

Modeling of Heat Transfer through Furnace Insulation and its Computational Solution

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Abstract: Proper design of furnace insulation is imperative both for cutting operating costs and to minimize their carbon footprint. A model for heat losses through the furnace insulation has been developed and the equations have been solved computationally using Newton-Raphson method.

Keywords: Heat Transfer, Furnace, Insulation, Newton-Raphson

1. Introduction

Furnaces and Kilns are commonly used in most industries for a variety of manufacturing and heat treatment processes. In-fact most manufactured items are required to be treated at a high temperature during some part of its manufacturing cycle. This makes these furnaces a major consumer of energy in the industry which can be in the form of fossil fuels like natural gas, diesel oil or direct electrical energy in resistance or induction heating furnaces. It is imperative that these furnaces, specially their insulation is designed properly to minimize energy losses with proper material selection, not only to cut operating costs but also to reduce their carbon footprint, to the extent possible. In this paper, I develop the heat transfer equations using the boundary conditions and energy conservation principals (in section – 3) and then develop a solution for them using numerical analysis by the Newton-Raphson Method (in section – 5).

The study culminates by developing a tool in excel for optimizing design of furnace insulation, allowing for proper material selection, predicting shell skin temperatures and calculating surface heat losses before the actual design is implemented. The tool also allows for making quick comparisons of alternate designs, doing cost benefit and payback analysis. The tools results and its co-relationship with real life examples is shown in section (6). In the following section (2), I try and define the problem to be solved in figure (1) and define all the variables and parameters used in the calculations in the following section (3).

2. Problem Definition

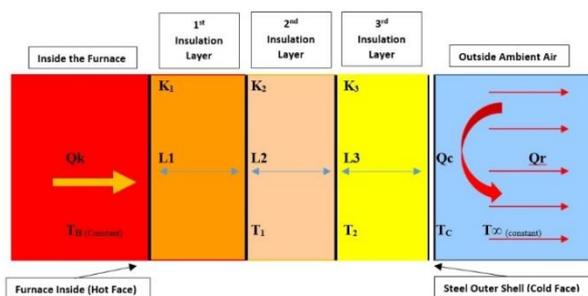


Figure 1

The variables and parameters used in the heat transfer equations are defined below

Q_k - Conductive heat flux from inside the furnace to the outside surface.

Q_{k1}, Q_{k2}....- Conductive heat flux through each insulation layer.

Q_c - Convective heat transfer from outer steel surface to ambient air.

Q_r -Radiative heat transfer from outer steel surface to ambient air.

T_h - Temperature of hot face or inside the furnace which is kept constant.

T_c - Temperature of cold face or skin temperature of outer steel surface.

T₁, T₂, T₃ ...T_n -Temperature in between 1st and 2nd layer, 2nd and 3rd layer and so on.

T_∞ - Ambient Air Temperature which is assumed to be constant.

K₁, K₂, K₃...K_n -Thermal Conductivity of each insulation layer.

K' - Equivalent Thermal Conductivity of all the layers in combination.

H_c -Coefficient of convection heat transfer from still air to vertical surface

L₁, L₂, L₃...L_n - Thickness of each insulation layer in meters.

L_{total} - Total thickness of all the layers.

σ - Stephen Boltzmann's constant = $5.670373 \times 10^{-8} \text{ W/m}^2/\text{K}^4$

3. Heat Transfer Equations

In the problem defined in Section (II), the heat flux from inside the furnace to the outside the furnace shell is primarily by conduction through the insulation layers. This is denoted by **Qk** which, is the heat transfer or loss per unit area per unit time. We will be working in SI system, so units of **Qk** will be **J/m². s**

Now heat flux due to conduction through any material is given by,

$$Qk = \frac{K \cdot \Delta T}{L}$$

where,

Qk is the Conductive heat flux through the material, **K** is the Thermal conductivity of the material, **L** is the Thickness of each insulation layer in meters and **ΔT** is the temperature difference on either side of the material.

Now we want to find the steady state solution of the problem, so **Th** and **T∞** will be assumed to be constant. Since the system is in steady state, by the law of conservation of energy, the heat flux through each layer will be same and there is no accumulation of energy at any layer of insulation.

