lemma 3.4 Let (X, μ_1, μ_2) be a BGTS. A and B are subsets of X. If A⊆B, then c_{s_u}(A) ⊆ c_{s_u}(B).

Proof: Let A⊆ B. Then, A ⊆ B ⊆ c_{s_u}(B). This implies that c_{s_u}(B) is a μ-semi closed set containing A. However, A⊆ c_{s_u}(A) and that c_{s_u}(A) is the smallest semi-closed set containing A. Therefore, c_{s_u}(A) ⊆ c_{s_u}(B).

Theorem 3.5 Let (X, μ_1, μ_2) be a BGTS and A ⊆ X. Then A is $\tilde{\mu}_{(m,n)}$ open if and only if A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)).

Proof: Let A be $\tilde{\mu}_{(m,n)}$ open in (X, μ_1, μ_2). Then there exists a μ_m-open set U such that U⊆A ⊆ $c_{s_{\mu_n}}$(U). Since U is μ_m-open then U = i_{μ_m}(U) ⊆ i_{μ_m}(A). Then A⊆ $c_{s_{\mu_n}}$(U) ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(U)) ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)). Thus A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)).
Conversely, let A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)) and take U = i_{μ_m}(A). Then i_{μ_m}(A) ⊆ A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)). Hence A is $\tilde{\mu}_{(m,n)}$ open in X, μ_1, μ_2).

Theorem 3.6 Every μ_m-open set in X is $\tilde{\mu}_{(m,n)}$-open in a BGTS (X, μ_1, μ_2).

Proof : Let A be a μ_m-open set in X. Then i_{μ_m}(A) = A. Since A ⊆ $c_{s_{\mu_n}}$(A) then A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)). By Theorem 3.5, A is $\tilde{\mu}_{(m,n)}$ open in (X, μ_1, μ_2).

Remark 3.7 The converse of Theorem 3.6 is not true.

To see this, consider Example 3.2. Note that X and {a,b} are $\tilde{\mu}_{(1,2)}$-open sets but not μ_1-open sets.

Definition 3.8 [1] A subset A of a BGTS (X, μ_1, μ_2) is said to be $\mu_{(m,n)}$-semi-open if A ⊆ c_{μ_n}(i_{μ_m}(A)).

Theorem 3.9 Every $\tilde{\mu}_{(m,n)}$-open set is $\mu_{(m,n)}$-semi-open.

Proof : Let A be a $\tilde{\mu}_{(m,n)}$-open set. By Theorem 3.5, A⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)) where $c_{s_{\mu_n}}$(i_{μ_m}(A)) is μ_n-closed. Since every μ_n-closed set is μ_n-semi-closed and that $c_{s_{\mu_n}}$(i_{μ_m}(A)) is the smallest μ_n-semi-closed set containing i_{μ_m}(A), then $c_{s_{\mu_n}}$(i_{μ_m}(A)) ⊆ c_{μ_n}(i_{μ_m}(A))This implies that A⊆ c_{μ_n}(i_{μ_m}(A))Thus A is $\mu_{(m,n)}$-semi-open.

Remark 3.10 The converse of the above theorem need not be true as shown in the following example.

Example 3.11 Consider the GTs in Example 3.2. Then the $\mu_{(2,1)}$-semi-open sets are ∅, X, {a}, {b} and {a,b}. Notice that {b} is $\mu_{(2,1)}$-semi-open but not $\tilde{\mu}_{(2,1)}$-open.

Lemma 3.12 Let (X, μ) be a GTS and $\{A_\alpha : \alpha \in$ J} be the collection of A⊆X. Then $\bigcup_{\alpha\in J} c_{s_\mu}(A_\alpha) \subseteq c_{s_\mu}(\bigcup_{\alpha\in J} A_\alpha)$.

Proof: Let x∈ A_α. Since $A_\alpha \subseteq \bigcup_{\alpha\in J}(A_\alpha)$, it follows that x ∈ $\bigcup_{\alpha\in J}(A_\alpha)$. Note that x∈ $c_{s_\mu}(A_\alpha)$. Thus x∈ $\bigcup_{\alpha\in J} c_{s_\mu}(A_\alpha)$. By Lemma 3.4, $c_{s_u}(A_\alpha) \subseteq c_{s_u}(\bigcup_{\alpha\in J}(A_\alpha))$. This implies that x∈ $c_{s_u}(\bigcup_{\alpha\in J}(A_\alpha))$. This shows that,$\bigcup_{\alpha\in J} c_{s_\mu}(A_\alpha) \subseteq c_{s_\mu}(\bigcup_{\alpha\in J} A_\alpha)$.

