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Vibration-Based Methods for Detecting a Crack in a Fixed Free Beam

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Abstract: ***The phenomenon of vibration can be applied to identify the crack size and location. In particular, cracks decrease the stiffness and the natural frequency thus, causing specimens to fail under normal working conditions. This paper presents the application of the vibration-based technique for detecting the location and the size of a crack in structures. The crack is modelled by a rotational spring. The predicted crack depth and location are compared with the actual data obtained from finite element models.***

Keywords: Eigen value problem, rotational spring stiffness, crack, MATLAB

1. Introduction

Cracks are one of the main causes of structural failure. In order to reduce or eliminate the sudden failure of structures, they should be regularly checked for cracks. The crack present in the component imparts local flexibility to the element, which leads to reduction in natural frequencies and mode shapes. It is possible to estimate the location and the size of a crack by measuring changes in the vibration parameters. Crack is modelled as rotational spring having stiffness ks and is added to global stiffness matrix where rotational degrees of freedom are there. And no change in mass matrix is assumed because of small size of crack.

2. Literature Review

The formation of cracks in a structure affects the local stiffness and flexibility of the structure. This problem has been a subject of investigation in many papers. In the present study an attempt has been made to the reviews on the isotropic cracked cantilever beam.

Adams et al [1] have presented a method for damage detection in a one-dimensional component using the natural frequencies of longitudinal vibrations. They modelled the damage by a linear spring. Petroski [2] has proposed a tech-nique by which the section modulus is appropriately reduced to model a crack. Chondros and Dimarogonas [3] have used the concept of a rotational spring to model the crack and proposed a method to identify cracks in welded joints. Rizos et al [4] have applied this technique to a cantilever beam and detected the crack location through the measurement of amplitudes at two points of component vibrating at one of its natural modes. Liang et al [5] have proposed a similar method, but it required measurements of the three fundamental fre- quencies of the beam. Dimarogonas and Paipetis [6] calculated the rotational spring constant of a beam of a rectangular cross section from the crack strain energy function. Ostachowicz and Krawczuk [7] obtained the relationships between the reduced stiffness of the cracked section and the crack size of a beam of rectangular cross section from the decrease in the elastic deformation energy of the crack expressed in terms of the stress intensity factor. Barad et al [8] used relations between the rotational spring constant, crack size and location and proposed a method for detecting a crack in a cantilever beam. In most cases, the proposed methods are applied to a cantilever beam although the simply supported beam is commonly used.

3. Theory

For free vibration the equation can be written as,

$$[M]\{q\&\&\}+ [K]\{q\} = 0 \qquad eq(1)$$

The equation (1) represents an Eigen value problem and the roots of the equation gives rise to square of the natural frequency given by the equation,

$$[K] - \omega n^2 [M] = 0. \qquad eq(2)$$

Where

$$K = \begin{bmatrix} 12EI/L3 & 6EI/L2 & -12EI/L3 & 6EI/L2 & 0 & 0 \\ 6EI/L2 & 4EI/L & -\frac{6EI}{L2} + ks & 2EI/L & 0 - ks & 0 \\ -12EI/L3 & -6EI/L2 & \frac{12EI}{L3} + \frac{12EI}{L13} & -\frac{6EI}{L2} + \frac{6EI}{L12} & -12EI/L13 & 6EI/L12 \\ 6EI/L2 & 2EI/L & -\frac{6EI}{L2} + \frac{6EI}{L12} - ks & \frac{4EI}{L} + \frac{4EI}{L1} & -\frac{6EI}{L12} + ks & 2EI/L1 \\ 0 & 0 & -12EI/L13 & -6EI/L12 & 12EI/L13 & -6EI/L12 \\ 0 & 0 & 6EI/L12 & 2EI/L1 & -6EI/L12 & 4EI/L1 \end{bmatrix}$$

I=moment of inertia of the section with respect to z-axis
E = Young's modulus of elasticity
ks=stiffness of rotational spring=ks=0.70*((h/dcr).^1.2 - 1)*E*I/h
and dcr = depth of crack and
h = total height of beam

Crack is modelled as rotational spring having stiffness ks and is added to global stiffness matrix where rotational degrees of freedom is there. No changes in mass matrix is assumed due to crack. And the mass matrix is where L is the crack location from fixed end and L1 is the remaining length of beam after crack.

