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On Various Properties of $\delta -$ Convergence of Nets

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Abstract: ***The notion of convergence is most important concepts in topology. Nets and Filters are two notions to fill this need for introducing the concept of convergence in more general setting of topological space. By introducing the concept of δ − convergence of net we have proved various results.***

1. Introduction

In 1922, E.H.Moore and H.L. Smith developed a generalized version of sequence to produce the notion of nets. Although they lead to essentially equivalent theories, the nets have the advantage that they are very natural and are direct generalization of sequences with which, we are all too familiar.

Jong Suh Park in the paper [7] has got many interesting results related with H-closed spaces. By using the notion of **σ**-continuous maps, -closure, **ω**-accumulation point etc. various results are proved concerned with these concepts

In this paper we have introduced the concept of $\boldsymbol{\delta} -$ convergence and have got many results. While proving the theorems many concepts like $\boldsymbol{\delta} -$ convergence of nets, $\boldsymbol{\delta} -$ cluster point of nets.

Throughout the paper, spaces are topological spaces, symbols X, Y, Z are used for topological spaces and f, g, h are used for maps between topological spaces. For terms and notation not explained here we refer the reader to [3, 8, 9].

2. δ − convergence of nets

The section begins with the following definition.

2.1 Definition

Let X be a space . A net (x_i) in X is said to be $\boldsymbol{\delta} -$ accumulate to a point x of X denoted by $x_i \underset{\alpha}{\overset{\delta}{}} x$ if for any neighbourhood U of X and here is an $i_1 \geq i$ such that such that x_{i_1} ∈ Int Cl U.

2.2 Definition

Let X be a space. A net (x_i) in X is to $\boldsymbol{\delta} -$converge to a point x of X denoted by $x_i \overset{\delta}{\rightarrow} x$ if for each neighbourhood U of x there is an i_1 such that $x_i \in$ Int Cl U for $i \geq i_1$.

2.3 Definition

A space X is called δ-compact spaces if for each open cover $\{U_i\}$ of X there are finitely many i_k such that X=$\bigcup_{K=1}^{n} Int\ Cl\ \ (U_{i_k})$.

2.4 Definition

Let X be a space .For a subset A of X the weak closure of A denoted by Cl$\boldsymbol{\omega}^*$ (A) is defined by the set
Cl$\boldsymbol{\omega}^*$ (A)={x ∈ X |A ∩ Int Cl U $\neq \phi$ for all open neighbourhoods U of x}.

2.5 Lemma

Let X be a space and A$\subseteq X$.Then x ∈Cl$\boldsymbol{\omega}^*$ (A) if and only if there is a net (x_i) of points of A, $\boldsymbol{\delta} -$converging to a point x ∈ X.

Proof: Assume that x ∈Cl$\boldsymbol{\omega}^*$ (A).Then A ∩ Int Cl U_i of x in X. Consider the family $\eta_x$of all neighbourhoods of x with the reverse order inclusion and define a net in X as follows:
S:$\eta_x \rightarrow X$ by
S $(U_i)=x_i$ where $x_i \in$ A ∩ Int Cl U_i.
Then (x_i) is a net of points of A and $x_i \overset{\delta}{\rightarrow} x$.Conversely assume that $x_i \overset{\delta}{\rightarrow} x$.For a neighbourhood U of x, there exists i_1 such that $x_i \in$ Int Cl U_i for all $i \geq i_1$.Since $x_i \in$ A for all i, we have
A ∩ Int Cl U $\neq \phi$.Thus x ∈Cl$\boldsymbol{\omega}^*$ (A).

2.6 Definition

A space X is called $\delta - Hausdorff$ if for any two distinct points x and y of X there are open neighbourhoods U of x and V of y such that
Int Cl U∩Int Cl V= ϕ

2.7 Lemma

Let X be a δ-compact space.Then for each net (x_i)
In X there is an x ∈ X such that $x_i \underset{\alpha}{\overset{\delta}{}} x$.

