

# Vibration-Based Methods for Detecting a Crack in a Fixed Free Beam

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**Abstract:** The phenomenon of vibration can be applied to identify the crack size and location. In particular, cracks decrease the stiffness and the natural frequency thus, causing specimens to fail under normal working conditions. This paper presents the application of the vibration-based technique for detecting the location and the size of a crack in structures. The crack is modelled by a rotational spring. The predicted crack depth and location are compared with the actual data obtained from finite element models.

**Keywords:** Eigen value problem, rotational spring stiffness, crack, MATLAB

## 1. Introduction

Cracks are one of the main causes of structural failure. In order to reduce or eliminate the sudden failure of structures, they should be regularly checked for cracks. The crack present in the component imparts local flexibility to the element, which leads to reduction in natural frequencies and mode shapes. It is possible to estimate the location and the size of a crack by measuring changes in the vibration parameters. Crack is modelled as rotational spring having stiffness  $k_s$  and is added to global stiffness matrix where rotational degrees of freedom are there. And no change in mass matrix is assumed because of small size of crack.

## 2. Literature Review

The formation of cracks in a structure affects the local stiffness and flexibility of the structure. This problem has been a subject of investigation in many papers. In the present study an attempt has been made to the reviews on the isotropic cracked cantilever beam.

Adams et al [1] have presented a method for damage detection in a one-dimensional component using the natural frequencies of longitudinal vibrations. They modelled the damage by a linear spring. Petroski [2] has proposed a technique by which the section modulus is appropriately reduced to model a crack. Chondros and Dimarogonas [3] have used the concept of a rotational spring to model the crack and proposed a method to identify cracks in welded joints. Rizos et al [4] have applied this technique to a cantilever beam and

detected the crack location through the measurement of amplitudes at two points of component vibrating at one of its natural modes. Liang et al [5] have proposed a similar method, but it required measurements of the three fundamental frequencies of the beam. Dimarogonas and Paipetis [6] calculated the rotational spring constant of a beam of a rectangular cross section from the crack strain energy function. Ostachowicz and Krawczuk [7] obtained the relationships between the reduced stiffness of the cracked section and the crack size of a beam of rectangular cross section from the decrease in the elastic deformation energy of the crack expressed in terms of the stress intensity factor. Barad et al [8] used relations between the rotational spring constant, crack size and location and proposed a method for detecting a crack in a cantilever beam. In most cases, the proposed methods are applied to a cantilever beam although the simply supported beam is commonly used.

## 3. Theory

For free vibration the equation can be written as,

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \quad \text{eq(1)}$$

The equation (1) represents an Eigen value problem and the roots of the equation gives rise to square of the natural frequency given by the equation,

$$[K] - \omega^2 [M] = 0. \quad \text{eq(2)}$$

Where

$$K = \begin{bmatrix} 12EI/L^3 & 6EI/L^2 & -12EI/L^3 & 6EI/L^2 & 0 & 0 \\ 6EI/L^2 & 4EI/L & -\frac{6EI}{L^2} + k_s & 2EI/L & 0 - k_s & 0 \\ -12EI/L^3 & -6EI/L^2 & \frac{12EI}{L^3} + \frac{12EI}{L^3} & -\frac{6EI}{L^2} + \frac{6EI}{L^2} & -12EI/L^3 & 6EI/L^2 \\ 6EI/L^2 & 2EI/L & -\frac{6EI}{L^2} + \frac{6EI}{L^2} - k_s & \frac{4EI}{L} + \frac{4EI}{L} & -\frac{6EI}{L^2} + k_s & 2EI/L \\ 0 & 0 & -12EI/L^3 & -6EI/L^2 & 12EI/L^3 & -6EI/L^2 \\ 0 & 0 & 6EI/L^2 & 2EI/L & -6EI/L^2 & 4EI/L \end{bmatrix}$$

$I$  = moment of inertia of the section with respect to  $z$ -axis

$E$  = Young's modulus of elasticity

$k_s$  = stiffness of rotational spring =  $k_s = 0.70 \cdot ((h/dcr)^{1.2} - 1) \cdot E \cdot I/h$

and  $dcr$  = depth of crack and

$h$  = total height of beam

Crack is modelled as rotational spring having stiffness  $k_s$  and is added to global stiffness matrix where rotational degrees of freedom is there. No changes in mass matrix is assumed due to crack. And the mass matrix is where  $L$  is the crack location from fixed end and  $L_1$  is the remaining length of beam after crack.

