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Flow of Non-Newtonian Fluid between Two Parallel Plates Due to a Suddenly Moved Plate

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Abstract: ***The aim of this paper is to study the flow of a visco – elastic liquid due to a plate which suddenly starts oscillating in the presence of another parallel stationary plate. The Visco – elastic liquid considered is of Oldroyd type.***

Keywords: Visco – elastic liquid, Oldroyd type, parallel stationary plate

1. Introduction

The abundance of literature deals with the flows of viscous fluid by employing Navier – Stokes equation. But there are many fluids with complex Microstructure that cannot be described by the Navier–Stokes equation. These fluids which exhibit a non- linear relationship between the stresses and the rate of strain are called non- Newtonian fluids. Non-Newtonian fluids such as paints, grease, oils, blood, liquid polymers glycerin etc. are frequently encountered in many disciplinary fields including chemical engineering, foodstuff, biomedicine etc. in comparison with Newtonian fluids. The departure from the Newtonian behavior manifests itself in a variety of ways [6]. Non- Newtonian viscosity, stress – relaxation, non- linear creeping, development of normal stress differential and yield stress. Numerous models have been proposed to describe the response characteristic of these fluids. Among them the Oldroyed –B model had some success in describing the response of some polymeric liquid. This model is more amenable to analysis and more importantly experimental. As a result many papers regarding these fluids have appeared in the last time [1-5],Rajgopal [7], and Wood [9].The aim of this paper is to study the flow of a visco – elastic liquid due to a plate which suddenly starts oscillating in the presence of another parallel stationary plate. The Visco – elastic liquid considered is of Oldroyd type. Oldroyd considered non –Newtonian liquids for which the stress tensor S_{ik} and the rate of strain tensor e_{ik} are related

$$S_{ik} = p_{ik} - pg_{ik} \tag{3.1.1}$$

$$p^{ik} + \lambda_1 \frac{\delta}{\delta t} p^{ik} = 2\eta_0 \left(e^{ik} + \lambda_2 \frac{\delta}{\delta t} e^{ik} \right) \tag{3.1.2}$$

Where P is an isotropic pressure, g_{ik} the metric tensor, and t time.η_0 is a constant having the dimensions of viscosity and λ_1 and λ_2 are constant having the dimension of time . The derivative denoted by $\delta/\delta t$ is the 'convected derivative' and for any second order contra-variant tensor b^{ik} is

$$\frac{\delta}{\delta t} b^{ik} = \frac{\partial b^{ik}}{\partial t} + v^j b^{ik}_j - b^{ij} v^k_j - b^{jk} v^i_j + b^{ik} v^j_j$$

The Equation of conservation of mass and momentum for an incompressible liquid of density ρ are

$$e^i_i = 0 \ (3.1.3)$$

$$\rho \left(\frac{\partial v^i}{\partial t} + v^k v^i_k \right) = s^{ik}_k \quad . \tag{1.1.4}$$

In this paper , the flow of a visco- elastic fluid between two parallel plate has been studied when one plate is stationary and the other is suddenly start oscillating. Both finite Fourier sine transform and Laplace transform method have been applied to solve the basic differential equations. The flow phenomenon has been characterized by the parameter α, β and ω and the effect of theses on the flow characteristics have been studied through several graphs.

2. The equation of motion

Cartesian co- ordinate x, y, z are chosen such that x – axis is taken along one of the plate and the y -axis is normal to it. The plate y = 0 suddenly starts oscillating from rest in its own plane while the plate y = h is at rest. Since the plates are infinitely long all physical quantities are independent of x. So the velocity field, Consistent with the continuity equation is

$$u = u(y,t),\ v = 0 \tag{1.2.1}$$

The stress – strain – rate equation gives

$$p_{xx} + \lambda_1 \left[\frac{\partial p_{xx}}{\partial t} - p_{xy} \frac{\partial u}{\partial y} \right] = -2\eta_0 \lambda_2 \left(\frac{\partial u}{\partial y} \right)^2 \tag{1.2.2}$$