Proof: Suppose that (x_i) has no $\boldsymbol{\delta} -$ accumulate point in X .Then for all x ∈ X. For each x ∈ X there is a neighbourhood U_x of x and i_x such that $x_i \notin$ Int Cl U_x for all $i \geq i_x$.

Then $\{U_x | x \in X\}$ is an open cover of X.Since X is $\boldsymbol{\delta} -$compact, there are finitely many x_k such that
X=$\bigcup_{K=1}^{n} Int\ Cl\ U_{X_K}$.
Choose j such that j≥ $i_{x_k}$for all k=1, 2, …., n. Conclude from above that $x_j \notin \bigcup_{K=1}^{n} Int\ Cl\ U_{X_K}$for all k=1, 2, …., n or $x_j \notin \bigcup_{K=1}^{n} Int\ Cl\ U_{X_K}$ .This contradiction shows that (x_i) has necessarily a $\boldsymbol{\delta}$-cluster point in X.

2.8 Definition

A function f: $X \to Y$ is called $\boldsymbol{\delta}$-continuous at a point if for each neighbourhood U of f (x) there is a neighbourhood V of x such that f (Int Cl V)⊂Int Cl U.

2.9 Theorem

Let X, Y be a spaces. A function f: $X \to Y$is $\boldsymbol{\delta}$-continuous at x ∈ X if and only if for any net (x_i) in X satisfying $x_i \overset{\delta}{\to} x$, the net f (x_i) $\overset{\delta}{\to}$ (x) in Y.

Proof: Given any neighbourhood U of f (x), there is a neighbourhood V of x such that f (Int Cl V)⊂Int Cl U.Also there is an i_1 such that x_i∈Int Cl V for all i≥i_1 we have
f (x_i) $\overset{\delta}{\to}$ (x).

Conversely, assume that f is not $\boldsymbol{\delta}$-continuous at x. Then there is a neighbourhood U of f (x) such that f (Int Cl V)⊄Int Cl U for all neighbourhood V of x. Let (V_i) be the family of neighbourhood of x with the reverse inclusion order. For each i,
f (Int Cl V)⊄Int Cl U

There is an x_i∈ Int Cl V_i such that f (x_i) ∉ Int Cl U. Then the net (x_i) in X $\boldsymbol{\delta}$-converges to x but the net (f (x_i)) in Y does not $\boldsymbol{\delta}$-converge to f (x).Thus we have a contradiction. Hence f is $\boldsymbol{\delta}$-T

2.10 Definition

Let X, Y be spaces. A function f:X→Y is said to have $\boldsymbol{\omega}$*-closed graph if its graph G (f)={ (x, f (x))|x ∈ X} is a $\boldsymbol{\omega}$*-closed subset of X × Y.

2.11 Theorem

Let X and Y be a spaces. A function f: $X \to Y$has a $\boldsymbol{\omega}$*-closed graph if and only if any net (x_i) in X, $x_i \overset{\delta}{\to} x$ ∈ X and f (x_i) $\overset{\delta}{\to}$y ∈ Y implies y= f (x).

Proof: Assume that f: $X \to Y$ has $\boldsymbol{\omega}$*-closed graph. Since (x_i, f (x_i)) is a net in G (f) and (x_i, f (x_i))$\overset{\delta}{\to}$ (x, y), we have
(x, y) ∈ Cl$\boldsymbol{\omega}$* G (f)= G (f).
Thus y= f (x).
Conversely, assume that (x, y) ∈ Cl$\boldsymbol{\omega}$* G (f)
Then there is a net (x_i) in X such that (x_i, f (x_i))$\overset{\delta}{\to}$ (x, y).
Since $x_i \overset{\delta}{\to} x$and f (x_i) $\overset{\delta}{\to}$f (x), y=f (x).
Thus (x, y) ∈ G (f).
Hence G (f) $\boldsymbol{\omega}$*-closed.