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$$M = \rho \omega A / 420 \begin{bmatrix} 156L & 22L2 & 54L & -13L2 & 0 & 0 \\ 22L2 & 4L3 & 13L2 & -3L3 & 0 & 0 \\ 54L & 13L2 & 156L + 156L1 & -22L2 + 22L12 & 54L1 & -13L12 \\ -13L2 & -3L3 & -22L2 + 22L12 & 4L3 + 4L13 & 13L12 & -3L13 \\ 0 & 0 & 54L1 & 13L12 & 156L1 & -22L12 \\ 0 & 0 & -13L12 & -3L13 & -22L12 & 4L13 \end{bmatrix}$$

$\rho$  = mass density of the material  
 $A$  = cross sectional area of the beam element

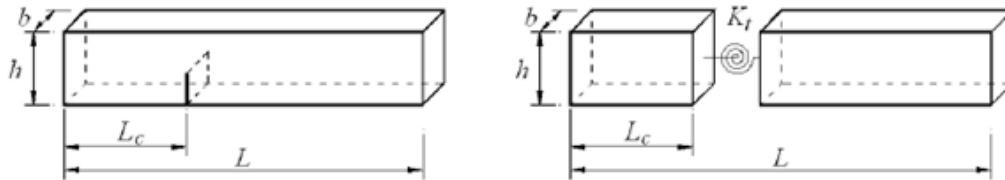


Figure 1: A beam with a crack and representation of crack with a rotational spring

#### 4. Numerical Examples

The numerical analysis is carried out for a simply supported beam of rectangular cross section with a single crack. The natural frequencies of transverse vibration at different crack location and size are calculated by the finite element method, using a computer program (ANSYS). The beam is discretized by 8-node solid185 elements. The line is divided into 20 parts. Rectangular area is created first to the length and height of beam and crack area is subtracted. Remaining area is extruded to the required width of the beam. Boundary conditions are taken as one end fixed all degrees of freedom and the other end only rotation about x-axis is constrained.

The dimensions of the beam are: Length,  $L = 0.7$  m, height,  $h = 0.01$  m, width,  $b = 0.05$  m. The material properties are: Modulus of elasticity,  $E = 2.1 \times 10^{11}$  N/m<sup>2</sup>, density,  $\rho = 7850$  kg/m<sup>3</sup>, Poisson's Coefficient,  $\mu = 0.3$ . The natural frequencies for the studied cases are shown in Table 10.

The corresponding results with beam theory are shown in Table 11.

