Drag on a Micropolar Flow Past a Cylinder Specifying Uniform Velocity away from the Boundaries

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Abstract: A study of the effect of drag force on an micropolar flow past a cylinder specifying uniform velocity away from the boundaries. We find a similarity solution, assuming the fluid outside the cylinder and satisfies the Eringen’s micro polar equations and applying no slip condition at the cylinder of the surface. An appearance for drag force is obtained. It is found that the increase in the coupling parameter with fixed coupling stress parameter is to increase drag. Further a reversed behavior is noticed that the drag is decreases and the same is represented graphically.

Keywords: micropolar flow, coupling stress parameter, Drag, Eringen’s micropolar equations

1. Introduction

The earliest formulation of a general theory of microcontinua is accredited to Eringen [1] has considered as fluids with deformable microelements. Eringen’s [2] ‘micropolar fluid theory’ is based on the assumptions that the deformation of the fluid microelements is very small. This theory is still capable of taking into account the effect of microrotational surface and body couples. The evaluation of uniform flow past a cylindrical shell in Newtonian stokes flow has been extensively investigated in the literature, because of its application in lubrication theory, transpiration cooling and other important applications. The stokes uniform flow past a porous sphere has also been investigated by several authors with the assumption of axisymmetric flow (Padmavati et al [3], Berman [4], Rudraiah et al [5]).

Although researchers started working on this area when Happel[6, 7] and Kuwabara [8] take geometry of cell exterior and to be the identical (cylindrical/spherical) and provide disappearance of shear stress and disappearance of revolution at cell surface correspondingly. Micropolar fluids are of attention in the general context of non-Newtonian fluid mechanics. Some purpose ranges from flow of blood and blood-like fluids [9], suspensions of rigid particles in Newtonian fluids [10, 11], liquid crystals, granular fluids [12] and hydrodynamic turbulence [13].

However in environmental pollution problems, particularly in water pollution problem, it is central to consider the effects of suspended particles on the flow past a cylinder. The effect of these suspended particles may be taken into account either using Eringen’’s micropolar fluid model or using Saffman dusty fluid model. The Saffman dusty fluid model does not much importance of the effect of micro rotation of balanced particles unless we consider principal of angular momentum in addition to linear momentum. The micropolar fluid model has built in mechanism of taking care of micro rotation.

The recently Deo, Yadav and Tiwari [14] studied flow past a swarm of particles of cylindrical geometry with Happel’s formulation and also studied the cause of different parameters such as particle volume fraction on flow pattern. Complete study of cell model functional to the micropolar flow along and at right angles to cylindrical particles is given in the work of Sherief et al. [15]. The core of the cell was considered to be solid, slip conditions for linear and angular velocities were set on its surface. The growth of the cell model to the viscous spheroid in micropolar spheroidal shell is agreed in the work [16].

Presently the analytical study of micropolar fluid flow past an impermeable cylinder specifying identical velocity far from the boundaries. The expression for drag force is determined. The evaluation of drag coefficient on non-dimensional coupling parameter N1 and coupling stress parameter N3 is discussed and presented graphically

2. Mathematical Formulation

Consider a steady incompressible micropolar fluid flow past an impervious cylinder of radius ‘a’ embedded in a sparsely packed porous medium. The schematic representation is show in figure 1. Under assumptions and approximations made together with governing by the equations of continuity, conservation of momentum and Conservation of angular momentum

\[ \nabla \cdot \hat{q} = 0 \quad (1) \]

\[-\nabla p + \zeta (\nabla \times \vec{\omega}) + (\zeta + \eta) \nabla^2 \hat{q} = 0 \quad (2) \]

\[\zeta (\nabla \times \hat{q}) - 2\zeta \hat{\omega} + (\zeta + \eta) (\nabla \cdot \hat{\omega}) + \eta \nabla^2 \hat{\omega} = 0 \quad (3) \]
3. Boundary Conditions

To solve the above governing equation we considered the boundary conditions as no-slip condition given by

$$ \frac{\partial}{\partial r} \psi(r, \theta) + \frac{\partial}{\partial \theta} \psi(r, \theta) = 0 \quad \text{at} \quad r = 1. \quad (8) $$

Extreme away from the cylinder the flow has uniform velocity, given by

$$ \psi(r, \theta) \sim r \sin \theta \quad \text{as} \quad r \rightarrow \infty \quad (9) $$

4. System of Solution

The boundary condition from equation (9) suggests the following similarity solution

$$ \psi(r, \theta) = f(r) \sin \theta. \quad (10) $$

Substituting equation (10) in (7) the functions $\psi(r, \theta)$ reduces to fourth order ordinary differential equation in $f(r)$ as follows:

$$ f''''(r) + \frac{2}{r} f'''(r) - \frac{3}{r^2} f''(r) - \frac{3}{r^3} f'(r) - \frac{3}{r^4} f(r) = 0 \quad (11) $$