$$M = rhoA/420 \begin{bmatrix} 156L & 22L2 & 54L & -13L2 & 0 & 0 \\ 22L2 & 4L3 & 13L2 & -3L3 & 0 & 0 \\ 54L & 13L2 & 156L+156L1 & -22L2+22L12 & 54L1 & -13L12 \\ -13L2 & -3L3 & -22L2+22L12 & 4L3+4L13 & 13L12 & -3L13 \\ 0 & 0 & 54L1 & 13L12 & 156L1 & -22L12 \\ 0 & 0 & -13L12 & -3L13 & -22L12 & 4L13 \end{bmatrix}$$

rho = mass density of the material
A = cross sectional area of the beam element

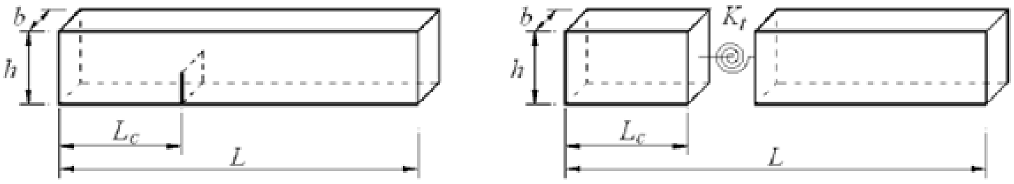

Figure 1: A beam with a crack and representation of crack with a rotational spring

4. Numerical Examples

The numerical analysis is carried out for a simply supported beam of rectangular cross section with a single crack. The natural frequencies of transverse vibration at different crack location and size are calculated by the finite element method, using a computer program (ANSYS). The beam is discretized by 8-node solid185 elements. The line is divided into 20 parts. Rectangular area is created first to the length and height of beam and crack area is subtracted. Remaining area is extruded to the required width of the beam. Boundary conditions are taken as one end fixed all degrees of freedom and the other end only rotation about x-axis is constrained.

The dimensions of the beam are: Length, L =0.7 m, height, h = 0.01 m, width, b = 0.05 m. The material properties are: Modulus of elasticity, E = 2.1×1011 N/m2, density, rho= 7850 kg/m3, Poisson's
Coefficient, μ= 0.3. The natural frequencies for the studied cases are shown in Table 10.

The corresponding results with beam theory are shown in Table 11.

5. Results

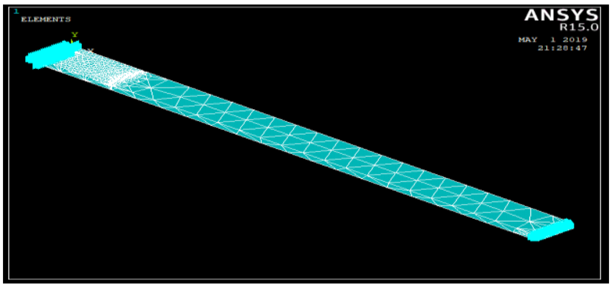

Figure 2: Crack at 0.1m location where depth of crack dc=0.001mm

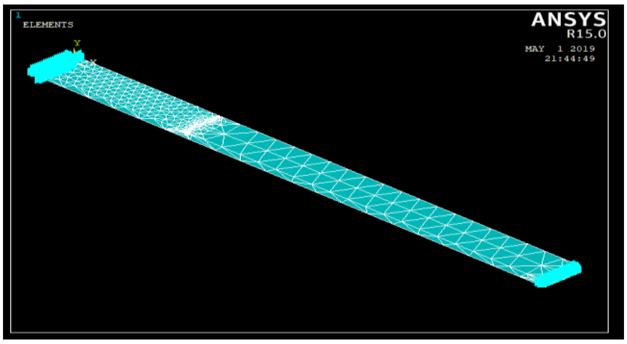

Figure 3: Crack at 0.2 m location where depth of crack dc=0.001mm

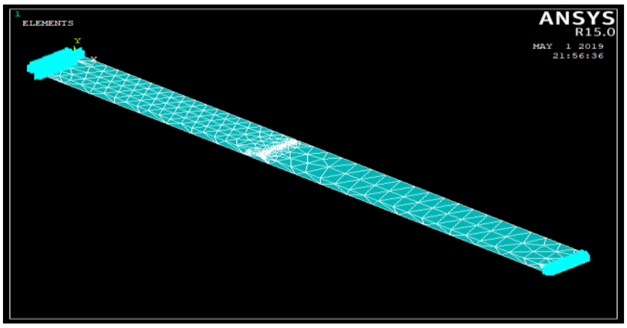

Figure 4: Crack at 0.3 m location where depth of crack dc=0.001mm

Table 1: Lc=0.1; dc=0.001; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	72.643
2	188.28
3	422.15

