Automatic Control and System Theory 1

Ninad Kiran Magdum

Abstract: ***System is a structure that consists of interrelated and interdependent elements, which influence one another to maintain their working and the existence of the system, in order to achieve the goal of the system. All systems have input, output and feedback in order to maintain an internal steady-state even if external environment tends to change. System is generally used in many sectors, like process control, data elaboration, biology, economy, ecology, management, traffic control, etc.***

Keywords: automation

1. History

There is fascinating history for the use of feedback to control. Float regulator mechanism in Greece appeared to be first application of feedback control in the period 300 to 1 B.C. [1, 2, 3]. An oil lamp devised by Philon in approximately 250 B.C. used a float regulator in an oil lamp for maintaining a constant level of fuel oil. A book entitled 'Pneumetica' outlined several forms of water level mechanisms using float regulators, published by Heron of Alexendria, who lived in first century A.D.

The temperature regulator was the first feedback system to be invented in modern Europe of Cornelis Drebbel (1572-1633) of Holland. [1] Dennis Papin invented the first pressure regulator for steam boilers in 1681.

James Watt's 'Fly ball Governor' is agreed to be first automatic feedback controlled used in industrial process for controlling the speed of a steam engine. Russia claimed that the first historical feedback system is the water level float regulator invented by I.Polzunov. The next century was characterized by the development of automatic control through intuition and invention. To increase the accuracy of the control system many efforts led to slower attenuation of the transient oscillation and event to unstable systems. It then became compulsory to develop the theory of Automatic Control. In 1868, J.C. Maxwell derived a theory using differential equation model of a governor. Maxwell concentrated on the study of effect of various system parameters on the system performance.

2. System

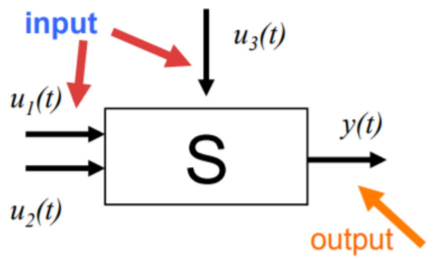
2.1 What is system?

System is a structure that consists of interrelated and interdependent elements, which influence one another to maintain their working and the existence of the system, in order to achieve the goal of the system. All systems have input, output and feedback in order to maintain an internal steady-state even if external environment tends to change. System is generally used in many sectors, like process control, data elaboration, biology, economy, ecology, management, traffic control, etc.

2.2 System Theory

System theory is one of the theories that focus on the arrangement of and relations between the parts which connect an entity and its properties, rather than reducing an entity to the properties of its parts or elements. Goal of system theory is to mathematically represent system in order to understand system's main physical properties and to design a proper control system.

Consider an oriented system



Above system consists of input variables (causes) and output variables (effects).

3. System and Models

Static and Dynamic system

3.1 Static System

Static system is defined as system in which output at any instant of time depends on input sample at the same time. Eg:- Y(n)= 5X(n).

Here, 5 is constant which multiplies input X(n). Y(n) depends on the input at (nth) time instant X(n). Hence, it is known as static system. Static system is memory less system. Observe above example i.e input output relation of static sytem, output does not depend on delayed [x (n-k)] or advanced[x (n+k)] input signals. It depends on (nth) input signal. Signals should be stored in memory to calculate the output at nth instant if output depends upon delayed input signals. Hence, for static systems, memory is not required and static system is memory less.

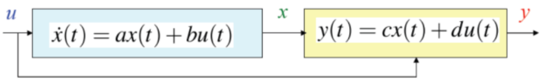
3.2 Dynamic System

Dynamic system is defined as system in which output at any instant of time depends on input sample at the same time as well as at other times. Here other time is nothing but time different than present time instant which may be past or future time instant.

If X(n)= input signal at present instant, then
1) X(n-k)= Delayed input signal or past signal.
2) X(n=k)= Advanced input signal or future signal.

If the output depends on the present time instant only, then a continuous time system is said to be static system.

$$\dot{x}(t) = ax(t) + bu(t)$$
$$y(t) = cx(t) + du(t)$$



Given two equation shows the description of the system, a differential and an algebraic one.