#### 5. Results

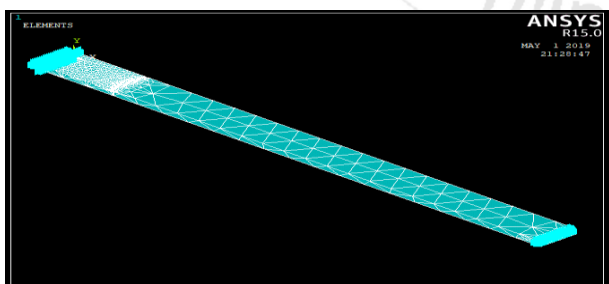


Figure 2: Crack at 0.1m location where depth of crack  $dc=0.001$ mm

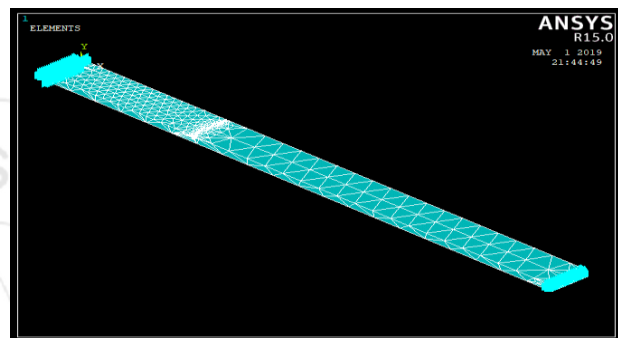


Figure 3: Crack at 0.2 m location where depth of crack  $dc=0.001$ mm

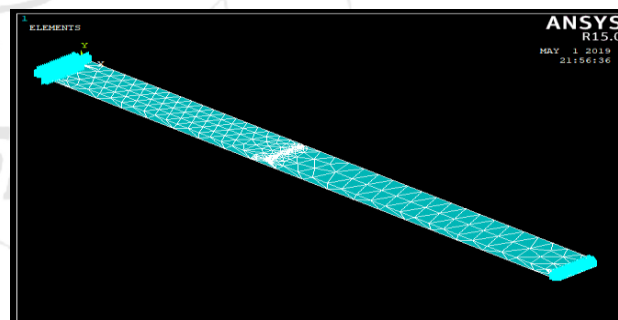


Figure 4: Crack at 0.3 m location where depth of crack  $dc=0.001$ mm

Table 1:  $L_c=0.1$ ;  $dc=0.001$ ;  $L=0.7$ ;  $\rho=7850$ kg/m<sup>3</sup>;  $w=0.05$ m;  $h=0.01$ m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 72.643              |
| 2           | 188.28              |
| 3           | 422.15              |

Table 2:  $L_c=0.2$ ;  $dc=0.001$ ;  $L=0.7$ ;  $\rho=7850$ kg/m<sup>3</sup>;  $w=0.05$ m;  $h=0.01$ m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 68.136              |
| 2           | 184.16              |
| 3           | 387.48              |

Table 3:  $L_c=0.3$ ;  $dc=0.001$ ;  $L=0.7$ ;  $\rho=7850$ kg/m<sup>3</sup>;  $w=0.05$ m;  $h=0.01$ m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 66.977              |
| 2           | 184.65              |
| 3           | 359.28              |

**Table 4:** Lc=0.1; dc=0.002; L=0.7; rho=7850kg/m<sup>3</sup>; w=0.05m; h=0.01m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 71.198              |
| 2           | 185.46              |
| 3           | 422.88              |

**Table 5:** Lc=0.2; dc=0.002; L=0.7; rho=7850kg/m<sup>3</sup>; w=0.05m; h=0.01m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 67.962              |
| 2           | 184.24              |
| 3           | 386.61              |

**Table 6:** Lc=0.3; dc=0.002; L=0.7; rho=7850kg/m<sup>3</sup>; w=0.05m; h=0.01m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 67.031              |
| 2           | 184.11              |
| 3           | 357.34              |

**Table 7:** Lc=0.1; dc=0.003; L=0.7; rho=7850kg/m<sup>3</sup>; w=0.05m; h=0.01m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 69.800              |
| 2           | 184.94              |
| 3           | 420.15              |

**Table 8:** Lc=0.2; dc=0.003; L=0.7; rho=7850kg/m<sup>3</sup>; w=0.05m; h=0.01m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 68.266              |
| 2           | 184.85              |
| 3           | 387.04              |

**Table 9:** Lc=0.3; dc=0.003; L=0.7; rho=7850kg/m<sup>3</sup>; w=0.05m; h=0.01m

| Mode number | Frequency (rad/sec) |
|-------------|---------------------|
| 1           | 66.981              |
| 2           | 184.65              |
| 3           | 375.00              |

**Table 10**

| ANSYS results in Tabular form cantilever | ANSYS    |           | Radians/s | Radians/s | Radians/s |
|--|----------|-----------|-----------|-----------|-----------|
|  | Location | crackSize | w1        | w2        | w3        |
| uncracked                                | 0        | 0         | 0         | 0         | 0         |
| 1  | 0.1      | 0.1       | 72.643    | 188.28    | 422.15    |
| 2  | 0.1      | 0.2       | 71.198    | 185.46    | 422.88    |
| 3  | 0.1      | 0.3       | 69.8      | 184.94    | 420.15    |
| 4  | 0.1      | 0         | 0         | 0         | 0         |
| 5  | 0.2      | 0.1       | 68.136    | 184.16    | 387.48    |
| 6  | 0.2      | 0.2       | 67.962    | 184.24    | 386.61    |
| 7  | 0.2      | 0.3       | 68.266    | 184.85    | 387.04    |
| 8  | 0.2      | 0         | 0         | 0         | 0         |
| 9  | 0.3      | 0.1       | 66.977    | 184.65    | 359.28    |
| 10                                       | 0.3      | 0.2       | 67.031    | 184.11    | 357.34    |
| 11                                       | 0.3      | 0.3       | 66.981    | 184.65    | 375       |

Frequencies and mode shapes are obtained by solving the Eigen Value Problem (EVP)  $[K] - \omega^2 [M] = 0$ .