Theorem 3.13 Let $\{A_\alpha : \alpha \in$ J} be the collection of$\tilde{\mu}_{(m,n)}$-open sets in (X, μ_1, μ_2). Then $\bigcup_{\alpha\in J} A_\alpha$ is also$\tilde{\mu}_{(m,n)}$-open set in (X, μ_1, μ_2).

Proof: Since A_α is $\tilde{\mu}_{(m,n)}$-open, there exists a μ_m-open set U_α of X such that $U_\alpha \subseteq A_\alpha \subseteq c_{s_{u_n}}(U_\alpha)$. This implies that

$$\bigcup_{\alpha\in J} U_\alpha \subseteq \bigcup_{\alpha\in J} A_\alpha \subseteq c_{s_{u_n}}(U_\alpha) \subseteq c_{s_{u_n}}(\bigcup_{\alpha\in J} U_\alpha).$$

Since U_α is μ_m-open, $\bigcup_{\alpha\in J} U_\alpha$ is also μ_m-open. Then, $\bigcup_{\alpha\in J} A_\alpha$ is $\tilde{\mu}_{(m,n)}$-open set in (X, μ_1, μ_2).

Corollary 3.14 The family of $\tilde{\mu}_{(m,n)}$-open sets forms a generalized topology on X.

Proof: Follows from Remark 3.3(ii) and Theorem 3.13

Remark 3.15 If A and B are two $\tilde{\mu}_{(m,n)}$-open sets in (X, μ_1, μ_2), then A∩B need not be $\tilde{\mu}_{(m,n)}$-open in (X, μ_1, μ_2) as shown in the following example.

Example 3.16 Let X= {a,b,c} and consider the following GTs: μ_1 = {∅, X {a,b}, {b,c}} and μ_2 = {∅, {b}, {c}, {b,c}}. Then the $\tilde{\mu}_{(1,2)}$-open sets are ∅, X, {a,b} and {b,c}. See that {a,b}∩{b,c}= {b} but {b} is not $\tilde{\mu}_{(1,2)}$-open in (X, μ_1, μ_2).

Theorem 3.17 Let A be a $\tilde{\mu}_{(m,n)}$-open set and let B be any set such that A⊆B⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)). Then B is also a$\tilde{\mu}_{(m,n)}$-open set in (X, μ_1, μ_2).

Proof: If A is a $\tilde{\mu}_{(m,n)}$-open set then by Theorem 3.5 A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(A)). Since A⊆B, it implies that $c_{s_{\mu_n}}$(i_{μ_m}(A)) ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(B)). This shows that B is$\tilde{\mu}_{(m,n)}$-open.

Let the family of $\tilde{\mu}_{(m,n)}$-open sets (resp. $\tilde{\mu}_{(m,n)}$closed sets) of X be denoted as $\tilde{\mu}_{(m,n)}$O(X) (resp. $\tilde{\mu}_{(m,n)}$C(X)).

Remark 3.18$\tilde{\mu}_{(1,2)}$O(X) is generally not equal to $\tilde{\mu}_{(2,1)}$O(X).

To see this, consider Example 3.2. Then $\tilde{\mu}_{(1,2)}$O(X) = {∅, X, {a}, {a,b}, {a,c}, {c}} and $\tilde{\mu}_{(2,1)}$O(X) = {∅, X, {a}, {a,b}}. Thus $\tilde{\mu}_{(1,2)}$O(X)≠ $\tilde{\mu}_{(2,1)}$O(X).

Theorem 3.19 Let (X, μ_1, μ_2) be a BGTS and let A⊆X. A is $\tilde{\mu}_{(m,n)}$-closed if and only if $i_{s_{\mu_n}}$(c_{μ_m}(A)) ⊆A.

Proof: If A is $\tilde{\mu}_{(m,n)}$-closed then X\A is $\tilde{\mu}_{(m,n)}$-open. By Theorem 3.5 it follows that X\A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(X\A)). Thus X\A ⊆ $c_{s_{\mu_n}}$(i_{μ_m}(X\A)) = $c_{s_{\mu_n}}$(X\c_{μ_m}(A)) = X\

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$i_{s_{\mu_n}}(c_{\mu_m}(A))$. Hence, X\A⊆ X\ $i_{s_{\mu_n}}(c_{\mu_m}(A))$, which implies that $i_{s_{\mu_n}}(c_{\mu_m}(A))$ ⊆A. Conversely, suppose that $i_{s_{\mu_n}}(c_{\mu_m}(A))$ ⊆A. Then X\A⊆ X\ $i_{s_{\mu_n}}(c_{\mu_m}(A))$ = $c_{s_{\mu_n}}(X \backslash c_{\mu_m}(A))$= $c_{s_{\mu_n}}(i_{\mu_m}(X \backslash A))$. Thus X\A ⊆ $c_{s_{\mu_n}}(i_{\mu_m}(X \backslash A))$. Therefore, X\A is $\tilde{\mu}_{(m,n)}$-open implying that A is $\tilde{\mu}_{(m,n)}$-closed in (X, μ_1, μ_2).