$$p_{xy} + \lambda_1 \left[\frac{\partial p_{xy}}{\partial t} - p_{yy} \frac{\partial u}{\partial y} \right] = \eta_0 \left(\frac{\partial u}{\partial y} \right) + \eta_0 \lambda_2 \frac{\partial^2 u}{\partial y \partial t} \tag{1.2.3}$$

$$p_{yy} + \lambda_1 \frac{\partial p_{yy}}{\partial t} = 0 \tag{1.2.4}$$

A particular solution of equation (1.2.4) is $p_{yy} = 0$. Hence equation (1.2.3) gives

$$p_{xy} + \lambda_1 \left[\frac{\partial p_{xy}}{\partial t} \right] = \eta_0 \left(\frac{\partial u}{\partial y} \right) + \eta_0 \lambda_2 \frac{\partial^2 u}{\partial y \partial t} \tag{1.2.5}$$

Since the flow takes place due to the shearing action of the plate y = 0 , the momentum equation (3.1.4) reduces to

$$\rho \frac{\partial u}{\partial t} = \frac{\partial p_{xy}}{\partial y} \quad , \quad 0 = -\frac{\partial p}{\partial y} \tag{1.2.6}$$

Eliminating p_{xy} between (3.2.5)and (3.2.6)and introducing the non – dimensional parameters

$$\eta = \frac{y}{h}\ ,\tau = \frac{vt}{h^2}, \alpha = \frac{\lambda_1 v}{h^2}, \beta = \frac{\lambda_2 v}{h^2}, \omega = \frac{n h^2}{v}, w = \frac{u}{U} \tag{1.2.7}$$

We get

$$\frac{\partial w}{\partial \tau} + \alpha \frac{\partial^2 w}{\partial \tau^2} = \frac{\partial^2 w}{\partial \eta^2} + \beta \frac{\partial^3 w}{\partial \tau \partial \eta^2} \tag{1.2.8}$$

Where $v = \eta_0/\rho$. This equation is to be solved with the boundary conditions:

$$\tau \le 0: w = 0 \text{ for } 0 \le \eta \le 1\ , \tag{1.2.9}$$

$$\tau > 0: w = \begin{cases} \cos\omega t & for\ \eta = 0 \\ 0 & \eta = 1 \end{cases} \tag{1.2.10}$$

Further we assume that

$$\frac{\partial \omega}{\partial \tau} = 0 \text{ when } \tau = 0 \qquad (1.2.11)$$

3. Solution of equation

To solve equation (1.2.8) in an exact form we first apply Fourier sine transform . Following Sneddon [8] we define

$$\omega^*(m,\tau) = \int_0^1 \omega(\eta,\tau) \sin(m\pi\eta)\, d\eta$$

So that

$$\omega(\eta,\tau) = 2\sum_{m=1}^{\infty} \omega^* \sin(m\pi\eta) \qquad (1.3.1)$$

Multiplying equation (1.2.8) throughout by $\sin(m\pi\eta)$ and integrating with respect to η within the limit 0 and 1 , we get using the conditions (1.2.10),

$$\alpha \frac{\partial^2 \omega^*}{\partial \tau^2} + (1+m^2\pi^2\beta)\frac{\partial \omega^*}{\partial \tau} + m^2\pi^2\omega^* = m\pi[\cos\omega\tau - \beta\omega\sin\omega\tau] \qquad (1.3.2)$$

The boundary conditions (1.2.9 -1.2.11) gives,

$$\omega^*(m,0) = \frac{\partial \omega^*(m,0)}{\partial \tau} = 0 \qquad (1.3.3)$$