The corresponding $f(r)$ from equation (8) and (9) reduces to: No-slip condition at the surface of the porous cylinder is given by

$$ f(1) = 0, \quad f'(1) = 0 \quad \text{at} \quad r = 1 \quad (12) $$

Further, the uniform velocity extreme away from the boundary, from equation (10) reduces to:

$$ f(r) \sim \sim r \quad \text{as} \quad r \rightarrow \infty \quad (13) $$

We denote

$$ g(r) = f''(r) + \frac{1}{r} f'(r) - \frac{1}{r^2} f(r) \quad (14) $$

Substitution of equation (14) in equation (11), it reduces to second order ordinary differential equation in $g(r)$ as,

$$ g''(r) + \frac{1}{r} g'(r) - \left( N^2 + \frac{1}{r^2} \right) g(r) = 0 \quad (15) $$

The solution for the equation (15) is obtained as

$$ g(r) = B_1 K_1(Nr) + C_1 I_1(Nr) \quad (16) $$

From equation (10) on equation (16) we get $C_1 = 0$ and using (14) becomes

$$ B_1 K_1(Nr) = f''(r) + \frac{1}{r} f'(r) - \frac{1}{r^2} f(r) \quad (17) $$

Equation (17) is a linear differential equation with variable co-efficient; its general solution can be obtained by the method of variation of parameters and is given by:

$$ \frac{A}{r} + Br + CK_1(Nr) = f(r) \quad (18) $$

Where $A$, $B$ and $C$ are arbitrary constants to be determined using the boundary conditions (12) and (13) on (18) and hence obtained solution is:
\[ f(r) = r - \left( 1 + \frac{2K_0(N)}{N} \right) \frac{r}{r} + \frac{2}{N} \frac{K_0(N)}{K_0(rN)} \]  

Therefore equation (10) reduces to:

\[ \psi(r, \theta) = \left( r - \left( 1 + \frac{2K_0(N)}{N} \right) \frac{r}{r} + \frac{2}{N} \frac{K_0(N)}{K_0(rN)} \right) \sin \theta \]  

Equation (20) shows that function for the present problem a function of coupling parameter and coupling stress parameter

The shearing stress at any point on the surface of the cylinder is given by

\[ \tau_{r\theta} = \mu \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{r}{r} \frac{\partial v}{\partial r} \right) \]  

On dimensionless, equation (21) reduces to

\[ \frac{\tau_{r\theta}}{\mu U_\infty} = \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{r}{r} \frac{\partial v}{\partial r} \right) \]  

On the surface of the cylinder (i.e \( r = 1 \)), the shearing stress becomes

\[ \tau_{r\theta} = \frac{\mu U_\infty}{\alpha} \]  

\[ \tau_{r\theta} = 2(1 + S) \sin \theta. \]  

Where \( S = \frac{1 - 11N - 8N^2}{8N - 1} \)

5. Determination of the Drag force

The drag force \( F \) experienced by an impermeable cylinder of radius ‘a’ is defined as:

\[ F_i = 2\pi \int_0^\infty (r \sin \theta) R^2 d\theta d\phi \]  

Further, equation (23) reduces to as:

\[ F_i = 6\pi \mu U_\infty a \left( 1 + S \right) \]  

Also, the drag coefficient can be defined as

\[ C_D = \frac{-F_i}{\frac{1}{2} \rho U_\infty^2 a^2 \pi} \]  

Substitution of equation (26) in equations (27) reduces to:

\[ C_D = \frac{24}{R_e} \left( 1 + S \right) \]  

Where \( R_e \) is the Reynolds number and if \( S = 0 \), then equation (26) reduces to:

\[ F_i = 6\pi \mu U_\infty a \]  

This result for the drag reported earlier by Stokes [17]

Figure 2: Deviation of the Drag coefficient with coupling parameter \( N_1 \) for fixed coupling stress parameter \( N_3 = 1 \).
6. Results

In this paper we study the variation of drag coefficient with variation of coupling parameter and coupling stress parameter for the steady flow of viscous, incompressible fluid past an impermeable cylinder placed in a sparsely packed porous region. The drag experienced by a cylinder embedded in porous medium, using the no-slip condition at the solid surface and uniform shear flow far away from the region as the boundary conditions. Also, the expression for the drag co-efficient \( C_D \) is obtained.

From figure 2 we noticed that the drag co-efficient \( C_D \) increases with increase coupling parameter \( N_1 \) for fixed...
coupling stress parameter \( N_3 \) =1 and figure 3 obtain drag coefficient \( C_D \) decreases with increase \( N_1 \) between 0.2 to 0.5 for fixed \( N_3 = 5 \). But drag coefficient \( C_D \) increases with increase \( N_1 \) between 0.6 to 1 for fixed \( N_3 = 5 \) near the solid surface and maintains asymptotic behavior away from the surface. Further, figures 4 shows that the increase coupling stress parameter \( N_3 \), drag coefficient \( C_D \) decreases for fixed coupling parameter \( N_1 \) =0.7 and figure 5 obtain drag coefficient \( C_D \) decreases with increase \( N_1 \) between 1 to 5 for fixed \( N_3 \) =0.5. But drag coefficient \( C_D \) increases with increase \( N_3 = 6 \) an words for fixed \( N_1 = 0.5 \)

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