This means that,

$$Qk = Qk1 = Qk2 = Qk3 \dots \dots \dots = Qkn \quad (1)$$

Or

$$Qk = \frac{K1.(Th-T1)}{L1} = \frac{K2.(T1-T2)}{L2} = \frac{Kn.(Tn-1-Tc)}{Ln} \quad (2)$$

Now we can think of all the layer put together having some equivalent thermal conductivity = **K'**. So, for the system of multiple insulation layers of different materials we can write the equivalent conductive heat transfer equation as

$$Qk' = \frac{K'}{(L1+L2+L3\dots+Ln)} [Th - Tc] \quad (3)$$

This equation is easier to work with since both **Th** and **Tc** are easily measurable, whereas intermediate temperature of each layer **T1, T2, T3** are difficult to measure in an actual physical furnace.

Assuming just two layers for our calculations. The equations (2) and (3) can be written as

$$Qk = \frac{K1.(Th-T1)}{L1} = \frac{K2.(T1-Tc)}{L2} = \frac{K'.(Th-Tc)}{(L1+L2)} \quad (4)$$

From (4) we get

$$\frac{K1.Th}{L1} - \frac{K1.T1}{L1} = \frac{K2.T1}{L2} - \frac{K2.Tc}{L2} \quad (5)$$

$$\frac{K1.Th}{L1} + \frac{K2.Tc}{L2} = T1 \left(\frac{K1}{L1} + \frac{K2}{L2} \right) \quad (6)$$

Simplifying we get

$$T1 = \left(\frac{K1.L2.Th + K2.L1.Tc}{K1.L2 + K2.L1} \right) \quad (7)$$

Substituting the value of T1 back in equation (4) we get

$$Qk = \frac{K1}{L1} \cdot \left[Th - \left(\frac{K1.L2.Th + K2.L1.Tc}{K1.L2 + K2.L1} \right) \right] \quad (8)$$

Simplifying we get

$$Qk = \frac{K1.K2}{(K1.L2 + K2.L1)} \cdot [Th - Tc] \quad (9)$$

or can be re-written as

$$Qk = \frac{1}{\left(\frac{L1}{K1} + \frac{L2}{K2} \right)} \cdot [Th - Tc] \quad (10)$$

Comparing equation (10) with RHS of equation (4) we get the equivalent thermal conductivity as

$$K' = \frac{L1+L2}{\left(\frac{L1}{K1} + \frac{L2}{K2} \right)} \quad (11)$$

For multiple number of insulations layers the above equation can be generalized as

$$K' = \frac{L1+L2+L3+\dots+Ln}{\left(\frac{L1}{K1} + \frac{L2}{K2} + \frac{L3}{K3} + \dots + \frac{Ln}{Kn} \right)} \quad (12)$$

$$Qk' = \frac{K'}{(L1+L2+L3\dots+Ln)} [Th - Tc] \quad (13)$$

Now the heat transferred from inside the furnace to its outer skin or steel shell must be dissipated or transferred to the surrounding air. This happen both by conduction and radiation.

Heat transfer flux by convection is given by

$$Qc = Hc \cdot \Delta T = Hc (Tc - T\infty) \quad (14)$$

where,

Qc is the Convective heat flux from the surface of a material, **Hc** is the Co-efficient of Convective Heat Transfer between the material surface and the surrounding fluid and **ΔT** is the temperature difference between the material surface and the surrounding fluid.

The convective heat transfer co-efficient varies widely for various conditions depending on material surface, fluid velocity, fluid state (gas or liquid) etc. For our case the convection is from the furnace walls to the surrounding air which is considered still in an industrial setup. For such a case the convective heat transfer co-efficient is well documented in engineering handbooks and is typically in the range,

$$Hc = (1 - 5) \frac{BTU}{ft^2 \cdot hr \cdot F}$$

For still air. In SI units this is equal to

$$Hc = (5.78 - 28.90) \frac{W}{m^2 \cdot K} \quad (15)$$

Heat transfer flux through radiation is given by Stefan-Boltzmann Law for blackbody radiation and is given as –

$$Qr = \sigma \cdot (Tc^4 - T\infty^4) \cdot Fe \cdot Fa \quad (16)$$

Where,

σ is the Stephen Boltzmann's constant = 5.670373 X 10-8 W/m²/K⁴

Tc is the Temperature of cold face or skin temperature of outer steel surface, in Kelvin.