Taking Laplace transform of both sides of equation (1.3.2) and (1.3.3), we get

$$\overline{\omega^*}(m,p) = \frac{\pi m(p-\beta\omega^2)}{\alpha(p^2+\omega^2)(p-\alpha_1)(p-\alpha_2)} \qquad (1.3.4)$$

where α_1 and α_2 are the roots of

$$\alpha p^2 + (1+m^2\pi^2\beta)p + m^2\pi^2 = 0 \qquad (1.3.5)$$

Taking the inverse Laplace transform of (1.3.4) we get

$$\omega^*(m,\tau) = \pi m[A\cos\omega\tau + B\sin\omega\tau + Ce^{\alpha_1\tau} + De^{\alpha_2\tau}] \qquad (1.3.6)$$

Where

$$A = \frac{\pi^2m^2(1+\beta^2\omega^2) - (\alpha-\beta)\omega^2}{(\pi^2m^2-\alpha\omega^2)^2 + (1+\pi^2m^2\beta)^2\omega^2},$$

$$B = \frac{\omega(1+\alpha\beta\,\omega^2)}{(\pi^2m^2-\alpha\omega^2)^2 + (1+\pi^2m^2\beta)^2\omega^2},$$

$$C = -\frac{A(1+\pi^2m^2\beta+\alpha\alpha_1) + B\omega\alpha}{\sqrt{(1+\pi^2m^2\beta)^2 - 4\pi^2m^2\alpha}},$$

$$D = \frac{A(1+\pi^2m^2\beta+\alpha\alpha_1) + B\omega\alpha}{\sqrt{(1+\pi^2m^2\beta)^2 - 4\pi^2m^2\alpha}}$$

Using definition (1.3.1) and simplifying we get

$$\omega(\eta,\tau) = (1-\eta)\cos\omega\tau + 2\sum_{m=1}^{\infty} \pi m[A_1\cos\omega\tau + B\sin\omega\tau + Ce^{\alpha_1\tau} + De^{\alpha_2\tau}]\sin\pi m n \qquad (1.3.7)$$

Where

$$A_1 = \frac{\omega^2[\pi^2m^2(\alpha-\beta) - (1+\alpha^2\omega^2)]}{(\pi^2m^2-\alpha\omega^2)^2 + \pi^2m^2(1+\pi^2m^2\beta)^2\omega^2}$$

Introducing the non – dimensional shearing stress

$$T_{xy} = \frac{hp_{xy}}{\eta_0 h_0}$$

We get from equation (3.3.7) the relation

$$T_{xy} + \alpha\frac{\partial T_{xy}}{\partial \tau} = \frac{\partial \omega}{\partial \eta} + \beta\frac{\partial^2 \omega}{\partial \tau \partial \eta} \qquad (1.3.8)$$

Taking the Laplace transform of equation (1.3.8) and assuming that

$T_{xy} = 0$ when $\tau = 0$,

we get

$$\overline{T}_{xy} = \frac{1+\beta p}{1+\alpha p}\frac{d\overline{\omega}}{d\eta} \qquad (1.3.9)$$

Taking the Laplace transform of equation (3.3.7) and then differentiating with respect to η , we get

$$\frac{d\overline{\omega}}{d\eta} = -\frac{p}{p^2+\omega^2} + 2\sum_{m=1}^{\infty}(\pi^2m^2)\left[\frac{A_1p}{p^2+\omega^2} + \frac{B\omega}{p^2+\omega^2} + \frac{C}{p-\alpha_1} + \frac{D}{p-\alpha_2}\right] \times \cos\pi m n \qquad (1.3.10)$$

Substituting equation (1.3.9) into (1.3.10) and inverting , we get

$$T_{xy} = -\frac{1}{1+\alpha^2\omega^2}\Big[(1+\alpha\beta\omega^2)\cos\omega\tau + \omega(\alpha-\beta)\sin\omega\tau \; \frac{\alpha-\beta}{\alpha}e^{-(\tau/\alpha)}\Big] + (\pi^2m^2)\left[A_2\cos\omega\tau + B_2\sin\omega\tau + C_2e^{\alpha_1\tau} + A_2'e^{-(\tau/\alpha)}\right] \qquad (3.3.11)$$

