

Normal Gravity Field Computation for the Evaluation of Terrestrial Reference Systems for the Modeling of Gravity Anomalies in Cameroon

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Abstract: *The choice of a terrestrial reference model over another in view of the duplicity of reference systems within a single territory is discussed. To do this the calculation of the normal gravity fields of the GRS 80 and WGS 84 reference systems was performed in order to evaluate the accuracy of the two models on Cameroon. The study was done by descriptive statistical analysis using regional modeling and local modeling of the normal gravity field. The examination of some results shows an equivalence of some parameters for the two reference systems, notably the normal gravity averages estimated at 978121 ± 19.1 mGal for regional modeling and 978043 ± 0.13 mGal for local modeling. The standard error value of the normal gravity field difference between the two reference systems is evaluated to 0.0000007 mGal, which suggests that there is little difference between the GRS 80 and WGS 84 reference systems. Gravity anomalies calculated for the different reference systems at the local level reveal a minimum deviation for the WGS-84 reference, which is therefore the best formula of normal gravity in the territory.*

Keywords: Gravity anomaly, Normal gravity, Ellipsoid, Statistic

1. Introduction

The gravity acceleration on the surface of the Earth is the acceleration that undergoes any mass on this surface because of: The Newtonian attraction of all the masses of the Earth which creates the gravitational acceleration called gravity and the centrifugal acceleration due to the rotation of the Earth [1]. The gravity acceleration g varies from one point to the other on the surface of the Earth according to latitude, altitude, the attraction of the surrounding masses and the gravity tide or the variation of the lunar solar forces [2]. The values of gravity are used in several fields. Two types of gravity values are generally distinguished: the measured values (observed gravity) which are the measurements obtained using instruments such as gravimeters and altimetry satellites and the theoretical value (calculated gravity) obtained from calculation on a reference ellipsoid. Theoretical gravity, also called normal gravity, or reference gravity field, is the effect of gravity due to an ellipsoid of equipotential revolution. It represents the acceleration of gravity that would be generated by an ellipsoidal uniform earth [3]. According to [4], the theory of the equipotential ellipsoid was first established by P. Pizzetti in 1894; it was then developed by C. Somigliana in 1929 who developed the first formula for normal gravity. This theory served as a basis for the development of the International Gravity Formula (IGF) and the adoption of the GRS-30 Geodetic Reference System at the General Assembly of the International Union of Geodesy and Geophysics (IUGG) in Stockholm in 1930. Subsequently many other terrestrial references have emerged, all based on the geocentric equipotential ellipsoid theory, including the GRS-67

Geodetic Reference System, the use of which has been recommended to compensate for the lack of precision of the GRS-30 at the XIV General Assembly of the IUGG in 1967 in Lucerne, Switzerland and adopted in 1971 at the XV General Assembly of the IUGG in Moscow. In 1979, the XVII General Assembly of the IUGG meeting in Canberra adopted the geodetic reference system GRS-80 which served as a basis for the definition by the US Department of Defense, of the WGS-84 (World Geodetic System 1984) designed to be used as a reference system for GPS positioning (Global Position System). The gravity field obtained on a reference ellipsoid is of practical importance for many geodesic and geophysical applications because its conventional mathematical form simplifies many calculations, moreover the existing gaps between the Earth's gravity field and the theoretical gravity field or normal are weak. This division of the earth's gravity field into a "normal" field and a "disruptive" or "abnormal" field greatly simplifies many problems: the geoid determination for geodesists and the use of gravity anomalies to understand the interior of the Earth for geophysicists [5]. In Exploration Geophysics, the most important step in the processing of gravimetric data is the conversion of gravimetric measurements into gravity anomalies. [6] defined the gravity anomaly at a point on the physical surface of the Earth as the difference between the value of the Earth's normal gravity at a given latitude point and the gravity value observed at the same point, in other words, the gravity anomaly represents the gravimetric influence of the differences that exist between the real Earth and its model. From this point of view, the determination of the gravity anomaly requires precise values of normal gravity from the best ellipsoid