T∞ is the Ambient Air Temperature which is assumed to be constant, in Kelvin.

Fe is the emissivity factor and is equal to emissivity of the steel plate to surrounding air. It is considered 0.9 for our case.

Fa is the view factor and is considered 1.0 for a vertical plate surrounded by air. Now performing an energy balance around the boundary condition, which is $Qk' = Qc + QrorQk' - Qc - Qr = 0$ (17) using equations (13), (14) and (16) we get

$$\frac{K'}{(L1+L2+L3....+Ln)} [Th - Tc] - Hc (Tc - T\infty) - \sigma \cdot (Tc^4 - T\infty^4) \cdot Fe \cdot Fa = 0 \tag{18}$$

The above is the main operating equation and can be solved for **Tc** or the skin temperature of the furnace. All other parameters are known. Once **Tc** is known, we can calculate using equation (13) total heat lost through furnace insulation. **T1, T2, T3,....** are the interface temperature of each insulation layer and can be calculated using equation (2). These temperatures are required to check if our insulation material selection is correct or not.

Now equation (18) can be thought of a function of **Tc** which is the variable required to be calculated and rest of the parameters are known once the layers of insulation are selected and can be written as

$$f(Tc) = \frac{K'}{(L1+L2+L3....+Ln)} [Th - Tc] - Hc (Tc - T\infty) - 0.9 \cdot \sigma \cdot (Tc^4 - T\infty^4) \tag{19}$$

Here we have substituted the values of **Fe** and **Fa**. The above does not have a simple algebraic solution but can be solved computationally quite easily using some iterative technique. We will use a popular method called the Newton-Raphson method which is explained in the next section (IV)

4. Newton-Raphson Method

Newton Raphson method is a numerical analysis method of finding the roots of a real valued function by first making an initial guess of the roots and then, finding successively better approximation of the root by each iteration, where each next approximation is given by

$$Xn + 1 = Xn - \frac{f(Xn)}{f'(Xn)} \tag{20}$$

Where **X0** is the initial approximation or guess of the root of the function **f(x)**, **f(Xn)** is the value of the function at the nth approximation and **f'(Xn)** is the value of the derivative of the function at the nth approximation.

So,

$$X1 = X0 - \frac{f(X0)}{f'(X0)}$$

$$X2 = X1 - \frac{f(X1)}{f'(X1)}$$

$$X3 = X2 - \frac{f(X2)}{f'(X2)}$$

and

$$Xn + 1 = Xn - \frac{f(Xn)}{f'(Xn)}$$

If n is a large enough and the process is repeated until a sufficiently accurate value of the root is reached. This is indicated by a negligible or no change in the values of **Xn** and **Xn-1**, then the solution is said to have converged to the value of the root of the function.

This method requires that the function is differentiable and real. The convergence of the solution with each successive iteration is shown graphically in figures (2)-(5).