Where

$$A_2 = \frac{A_1(1+\alpha\beta\omega^2)^2 - B(\alpha\omega-\beta)}{(1+\alpha^2\omega^2)}, \quad B_2 = \frac{A_1\omega(\alpha-\beta) + B(1+\alpha\beta\omega)}{(1+\alpha^2\omega^2)}$$

$$C_2 = \frac{C(1+\alpha_1\beta)}{1+\alpha_1\alpha}, \quad D_2 = \frac{D(1+\alpha_2\beta)}{1+\alpha_1\alpha},$$

$$A_2' = \frac{B\alpha(\alpha\omega-\beta) - A_1(\alpha-\beta)}{\alpha(1+\alpha^2\omega^2)} - \frac{C(\alpha-\beta)}{\alpha(1+\alpha\alpha_1)} - \frac{D(\alpha-\beta)}{\alpha(1+\alpha\alpha_2)}$$

The skin friction S_1 and S_2 at the plates can be obtained by putting η=0 and η=1 in equation (3.3.11) respectively.

4. Conclusions

The flow of a non- Newtonian fluid between two parallel plates has been studied when one plate is stationary and the other suddenly starts oscillating. Solutions have been obtained for the velocity field and skin friction at the walls. The following conclusion have been drawn.An examination of fig-1.1 shows that in a thin liquid layer near the oscillating plate the velocity at any point decreases as the frequency parameter increases ; but beyond this liquid layer up to the stationary plate and opposite effect is observed .

Fig-1.2 shows that the effect of elasticity (α-β , being the measure of elasticity of the fluid) of the liquid is to decrease the velocity of the fluid particles a point near the oscillating plate , but beyond this layer in another thin liquid layer the elasticity of the liquid increases the velocity at a point . The value of η for which maximum values occur in curves for velocity distribution shift towards the oscillating plate as the elasticity of the liquid increases. Beyond this region of liquid again the elasticity of the liquid decreases the velocity of the liquid.

An examination of fig-1.3shows that in a thin liquid layer near the plate the velocity of the liquid decreases as the time elapses. But in thin liquid layer beyond this an opposite effect takes place and beyond this layer the velocity increases with the time measured from the instant the plate starts oscillating.

Fig-1.4 depicts the effect of elasticity of the liquid on the oscillating plate. An examination of this figure shows that the elasticity of the liquid decreases the skin- friction at this plate. Also it is seen that the shearing stress at the plate decreases as the frequency of oscillation increases.

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Finally, fig-1.5 shows that the elasticity of the liquid decreases the shearing stress at the stationary wall. For the higher values of the frequency the shearing stress as ω increase for comparably smaller values of ω, the shearing stress first decreases for smaller values of $\alpha - \beta$.