model, which best fits the local geoid model [7]. In Cameroon, local networks exist, these are referred to various experimental formulas of normal gravity due to different ellipsoid models, like the Clarke ellipsoid of 1880 whose origin is poorly known and used for a long time as the reference ellipsoidal surface [8]. Since the values of the normal gravity field are essential for the modeling of gravity anomalies, it is necessary to evaluate the accuracy of the terrestrial reference models used for the calculation of the normal gravity field. At present, the reference models GRS-80 and WGS-84 represent the most widely used geodetic reference systems for the calculation of the normal gravity field because they are very well suited to the calculation of the equipotential surfaces, the use of GPS emphasizes the use of WGS-84, while the IUGG recommends the use of GRS 80. Therefore, the objective of this study is to determine the normal gravity values of Cameroon for the GRS 80 and WGS-84 references, to evaluate their precision in order to make the choice of the most precise reference on this study area for gravity anomalies modeling.

2. Material and Method

2.1 Data

The data used for this study are measured gravity values from a ground geophysical survey carried out by the geophysicist team of the University of Yaoundé I in March 2015 [9]. The dataset corresponds to 223 gravimetric stations with irregular spacing between stations from 0, 5 to 2 km, collected in the locality of Kribi-Campo (Fig.1). Irregularity in the data distribution is mainly due to the inaccessibility of some sites as the survey site is in the dense equatorial forest and in the Campo National Park, where only open roads are accessible. The values of the variation of the gravitational attraction were measured using a Lacoste & Romberg G-823 gravimeter while the geographic coordinates (Longitude, Latitude, Altitude) are recorded using a GPS receiver (GPSMAP 64 s) from Garmin International. The measured gravity values were corrected for the effects of lunar tide and instrumental drift (assumed to be linear over time). In the framework of this study 172 stations out of the 223 existing ones were used in order to respect the maximum equidistance between the stations.

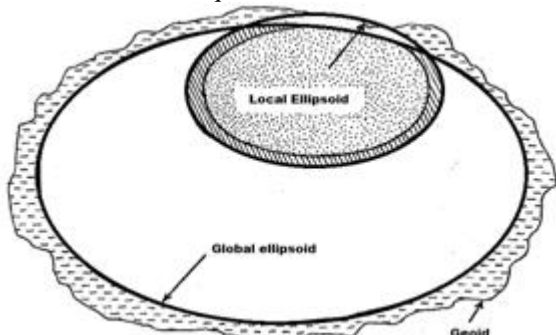


Figure 1: Representation of the Earth by a Geoid and an Ellipsoid

2.2 Methodology

In order to evaluate the accuracy of the geodetic reference systems, the values of the normal gravity fields of Cameroon were determined on the basis of GRS-80 and WGS-84

references and subjected to descriptive statistical analyzes using the Minitab 18 software. Modeling of the normal gravity field was done in two distinct ways, one local and one regional. Local modeling is the calculation of the normal gravity field over a relatively small area, such as a city scale. This type of modeling gives a local ellipsoidal surface (Fig. 2) that does not take into account the entire curvature of the earth. As part of this study, local modeling was done in localities in southwestern Cameroon, where measured gravity values were collected. On the other hand, the global modeling consists of the calculation of the normal gravity field for the Earth. In order to detect any local or regional anomalies, the actual gravity field of the earth is compared with that of a fluid body of the same animated mass with the same rotation (Clairaut spheroid).