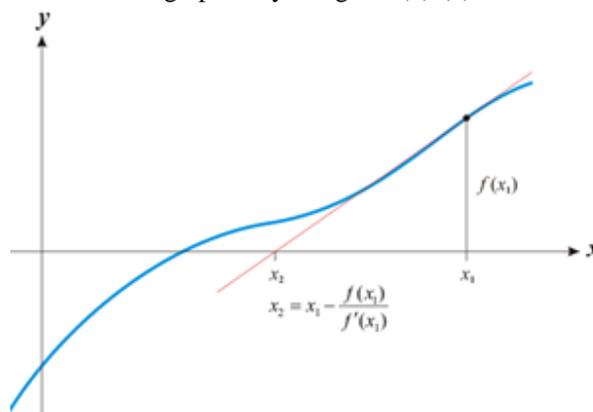


Figure 2

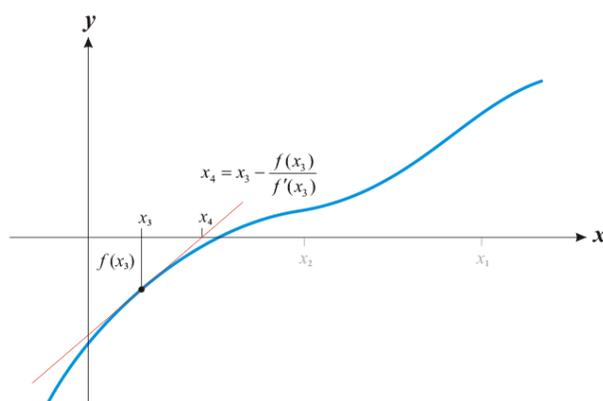


Figure 3

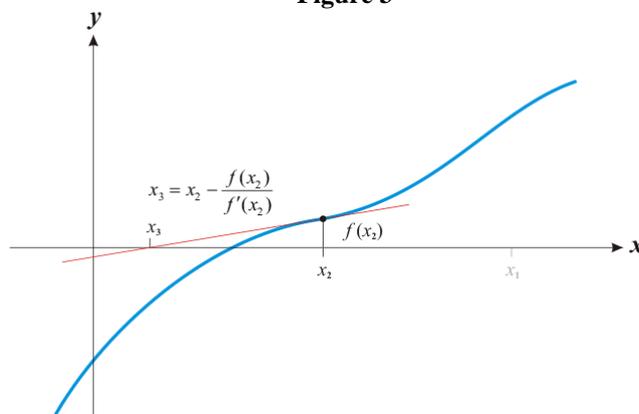


Figure 4

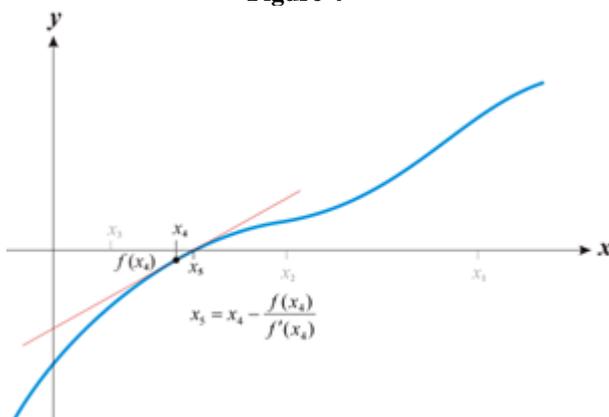


Figure 5

5. Solution of the Heat Transfer Equations using the Newton-Raphson method

Using the final equation (19) derived in section (III) and substituting $L_{total} = L_1 + L_2 + L_3 + \dots + L_n$,

$$f(T_c) = \frac{K'}{(L_{total})} [T_h - T_c] - H_c (T_c - T_\infty) - 0.9 \cdot \sigma \cdot (T_c^4 - T_\infty^4)$$

We can easily get its derivative

$$f'(T_c) = -\frac{K'}{(L_{total})} - H_c - 0.9 \cdot \sigma \cdot 4 \cdot T_c^3 \tag{21}$$

For the initial approximation or guess for T_c a good starting point is $T_{c0} = T_\infty$, or in other words the outside skin temperature of the furnace is same as the ambient air temperature. Using this and the technique described in the previous section, the above equation was solved for its roots. The solution was implemented in a tool in excel. The tool allows for selection of different layers of insulation of the furnace from a drop-down menu and then solves for the skin temperature T_c , Intermediate temperatures of each layer $T_1, T_2, T_3, \dots, T_n$ and computes the overall heat loss from the furnace Q_k .

The Table (1) below shows the raw calculations in excel of a specific case. In this case the initial guess is deliberately chose to be far away from the solution ($T_{c0} = 1000$ °C). Still the correct solution is arrived in just 7 iterations and we get the skin temperature as $T_c = 68.29$ °C. The energy balance is also cross checked and Q_k is found to be equal to $Q_c + Q_r$. Intermediate temperatures of each layer $T_1, T_2, T_3, \dots, T_n$ are also computed.