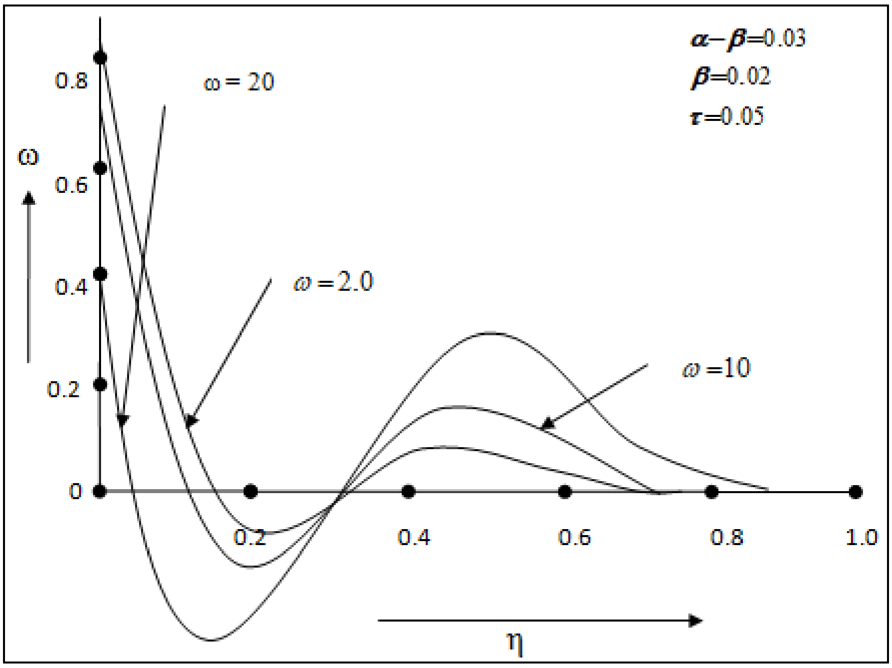

Figure 3.1: Velocity distribution for different values of the frequency parameter ω

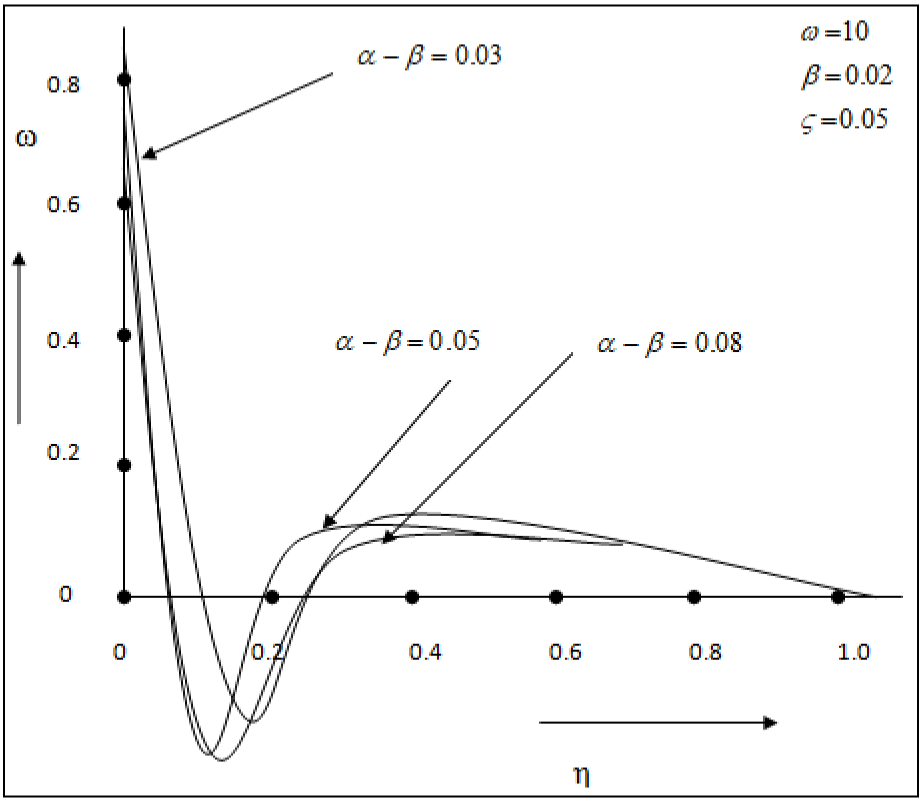

Figure 3.2: Velocity distribution for different values of elastic parameter $(\alpha - \beta)$