Theorem 3.20 Let (X, μ_1, μ_2) be a BGTS and let A⊆X. If $i_{s_{\mu_n}}(F)$ ⊆A⊆F, then A is $\tilde{\mu}_{(m,n)}$-closed for any μ_m-closed set F in (X, μ_1, μ_2).

Proof: Let $i_{s_{\mu_n}}(F)$ ⊆A⊆F, where F is μ_m-closed. The X\F ⊆X\A ⊆X\$i_{s_{\mu_n}}(F)$ ⊆ $c_{s_{\mu_n}}(X \backslash F)$. Let U=X\F. Then U is μ_m-open and U ⊆X\A⊆ $c_{s_{\mu_n}}(U)$. Then X\A is$\tilde{\mu}_{(m,n)}$-open. Hence, A is $\tilde{\mu}_{(m,n)}$-closed.

Remark 3.21 The converse of the above theorem is not true as can be seen in the following example.

Example 3.22 Let X = {a,b,c,d} with two generalized topologies μ_1 = {∅, {a}, {a,b}, {a,c}, {a,b,c}} andμ_2 = {∅, X, {a}, {b} {a,b}}. Then the $\tilde{\mu}_{(1,2)}$-closed sets are ∅, X, {b,c,d}, {c,d}, {d}, {c}, {b,d} and {b}. See that the $\tilde{\mu}_{(1,2)}$-closed set {b} is contained in μ_1-closed set {b,d} but the $i_{s_{\mu_2}}$({b,d}) is not contained in {b}.

Theorem 3.23 Let $\{A_\alpha : \alpha \in J\}$ be the collection of $\tilde{\mu}_{(m,n)}$-closed sets in the BGTS (X, μ_1, μ_2). Then $\bigcap_{\alpha \in J} A_\alpha$ is also $\tilde{\mu}_{(m,n)}$-closed in (X, μ_1, μ_2).

Proof: Let A_α be $\tilde{\mu}_{(m,n)}$-closed in (X, μ_1, μ_2). Then X\$A_\alpha$ is $\tilde{\mu}_{(m,n)}$-open. By Theorem 3.13, $\bigcup_{\alpha \in J} X \backslash A_\alpha$ is also$\tilde{\mu}_{(m,n)}$-open. This goes to show that$\bigcup_{\alpha \in J} X \backslash A_\alpha$ = X\$\bigcap_{\alpha \in J} A_\alpha$ is $\tilde{\mu}_{(m,n)}$-open. Hence $\bigcap_{\alpha \in J} A_\alpha$ is $\tilde{\mu}_{(m,n)}$-closed in (X, μ_1, μ_2).

Theorem 3.24 Let (X, μ_1, μ_2) be a BGTS and A⊆X. Then
(i) $i_{s_{\mu_n}}(c_{\mu_m}(A))$ is $\tilde{\mu}_{(m,n)}$-closed;
(ii) $c_{s_{\mu_n}}(i_{\mu_m}(A))$ is $\tilde{\mu}_{(m,n)}$-open.

Proof: Let A⊆X.
(i)Then the $i_{s_{\mu_n}}(c_{\mu_m}(i_{s_{\mu_n}}(c_{\mu_m}(A))))$⊆ $i_{s_{\mu_n}}(c_{\mu_m}(c_{\mu_m}(A))$.
By Theorem 3.19 $i_{s_{\mu_n}}(c_{\mu_m}(A))$ is $\tilde{\mu}_{(m,n)}$-closed.
(ii)The $c_{s_{\mu_n}}(i_{\mu_m}(A)) = c_{s_{\mu_n}}(i_{\mu_m}(i_{\mu_m}A)))$

$$\subseteq c_{s_{\mu_n}}(i_{\mu_m}(c_{s_{\mu_n}}(i_{\mu_m}A)))).$$

Thus by Theorem 3.5 $c_{s_{\mu_n}}(i_{\mu_m}(A))$ is $\tilde{\mu}_{(m,n)}$-open.

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