Table 1

Insulation Data			
Layer	K	L	Tn
1	1.95 W/mK	0.230 m	1024 Deg C
2	0.48 W/mK	0.115 m	868 Deg C
3	0.20 W/mK	0.110 m	516 Deg C
4	0.07 W/mK	0.050 m	68 Deg C

Parameters			
K'	0.32 W/mK	Th	1100 Deg C
Ltotal	0.505 m	T∞	33 Deg C
Hc	11.36 W/m ² /K	Fe	0.9
ε	5.67E-08 W/m ² /K ⁴	Qk	645.61 W/m ² /K
		Qc	400.72 W/m ² /K
		Qr	244.89 W/m ² /K
		Qk - Qc - Qr	0.00 W/m ² /K

Newton Raphson method			
Tc0	33 Deg C		
n	Tcn	F(Tc)	F'(Tc)
1	70.45 Deg C	667.69	-17.83
2	68.29 Deg C	-43.58	-20.25
3	68.29 Deg C	-0.17	-20.10
4	68.29 Deg C	0.00	-20.10
5	68.29 Deg C	0.00	-20.10
6	68.29 Deg C	0.00	-20.10
7	68.29 Deg C	0.00	-20.10
8	68.29 Deg C	0.00	-20.10
9	68.29 Deg C	0.00	-20.10
10	68.29 Deg C	0.00	-20.10

Table 2

Insulation Data			
Layer	K	L	Tn
1	1.95 W/mK	0.230 m	1024 Deg C
2	0.48 W/mK	0.115 m	868 Deg C
3	0.20 W/mK	0.110 m	516 Deg C
4	0.07 W/mK	0.050 m	68 Deg C

Parameters			
K'	0.32 W/mK	Th	1100 Deg C
Ltotal	0.505 m	T∞	33 Deg C
Hc	11.36 W/m ² /K	Fe	0.9
ε	5.67E-08 W/m ² /K ⁴	Qk	645.61 W/m ² /K
		Qc	400.72 W/m ² /K
		Qr	244.89 W/m ² /K
		Qk - Qc - Qr	0.00 W/m ² /K

Newton Raphson method			
Tc0	33 Deg C		
n	Tcn	F(Tc)	F'(Tc)
1	70.45 Deg C	667.69	-17.83
2	68.29 Deg C	-43.58	-20.25
3	68.29 Deg C	-0.17	-20.10
4	68.29 Deg C	0.00	-20.10
5	68.29 Deg C	0.00	-20.10
6	68.29 Deg C	0.00	-20.10
7	68.29 Deg C	0.00	-20.10
8	68.29 Deg C	0.00	-20.10
9	68.29 Deg C	0.00	-20.10
10	68.29 Deg C	0.00	-20.10

Table (2) shows the same calculations but this time I choose ($T_{c0} = T_\infty = 33$ °C). The calculations give the same answer ($T_c = 68.29$ °C) but this time the solution converges in just 3 iterations with negligible change between T_{c3} and T_{c4} .

6. Results and practical use

To analyze the results of the above technique we make use of a real-world example of a furnace that was built by AIF Engineering Pvt. Ltd under whom this was developed during my internship. AIF Engineering were happy to share the Furnace insulation design data and the actual skin temperatures measured at site.

The furnace is an Aluminium Melting and holding furnace supplied to National Aluminium Company (NALCO) a public sector enterprise in Orissa, India. To understand better what the furnace looks like please refer to figure (6) and (7) which shows the furnace and its insulation.



Figure 6



Figure 7

The area of the furnace which we were interested in, were the Upper Sidewall and the Lower Sidewall. Both these areas have varying insulation combinations. Both these areas are analyzed using the Excel tool and the screenshot of the results are presented below.