2.12 Lemma

Let X be a space. If an ultranet (x_i) of X. $\boldsymbol{\delta}$-accumulates to a point x of X then (x_i)$\boldsymbol{\delta}$-converges to x.

2.13 Theorem

Let X be a $\boldsymbol{\delta}$-compact space then every net in X has a $\boldsymbol{\delta}$-convergent subnet.

2.14 Theorem

A topological space is $\boldsymbol{\delta}$-Hausdorffiff limits of all nets in it are unique.
Proof: Suppose that X is a $\boldsymbol{\delta}$-Hausdorff, S:D→ X is a net in X and S:D→ $X$$\boldsymbol{\delta}$-converge to x and y in X. To show that x=y. Suppose on the contrary that is x≠ y, then there exist open sets U, V such that
$x \in U$, y∈ V and Int Cl U ∪ Int Cl V = ∅. Since S:D→ X $\boldsymbol{\delta}$-converge to both x and y, there exists m_1, m_2∈D such that for all n∈D, n≥$m_1$implies S (n) ∈ Int Cl U and n≥m_2 implies S (n) ∈ Int Cl V .Now because D is a directed set, there exists
n∈D such that, n≥m_1 and n≥m_2.
But then S (n) ∈Int Cl U ∩ Int Cl V, a contradiction .So x=y establishing the necessity of the condition.

Conversely, assume that the limits of all nets in a space X are unique .If X is not $\boldsymbol{\delta}$-Hausdorff then there exist two distinct points x, y in X which do not have mutually disjoint neighbourhoods in X. Letη_x, η_y be the neighbourhood system in X at x and y respectively .

Let D= $\eta_x \times \eta_y$ and for (U_1, V_1), (U_2, V_2) ∈ D, define
(U_1, V_1) ≥ (U_2, V_2) iff U_1⊂ U_2 and V_1⊂ V_2.This makes D is a directed set and we define a net S:D→ X as follows, for any
U ∈ η_x and V ∈ η_y
We know that
Int Cl U ∩ Int Cl V ≠ ∅.

Define S (U, V) to be any point in Int Cl U ∩ Int Cl V, we claim that the net S:D→ X so defined $\boldsymbol{\delta}$-converges to x. For let G be an open neighbourhood of x .Then (G, X) ∈ D. Now if
(U, V) ≥ (G, X) in D then U⊂ G and so
S (U, V) ∈ Int Cl U ∩ Int Cl V ⊂ Int Cl U ⊂ Int Cl G.
S (U, V) ∈ Int Cl G for (U, V) ≥ (G, X).

Thus S:D→ X so defined $\boldsymbol{\delta}$-converges to x. Similarly S:D→ X so defined $\boldsymbol{\delta}$-converges to y, also contradicting the hypothesis. So X is $\boldsymbol{\delta}$-Hausdorff.

2.16 Theorem

Let S:D→ X be a net in a topological space and let x∈X . Then x is a $\boldsymbol{\delta}$-cluster point of S:D→ X if there exists a subnet of S:D→ X which $\boldsymbol{\delta}$-converges to x in X.

Proof: It is given that S:D→ X be a net in a topological space and a subnet T:E → X of S:D→ X which $\boldsymbol{\delta}$-converges to x in X.
To show that x is a $\boldsymbol{\delta}$-cluster point of S, take a neighbourhood U of x∈X and let m_1∈ D be given .Then there exists p∈E such that for all m∈E, m ≥ p implies N (m) ≥ m_1 .Also because T:E → $X$$\boldsymbol{\delta}$-converges to x, there exists

q∈E such that for all m∈E, m ≥ q implies T (m) ∈Int Cl U i.e. S (N (m)) ∈Int Cl U.

Now choose m∈E such that m ≥ p and m ≥ q and let n=N (m).Then n ≥ m_1 and S (n) ∈Int Cl U. Since m_1 and U were arbitrary, it follows that x is a $\boldsymbol{\delta}$-cluster point of S.

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