Table 2: Lc=0.2; dc=0.001; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	68.136
2	184.16
3	387.48

Table 3: Lc=0.3; dc=0.001; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	66.977
2	184.65
3	359.28

Table 4: Lc=0.1; dc=0.002; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	71.198
2	185.46
3	422.88

Table 5: Lc=0.2; dc=0.002; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	67.962
2	184.24
3	386.61

Table 6: Lc=0.3; dc=0.002; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	67.031
2	184.11
3	357.34

Table 7: Lc=0.1; dc=0.003; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	69.800
2	184.94
3	420.15

Table 8: Lc=0.2; dc=0.003; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	68.266
2	184.85
3	387.04

Table 9: Lc=0.3; dc=0.003; L=0.7; rho=7850kg/m3; w=0.05m; h=0.01m

Mode number	Frequency (rad/sec)
1	66.981
2	184.65
3	375.00

Table 10

ANSYS results in Tabular form cantilever	ANSYS Location	crackSize	Radians/s w1	Radians/s w2	Radians/s w3
uncracked	0	0	0	0	0
1	0.1	0.1	72.643	188.28	422.15
2	0.1	0.2	71.198	185.46	422.88
3	0.1	0.3	69.8	184.94	420.15
4	0.1	0	0	0	0
5	0.2	0.1	68.136	184.16	387.48
6	0.2	0.2	67.962	184.24	386.61
7	0.2	0.3	68.266	184.85	387.04
8	0.2	0	0	0	0
9	0.3	0.1	66.977	184.65	359.28
10	0.3	0.2	67.031	184.11	357.34
11	0.3	0.3	66.981	184.65	375

Frequencies and mode shapes are obtained by solving the Eigen Value Problem (EVP) [K] – ω2 [M] = 0.

By this method the natural frequencies are

Table 11: Theoretical results for different crack location and size

cantilever	Theory (values)		Radians/s	Radians/s	Radians/s
	Location	crackSize	w1	w2	w3
uncracked	0	0			
1	0.1	0.1	69.62	122.56	935.87
2	0.1	0.2	68.54	120.48	630.52
3	0.1	0.3	67.14	118.06	495.26
4	0.1	0	0	0	0
5	0.2	0.1	70.48	110.59	685.85
6	0.2	0.2	68.69	105.63	564.22
7	0.2	0.3	68.53	100.39	483.78
8	0.2	0	0	0	0
9	0.3	0.1	73.89	110.96	432.96
10	0.3	0.2	73.27	101.07	414.18
11	0.3	0.3	72.08	92.17	398.67
12	0.3	0	0	0	0

Table 12: Comparison of analytical /Theoretical results with ANSYS results

Location	Size	Radians/s w1(Th)	Radians/s w1(ANSYS)	Radians/s w2(Th)	Radians/s w2(ANSYS)	Radians/s w3(Th)	Radians/s w3(ANSYS)
		0	0		0		0
0.1	0.1	69.62	72.643	122.56	188.28	935.87	422.15
0.1	0.2	68.54	71.198	120.48	185.46	630.52	422.88
0.1	0.3	67.14	69.8	118.06	184.94	495.26	420.15
0.1	0	0	0	0	0	0	0
0.2	0.1	70.48	68.136	110.59	184.16	685.85	387.48
0.2	0.2	68.69	67.962	105.63	184.24	564.22	386.61
0.2	0.3	68.53	68.266	100.39	184.85	483.78	387.04
0.2	0	0	0	0	0	0	0
0.3	0.1	73.89	66.977	110.96	184.65	432.96	359.28
0.3	0.2	73.27	67.031	101.07	184.11	414.18	357.34
0.3	0.3	72.08	66.981	92.17	184.65	398.67	375