For Furnace Upper Side Wall $T_c = 63.29^\circ\text{C}$

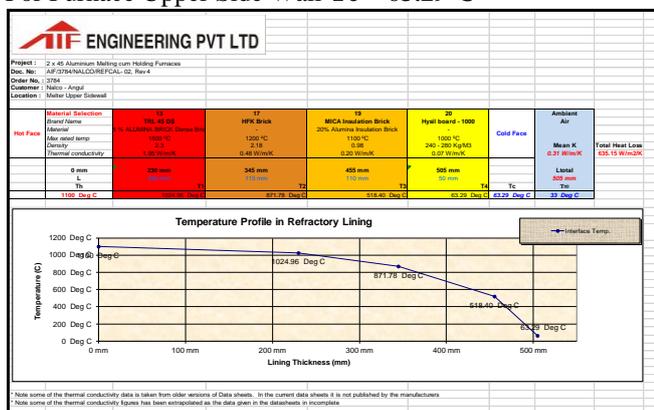


Figure 8

For Furnace Lower Side Wall $T_c = 72.87^\circ\text{C}$

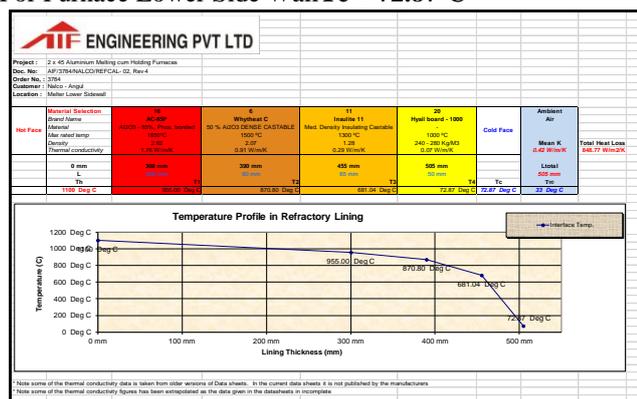


Figure 9

The above calculations show a good correlation with actual skin temperatures measured at 10 different locations on the furnace shown below both for the Upper Side Wall (Above Sill Level) and Lower Side Wall (Below Sill Level) areas, shown in Table (3).

Table 3

PG TEST CL NO- 9.5-SHELL SKIN TEMPERATURE TEST
As per Clause No- 9.5 the Furnace Skin Temperature shall not exceed (40°C + t) on walls above sill level and (55°C + t) on walls below Sill Level of the furnace where "t" is ambient temperature measured at a distance 5 mtr. for Furnace. The test shall be conducted along with temperature holding with stability test. The skin temperature shall be measured at minimum 10 mutually agreed locations, which will be 500 mm away from doors, burners and flue duct openings. (All Temperature in Deg. C)

TEST & DATE	Ambient Temp	AREA	Point-1	Point-2	Point-3	Point-4	Point-5	Point-6	Point-7	Point-8	Point-9	Point-10
Test -1 16.08.2017	33	Above Sill Level	63	63	63	61	71	58	58	63	63	62
		Below Sill Level	69	72	69	73	73	73	76	66	61	73
Test -2 16.08.2017	33	Above Sill Level	64	63	60	53	57	58	53	62	64	63
		Below Sill Level	70	72	71	68	77	77	67	72	74	75
Test -3 17.08.2017	33	Above Sill Level	61	62	61	54	68	62	56	66	67	65
		Below Sill Level	65	73	62	73	72	78	70	71	87	73

7. Conclusions and acknowledgements

Using the techniques described above I was able to develop a user-friendly tool in excel to optimize the design of furnace insulation, allowing for proper material selection, predicting shell skin temperatures and calculating surface heat losses before the actual design is implemented. The tool also allows the company to make quick comparisons of alternate designs in future projects, considerably reducing design time.

I would like to thank AIF Engineering for providing me with this opportunity to work on an actual practical problem, which not only allowed me to use concepts in Heat Transfer studied in my 11th grade, but also exposed me to higher level concepts like Numerical Analysis using Newton-Raphson method and writing this paper. I look forward to working with them again if the opportunity presents.

References

- [1] NCERT 11th Class Physics Text Book – Part 2 (Section 11.8 Page 286-290)
- [2] North American Combustion Handbook, 3rd Edition, Volume I.
- [3] https://en.wikipedia.org/wiki/Newton%27s_method (figure 2-5) taken from this link.
- [4] https://en.wikipedia.org/wiki/Thermal_radiation

Author Profile



team.

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