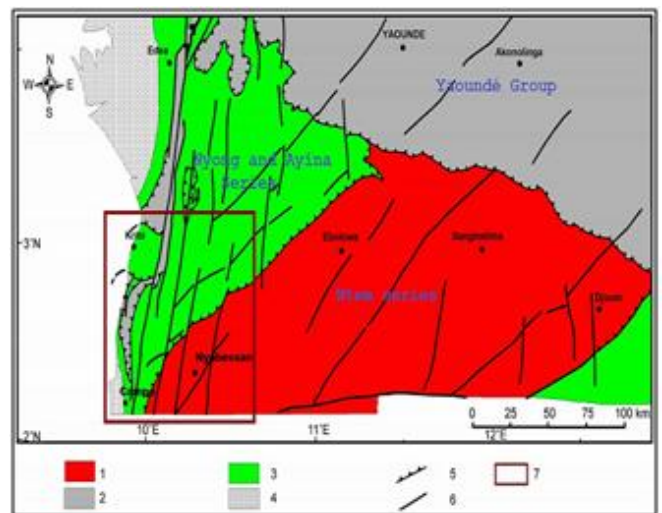


Figure 2: Simplified geological map of the SW-Cameroon [7]. 1: Archaean Basement; 2: Neoproterozoic cover; 3: Neoproterozoic cover; 4: Post Panafrican cover; 5: Thrust fault; 6: fault; 7: Study area.

Considering the globe sufficiently homogeneous, such a spheroid would be mistaken for the geoid. Since the surface of the globe is not uniform, consider the earth whose density varies radially and in rotation, this equipotential surface is an ellipsoid of revolution also called reference ellipsoid defined by:

$$f = \frac{a-c}{a} \tag{1}$$

f is the flattening of the earth, a and c are the radius of the earth at equator and the pole respectively.

The gravity on the reference ellipsoid can be derived from the gravitational potential U:

$$U_G = -\frac{GM}{r} + \frac{G}{r^3}(C - A) + \frac{(3 \cos^2 \theta - 1)}{2} - \frac{1}{2} \omega^2 r^2 \sin^2 \theta \tag{2}$$

Where G, M, r, ω are the gravitational constant, mass of the earth, radius of the spheroid and angular speed of the earth rotation respectively. A and C are the moments of inertia about the equatorial and polar axes.

To first order radius of spheroid is given by:

$$r = a(1 - f \sin^2 \phi) \tag{3}$$

Thus the acceleration of gravity on the reference ellipsoid is given by:

$$\vec{g} = -\vec{\nabla} U_g$$

$$|g| = \frac{GM}{r^2} + \frac{3GMa^2 J_2}{r^3} \frac{(3 \sin^2 \phi - 1)}{2} - \omega^2 r \cos^2 \phi \quad (4)$$

Where J_2 is the dynamic form factor.

The value of gravity on the ellipsoid is the normal gravity γ given by:

$$\gamma = \gamma_E (1 + \beta \sin^2 \phi + \beta' \sin^2 2\phi) \quad (5)$$

γ_E is gravity at equator.

$$\gamma = \frac{GM}{a^2} + \left[1 - \frac{3}{2} J_2 - m \right] = 9.780327 \text{ m/s}^2$$

$$\gamma = 9.780327 [1 + 0.0053024 \sin^2 \phi + 0.0000059 \sin^2 2\phi]$$

The normal gravity formulae derive from ellipsoidal model can now be written for GRS80 and WGS84:

$$\gamma(\phi)_{1980} = 978032.7 [1 + 0.0053024 \sin^2 \phi - 0.0000058 \sin^2 2\phi] \text{ mGal}$$

$$\gamma(\phi)_{1984} = 978032.67 [1 + 0.00527889 \sin^2 \phi + 0.000023462 \sin^2 2\phi] \text{ mGal}$$

The expression for gravity anomaly can be written as:

$$\Delta g = g_{obs} - \gamma(\phi) \quad (6)$$

Where g_{obs} is observed gravity.

3. Results and Discussion

The descriptive statistics results from the normal gravity field values of the geodetic reference systems GRS 80 and WGS 84 calculated for Cameroon in this study are presented in Tables 1 (regional modeling) and 2 (local modeling) respectively.