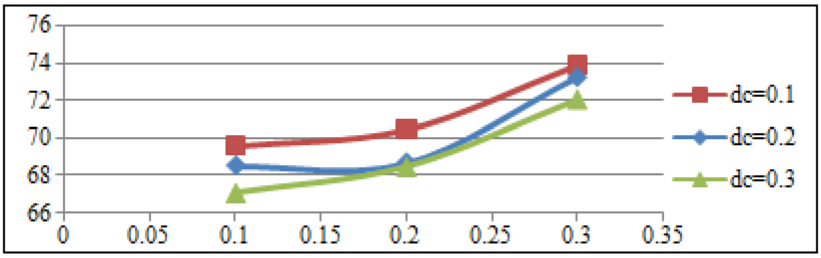

Figure 5: First natural frequency for different crack locations on x-axis

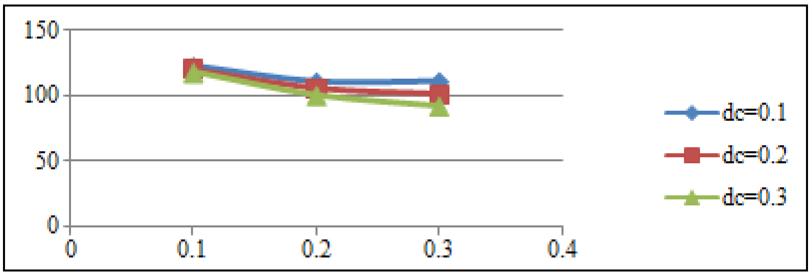

Figure 6: Second natural frequency for different crack locations Lc on x-axis

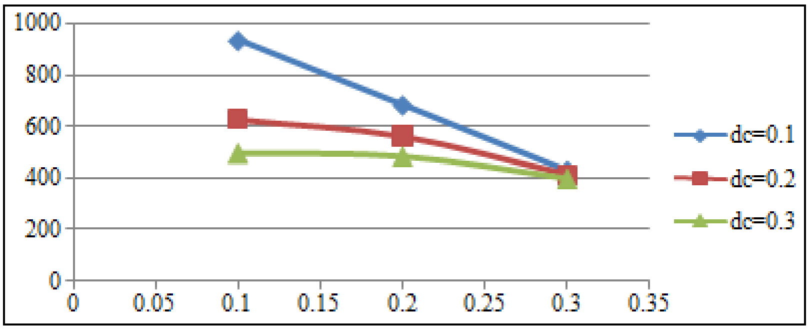

Figure 7: Third natural frequency for different crack locations on x-axis

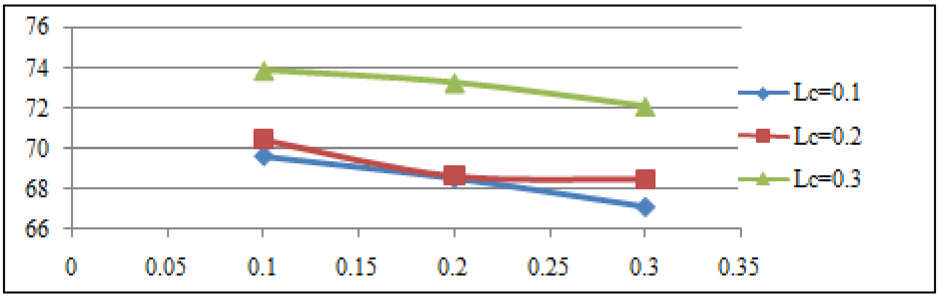

Figure 8: First natural frequency for different crack sizes on x-axis

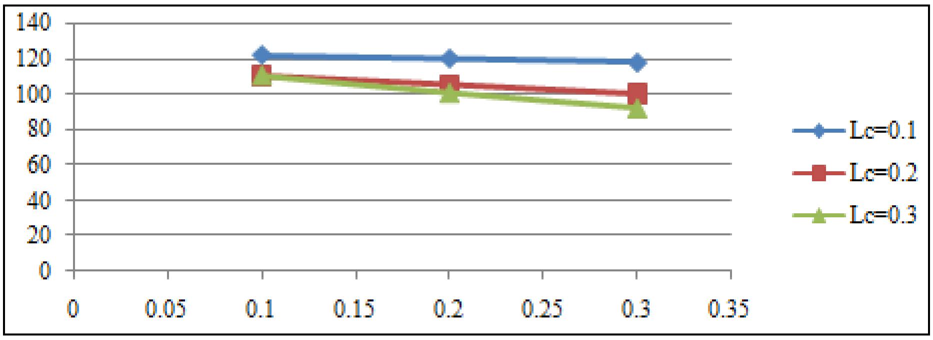

Figure 9: Second natural frequency for different crack sizes on x-axis

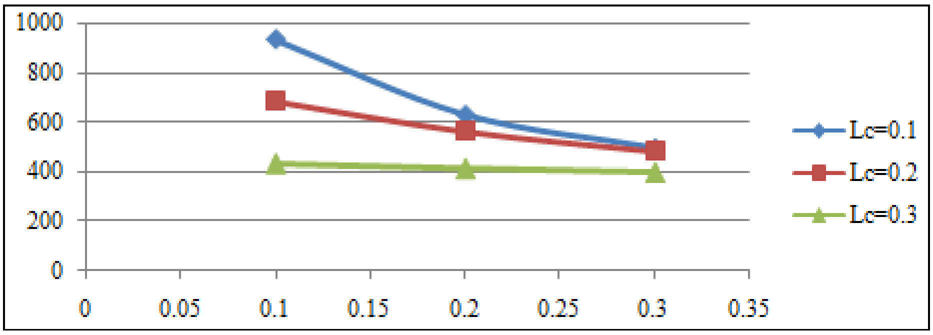

Figure 10: Third natural frequencies for different crack sizes on x-axis

6. Conclusion

The frequency of the cracked cantilever beam decreases with increase in the crack depth for all modes of vibration. When the crack location shifts towards the fixed end of the cantilever beam the natural frequency decreases in first mode of vibration. But for second and third modes of vibrations the frequency of the cracked beams for the same crack depth varies differently. The effect of crack is more near the fixed end than at far free end. The exact approx location of the crack and the depth can be computed by the natural frequencies of the beam,1st three natural frequencies are taken into consideration.

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Appendix – A

MATLAB program for calculating Eigen values:

```
%cantileverbeam-compliance matrix-ks
E=210E9;
L=0.700;%total length of beam
L1=0.300;%crack location from fixed end
Lc=L1;
L2=L-Lc;%remaining length after crack
b=0.050;% width of beam
h=0.010;% height of beam