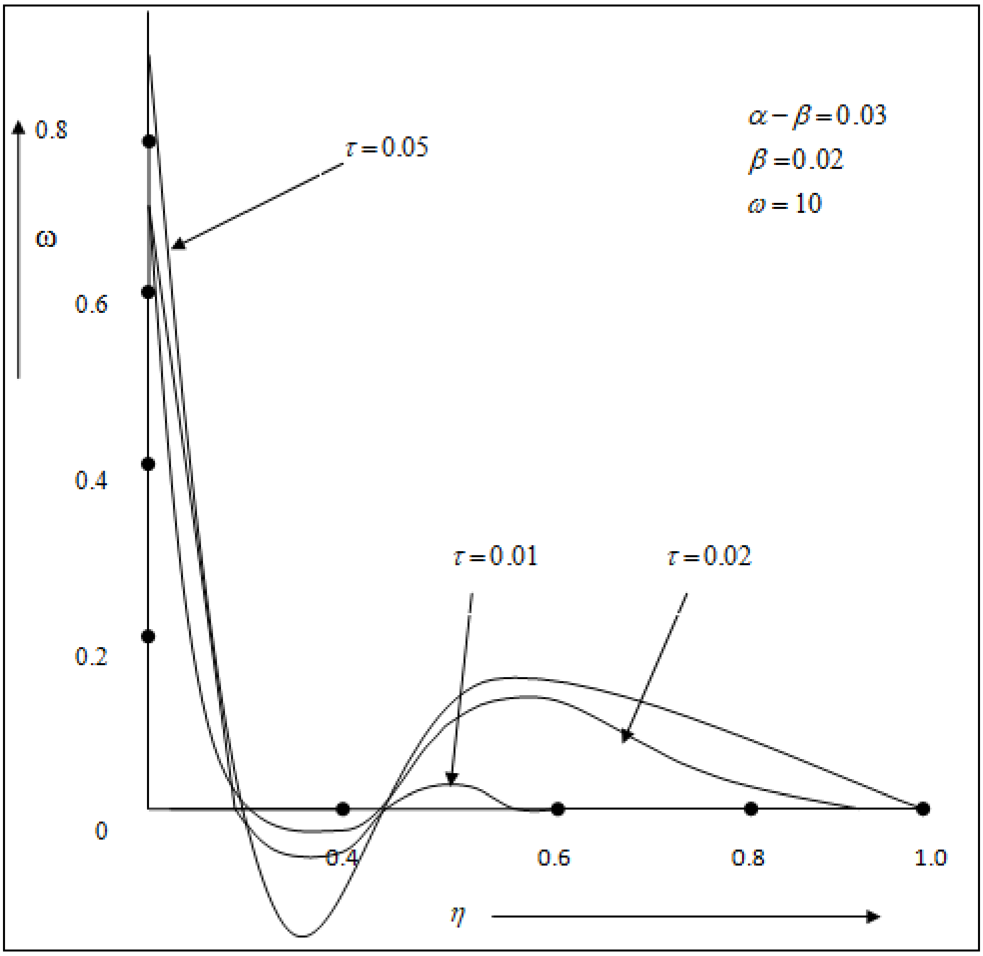

Figure 3.3: Velocity distribution for different value of time τ.