Table 1: Descriptive Statistics of normal gravity field from regional modeling

	Variable	
	GRS-80	WGS-84
N	28	28
Mean	978151	978151
SE Mean	19.1	19.1
StDev	101	101
Min	978034	978034
Median	978127	978127
Max	978356	978356

Table 2: Descriptive statistics of normal gravity field from local modeling

	Variable		
	GRS-80	WGS-84	WGS80 – GRS84
N	172	172	172
Mean	978043	978043	0.03060
SE Mean	0.13	0.13	0.0000007
StDev	1.67	1.67	0.000098
Min	978041	978041	0.03050
Median	978042	978042	0.03054
Max	978046	978046	0.03080

The examination of certain parameters (for example the mean) shows that for regional modeling the average values of normal gravity (978151 ± 19.1 gal) are equal for the two reference systems. The equivalence of values suggests that the choice of either approach is of minor importance for most geophysical exploration and geodesic applications [3]. The standard deviation values for the regional modeling

(101 mGal) are similar and high for both reference systems, suggesting that the datasets were calculated at identical intervals of a latitude change of $0.5^\circ \times 0.5^\circ$. The results of the local modeling of Table 2 of the two reference models show that the standard deviations are small. The small standard deviation values (1.67 mGal) is an indicator that the data used were calculated at close intervals, approximately 0.03° intervals.

The relationship between the GRS 80 and WGS 84 reference models is illustrated in Figures 3 and 4 through the normal curves superimposed on the histograms. The curves are skewed to the right for both reference models as well as for the difference between the two reference models, which reflects a similitude of the data distribution law. These relationships are also revealed in the box plot.

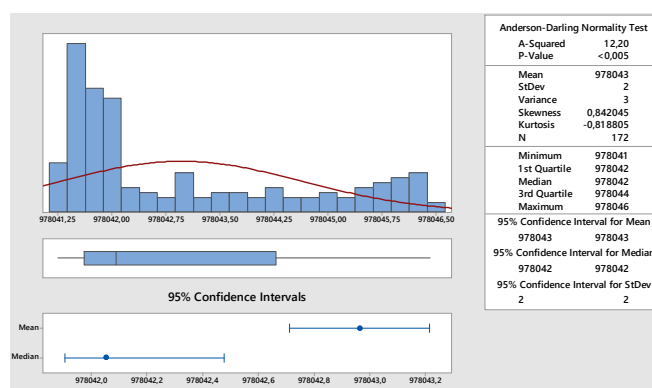


Figure 3: Descriptive statistics of normal gravity field from GRS80

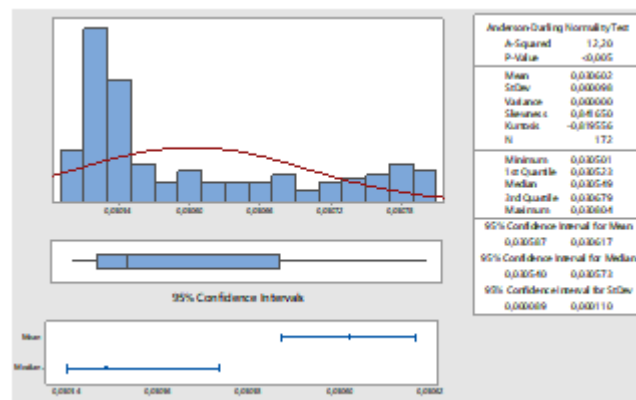


Figure 4: Descriptive statistics of normal gravity field from WGS84

The standard errors 19.1 mGal (regional modeling) and 0.13 mGal (local modeling) of the two reference models are equal. The large difference in standard error values for the normal gravity values of the regional model and the local model is explained by the spacing of stations that is larger for the regional model. An important result revealed in Table 2 is the standard error value (0.0000007 mGal) of the difference between the two reference systems. This relatively low value implies that there is little difference between the GRS 80 and WGS 84 reference systems. Indeed, the initial development committee of WGS 84 decided to closely follow the approach used by the International Union of Geodesy and Geophysics (IUGG), when it created and adopted the Geodetic Reference System