By this method the natural frequencies are

**Table 11:** Theoretical results for different crack location and size

| cantilever | Theory (values) |           | Radians/s | Radians/s | Radians/s |
|------------|-----------------|-----------|-----------|-----------|-----------|
|            | Location        | crackSize | w1        | w2        | w3        |
| uncracked  | 0               | 0         |           |           |           |
| 1          | 0.1             | 0.1       | 69.62     | 122.56    | 935.87    |
| 2          | 0.1             | 0.2       | 68.54     | 120.48    | 630.52    |
| 3          | 0.1             | 0.3       | 67.14     | 118.06    | 495.26    |
| 4          | 0.1             | 0         | 0         | 0         | 0         |
| 5          | 0.2             | 0.1       | 70.48     | 110.59    | 685.85    |
| 6          | 0.2             | 0.2       | 68.69     | 105.63    | 564.22    |
| 7          | 0.2             | 0.3       | 68.53     | 100.39    | 483.78    |
| 8          | 0.2             | 0         | 0         | 0         | 0         |
| 9          | 0.3             | 0.1       | 73.89     | 110.96    | 432.96    |
| 10         | 0.3             | 0.2       | 73.27     | 101.07    | 414.18    |
| 11         | 0.3             | 0.3       | 72.08     | 92.17     | 398.67    |
| 12         | 0.3             | 0         | 0         | 0         | 0         |

**Table 12:** Comparison of analytical /Theoretical results with ANSYS results

| Location | Size | Radians/s | Radians/s | Radians/s | Radians/s | Radians/s | Radians/s |
|----------|------|-----------|-----------|-----------|-----------|-----------|-----------|
|          |      | w1(Th)    | w1(ANSYS) | w2(Th)    | w2(ANSYS) | w3(Th)    | w3(ANSYS) |
|          |      | 0         | 0         |           | 0         |           | 0         |
| 0.1      | 0.1  | 69.62     | 72.643    | 122.56    | 188.28    | 935.87    | 422.15    |
| 0.1      | 0.2  | 68.54     | 71.198    | 120.48    | 185.46    | 630.52    | 422.88    |
| 0.1      | 0.3  | 67.14     | 69.8      | 118.06    | 184.94    | 495.26    | 420.15    |
| 0.1      | 0    | 0         | 0         | 0         | 0         | 0         | 0         |
| 0.2      | 0.1  | 70.48     | 68.136    | 110.59    | 184.16    | 685.85    | 387.48    |
| 0.2      | 0.2  | 68.69     | 67.962    | 105.63    | 184.24    | 564.22    | 386.61    |
| 0.2      | 0.3  | 68.53     | 68.266    | 100.39    | 184.85    | 483.78    | 387.04    |
| 0.2      | 0    | 0         | 0         | 0         | 0         | 0         | 0         |
| 0.3      | 0.1  | 73.89     | 66.977    | 110.96    | 184.65    | 432.96    | 359.28    |
| 0.3      | 0.2  | 73.27     | 67.031    | 101.07    | 184.11    | 414.18    | 357.34    |
| 0.3      | 0.3  | 72.08     | 66.981    | 92.17     | 184.65    | 398.67    | 375       |