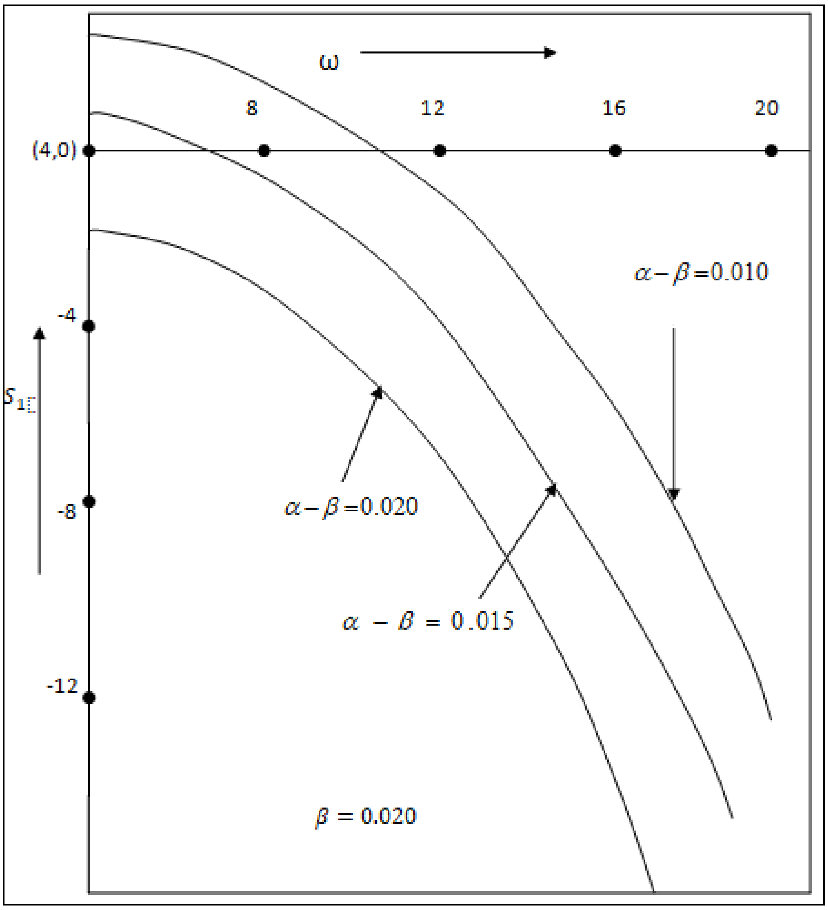

Figure 3.4: Skin friction near the moving wall

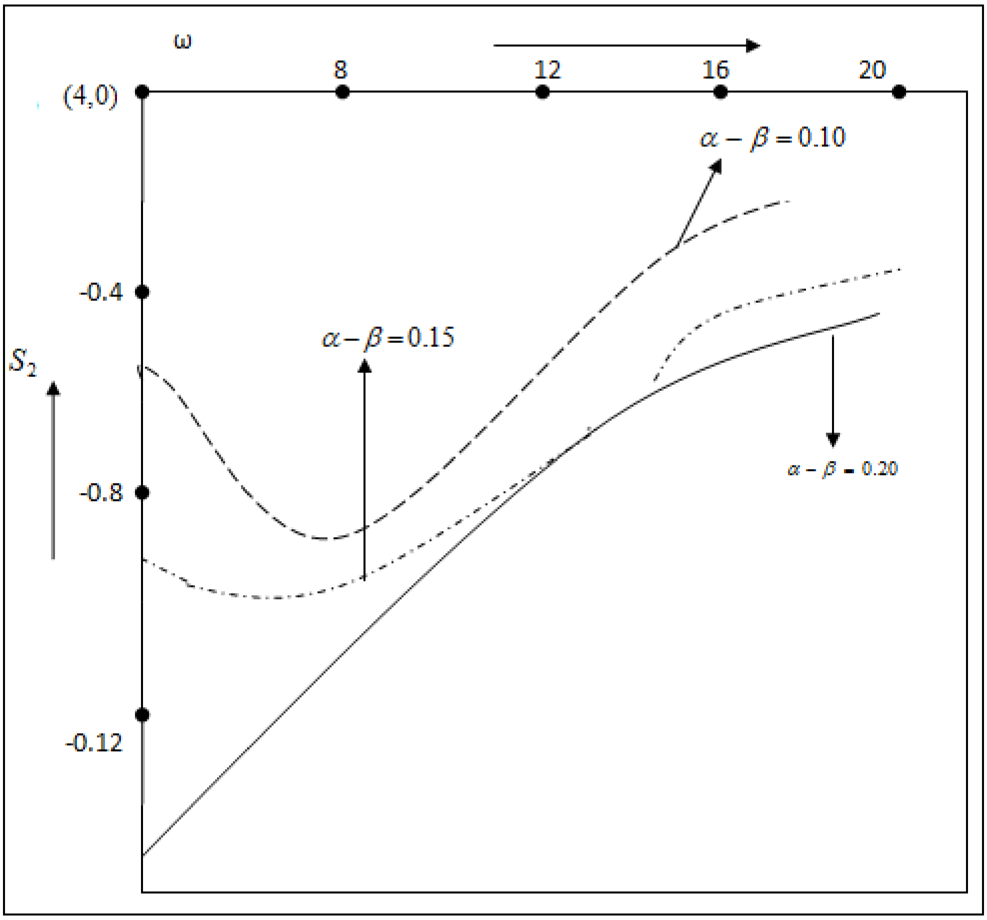

Figure 3.5: *Skin friction near the stationary wall*

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