of 1980 (GRS 80). The modifications that have produced the change from one reference system to the other is the refinement of certain parameters including the new value of the terrestrial gravitational constant (GM) recommended by the Defense Mapping Agency (DMA) and the new estimate of the dynamic value of the zonal coefficient of second degree J_2 (second degree zonal coefficient) evaluated during the joint NIMA / NASA project on the definition of the Earth Gravitational Model 1996 (Earth Gravitational Model 1996 EGM96). This last difference explains the gap of 0.0001 m found on the half minor axis b between the ellipsoids WGS84 and GRS80. Table 3 and figure 5 presents the descriptive statistics results of the difference between the observed gravity values measured by a gravimeter and the normal gravity values of the GRS-80 and WGS-84 reference systems calculated for points with the same coordinates.

Table 3: Descriptive Statistics of difference between gravity observed and normal gravity

	Variable	
	$g_{obs}-\gamma(\phi)_{1980}$	$g_{obs}-\gamma(\phi)_{1984}$
N	172	172
Mean	-48.38	-48.35
SE Mean	3.51	3.51
StDev	46.08	46.08
Min	-153.21	-153.53
Median	-25.67	-25.64
Max	-11.56	-11.53

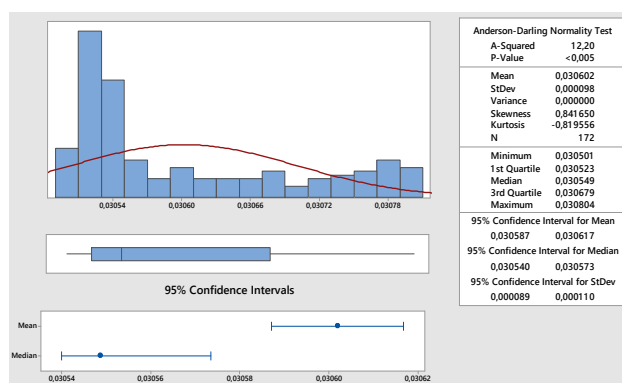


Figure 5: Descriptive statistics of the difference in values between GRS80 and WGS84

In first observation, the calculated values of the differences are all negative which justifies the average values of negative difference obtained, ie $(-48.4 \pm 3.51\text{gal})$ for the difference $g_{obs}-\gamma(\phi)_{1980}$ and $(-48.3 \pm 3.51\text{gal})$ for the difference $g_{obs}-\gamma(\phi)_{1984}$. These mean negative difference values reflect the fact that the undulations of the geoid models used are below the materialized surface of the ellipsoids GRS-80 and WGS-84. The difference between the averages of the two differences is of the order of a hundredth of mGal, ie 0.04 mGal. The average minimum anomaly value in this region is given by the difference between the observed gravity values and the normal gravity values of the WGS-84 reference, suggesting that the normal gravity formula of the geodetic reference system WGS-84 is the best on Cameroon in agreement with [2].

4. Conclusion

The practical use of gravity data for the many geodetic and geophysical applications requires that these data be equivalent and harmonized in a common reference system, so as to optimize the results of the gravity data processing. In this regard, this study attempted to evaluate the accuracy of two geodetic reference systems most widely used; the GRS 80 recommended by the U.G.G.I and the WGS-84 whose use is adapted to the Global Positioning System GPS. The evaluation of the normal gravity fields of these reference systems revealed slight differences. The minimal value of the standard error (0.0000007 mGal) of the difference between the two reference systems implies that there is little difference between the two systems, this interpretation is compatible with standard errors of 0.13 mGal resulting from the local modeling. The average minimum anomaly value is given by the difference between the observed gravity values and the normal gravity values of the WGS-84 reference system. There are slight differences between the GRS 80 and WGS 84 reference systems which have no practical consequence. However, the adoption of one or another system as a common reference affects gravity anomalies accuracy. This discrepancy can be minimized by an adequate choice in order to attain the requirements of high precision.

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