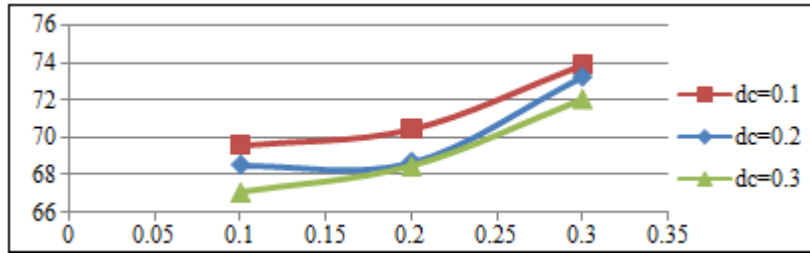


Figure 5: First natural frequency for different crack locations on x-axis

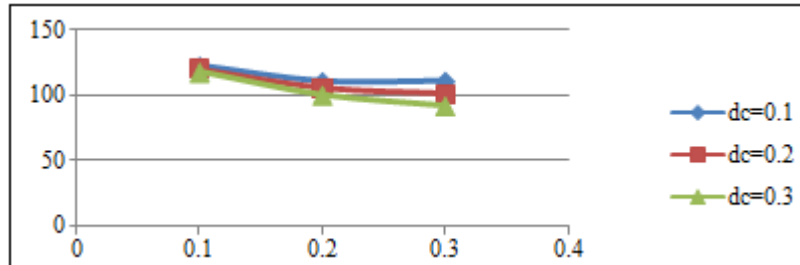


Figure 6: Second natural frequency for different crack locations Lc on x-axis

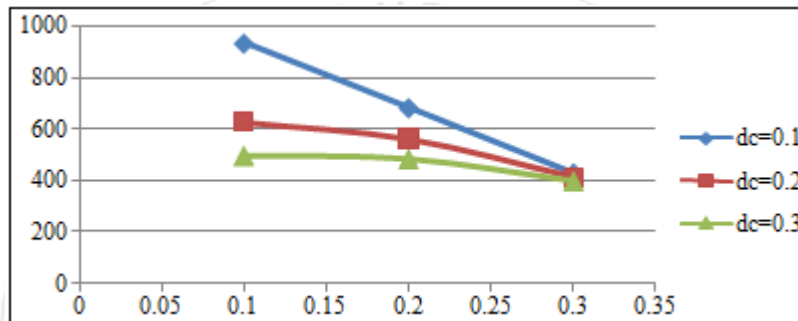


Figure 7: Third natural frequency for different crack locations on x-axis

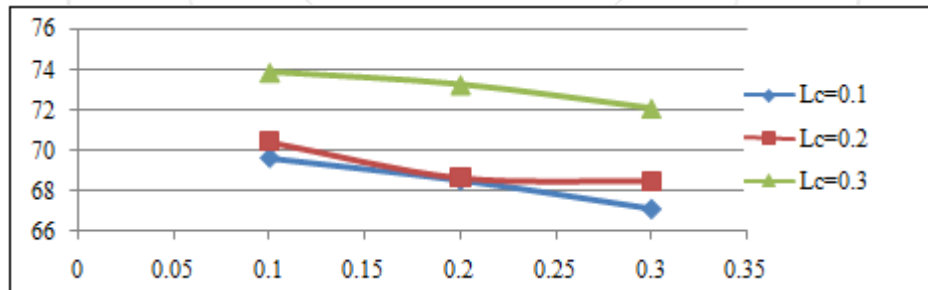


Figure 8: First natural frequency for different crack sizes on x-axis

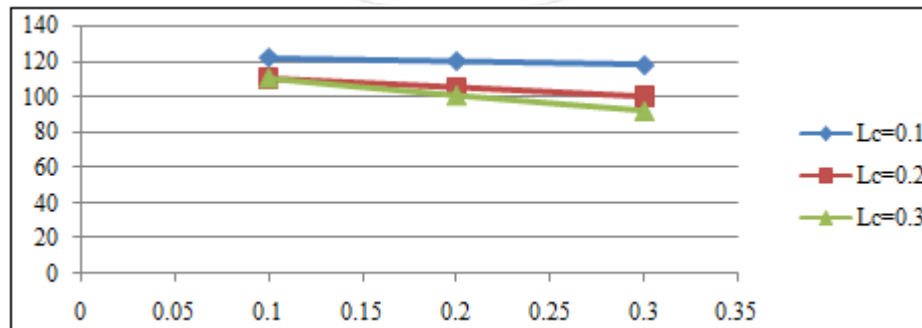


Figure 9: Second natural frequency for different crack sizes on x-axis

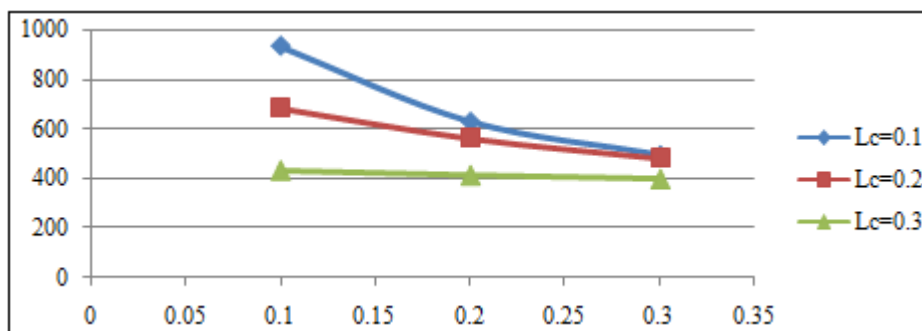


Figure 10: Third natural frequencies for different crack sizes on x-axis

## 6. Conclusion

The frequency of the cracked cantilever beam decreases with increase in the crack depth for all modes of vibration. When the crack location shifts towards the fixed end of the cantilever beam the natural frequency decreases in first mode of vibration. But for second and third modes of vibrations the frequency of the cracked beams for the same crack depth varies differently. The effect of crack is more near the fixed end than at far free end. The exact approx location of the crack and the depth can be computed by the natural frequencies of the beam, 1st three natural frequencies are taken into consideration.

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### Appendix – A

MATLAB program for calculating Eigen values:

```
%cantileverbeam-compliance matrix-ks
E=210E9;
L=0.700;%total length of beam
L1=0.300;%crack location from fixed end
Lc=L1;
L2=L-Lc;%remaining length after crack
b=0.050;% width of beam