The State Space Analysis

All possible sets of states of the system are called as state space of dynamic system. A state vector is used to identify the position and velocity of an object in any location on earth's surface. State space representation includes a mathematical model of a physical system as a set output and state variables. A set of variables used to describe the mathematical 'state' of a dynamic system are known as state variables. Future behaviour of the system in the absence of any external forces affecting the system can be determined with the help of the state. [5]

The state space representation of a system is given by two equations:

$$\dot{\mathbf{q}}(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{q}(t) + \mathbf{D}\mathbf{u}(t)$$

First equation is called as state equation, second equation is called as output equation. For an n^{th} order system (i.e., it can be represented by an n^{th} order differential equation) with *r* inputs and *m* outputs the size of each of the matrices is as follows:

- **q** is nx1 (n rows by 1 column); **q** is called the state vector, it is a function of time
- **A** is nxn; **A** is the state matrix, a constant
- **B** is nxr; **B** is the input matrix, a constant
- **u** is rx1; **u** is the input, a function of time
- **C** is mxn; **C** is the output matrix, a constant
- **D** is mxr; **D** is the direct transition (or feed through) matrix, a constant
- **y** is mx1; **y** is the output, a function of time

If we know the state of an object, then we know everything there is to know about it. When a designer describes a object, if he/she finds that its attributes are not sufficient to specify the state completely, then they might have overlooked attributes. [4]

Main Analysis Problem in System Theory

Motion Analysis- Motion analysis processing is the simplest case to find the motion of the system i.e to find the point in image where something is moving. More complex type of processing can be used to track a specific object over period of time.

Controllability Analysis- It is the analysis of the ability to achieve acceptable performances or how to have an effect on state motion which has a variable input. [6]

Observability Analysis- For suitable estimation of state of the system for advanced control over a course of time period observability analysis is used.

Sensitivity Analysis- Study of the influence on the state/ output evolution of changes in the initial state, of the input function, of the system's parameters.

Main synthesis problem in system theory:-
1) Input Synthesis
2) Synthesis of the input and of initial state
3) Control Synthesis

State

Dynamic System0: - It is the information of the internal condition of a dynamic system which helps to predict the effect of history of the system itself on its future behaviour which is needed in each instant.

The internal condition in physical system is recognized by and energy storage or momentum or mass. Hence states variables must be choose related to these storages. Eg- the voltage of a capacitor or an inductor in an electrical circuit or the velocity of mass in mechanical system. [2]

The state variables and state equations are NOT defined uniquely.

Considerations related to storage:-

- Electrical=Capacitor (C) and Inductor (L)
- Mechanical (linear) = Mass (M) and compliance (1/K)
- Mechanical (rotational) = Inertia (J) and rotational compliance (1/K)
- Fluid flow (hydraulic/pneumatic) = Fluid Capacitor (Cf) and Fluid Inductor (Lf)
- Thermal = Thermal Capacitor (Ct)

Domain	"capacitive" storage	"inductive" storage
electrical	$E = \frac{1}{2}Cv^2$	$E = \frac{1}{2}Li^2$
mechanical (linear)	$E = \frac{1}{2}Mv^2$	$E = \frac{1}{2}\frac{1}{K}f^2$
mechanical (rotational)	$E = \frac{1}{2}J\omega^2$	$E = \frac{1}{2}\frac{1}{K}c^2$
hydraulic/pneumatic	$E = \frac{1}{2}C_f p^2$	$E = \frac{1}{2}L_f q^2$
thermic	$E = C_t T$	Non present

The stored energy depends on the → Effort variables   Flow variables

References

[1] https://mitpress.mit.edu/books/origins-feedback-control
[2] Mayr, Otto. "The Origins of Feedback Control." *Scientific American*, vol. 223, no. 4, 1970, pp. 110–119., www.jstor.org/stable/24927641.
[3] Mayr, Otto. 1971. "Feedback Mechanisms in the Historical Collections of the National Museum of History and Technology." *Smithsonian Studies in History and Technology*. 1–133. https://doi.org/10.5479/si.00810258.12.1
[4] https://www.cs.uct.ac.za/mit_notes/software/htmls/ch05s08.html

[5] William J. Palm III (2010). System Dynamics (2nd ed.). p. 225.
[6] http://folk.ntnu.no/skoge/publications/thesis/2003_faanes/thesis_faanes_aug03.pdf