h=0.010;% height of beam
I=b*h*h*h/12;% moment of inertia of beam
%Ic=266;
%z=300;
rho=7850;%density of material
A=b*h;% cross sectional area of beam
dcr=0.001;% depth of crack
%Elements of stiffness matrix for first element
K111=12*E*I/L1^3;
K112=6*E*I/L1^2;
K113=-12*E*I/L1^3;
K114=6*E*I/L1^2;

K121=6*E*I/L1^2;
K122=4*E*I/L1;
K123=-6*E*I/L1^2;
K124=2*E*I/L1;

K131=-12*E*I/L1^3;
K132=-6*E*I/L1^2;
K133=12*E*I/L1^3;
K134=-6*E*I/L1^2;

K141=6*E*I/L1^2;
K142=2*E*I/L1;
K143=-6*E*I/L1^2;
K144=4*E*I/L1;
%Elements of stiffness matrix for second element
K211=12*E*I/L2^3;
K212=6*E*I/L2^2;
K213=-12*E*I/L2^3;
K214=6*E*I/L2^2;
K221=6*E*I/L2^2;
K222=4*E*I/L2;
K223=-6*E*I/L2^2;
K224=2*E*I/L2;
```

K231 = -12\*E\*I/L2^3;  
 K232 = -6\*E\*I/L2^2;  
 K233 = 12\*E\*I/L2^3;  
 K234 = -6\*E\*I/L2^2;

K241 = 6\*E\*I/L2^2;  
 K242 = 2\*E\*I/L2;  
 K243 = -6\*E\*I/L2^2;  
 K244 = 4\*E\*I/L2;

ks = 0.70\*((h/dcr).^1.2 - 1)\*E\*I/h %rotational stiffness due to crack

omgnc1 = 3.56\*sqrt(E\*I/(A\*rho\*L^4))%First natural frequency based on beam theory without crack

KG = [ 0 0 0 0 0 0;  
 0 0 0 0 0 0;  
 0 0 K133+K211+ks K134+K212 K213-ks K214;  
 0 0 K143+K221 K144+K222 K223 K224;  
 0 0 K231-ks K232 K233+ks K234;  
 0 0 K241 K242 K243 K244];

MG = rho\*A/420\*[0 0 0 0 0 0;  
 0 0 0 0 0 0;  
 0 0 156\*L1+156\*L2 -22\*L1\*L1+22\*L2 54\*L2 -13\*L2;  
 0 0 -22\*L1\*L1+22\*L2 4\*L1\*L1\*L1+4\*L2\*L2  
 13\*L2 -3\*L2\*L2;  
 0 0 54\*L2 13\*L2\*L2 156\*L2 -22\*L2\*L2;  
 0 0 -13\*L2\*L2 -3\*L2\*L2\*L2 -22\*L2\*L2  
 4\*L2\*L2\*L2];

EIG = eig(KG-MG);%Eigen values

omegan = sqrt(EIG);%Natural frequencies rad/sec

fprintf('omgn%24.18fn',omegan)