I=b*h*h*h/12;% moment of inertia of beam
%Ic=266;
%z=300;
rho=7850;%density of material
A=b*h;% cross sectional area of beam
dcr=0.001;% depth of crack
%Elements of stiffness matrix for first element
K111=12*E*I/L1^3;
K112=6*E*I/L1^2;
K113=-12*E*I/L1^3;
K114=6*E*I/L1^2;

K121=6*E*I/L1^2;
K122=4*E*I/L1;
K123=-6*E*I/L1^2;
K124=2*E*I/L1;

K131=-12*E*I/L1^3;
K132=-6*E*I/L1^2;
K133=12*E*I/L1^3;
K134=-6*E*I/L1^2;

K141=6*E*I/L1^2;
K142=2*E*I/L1;
K143=-6*E*I/L1^2;
K144=4*E*I/L1;
%Elements of stiffness matrix for second element
K211=12*E*I/L2^3;
K212=6*E*I/L2^2;
K213=-12*E*I/L2^3;
K214=6*E*I/L2^2;
K221 =6*E*I/L2^2;
K222 =4*E*I/L2;
K223=-6*E*I/L2^2;
K224 =2*E*I/L2;
```

```
K231 =-12*E*I/L2^3;
K232 =-6*E*I/L2^2;
K233=12*E*I/L2^3;
K234=-6*E*I/L2^2;

K241=6*E*I/L2^2;
K242=2*E*I/L2;
K243=-6*E*I/L2^2;
K244=4*E*I/L2;
ks=0.70*((h/dcr).^1.2 -1)*E*I/h %rotational stiffness due to
crack
omgnc1=3.56*sqrt(E*I/(A*rho*L^4))%First natural
frequency based on beam theory without crack

KG=[    0   0   0       0       0   0;
        0   0   0       0       0   0;
        0   0   K133+K211+ks    K134+K212   K213-ks K214;
        0   0   K143+K221   K144+K222   K223    K224;
        0   0       K231-ks     K232        K233+ks K234;
        0   0       K241        K242    K243    K244];

MG=rho*A/420*[0     0   0   0   0   0;
        0   0   0   0   0   0;
        0   0   156*L1+156*L2   -22*L1*L1+22*L2 54*L2   -
13*L2;
        0   0   -22*L1*L1+22*L2     4*L1*L1*L1+4*L2*L2
13*L2   -3*L2*L2;
        0   0   54*L2       13*L2*L2        156*L2  -22*L2*L2;
        0   0   -13*L2*L2       -3*L2*L2*L2     -22*L2*L2
4*L2*L2*L2];
EIG=eig(KG-MG);%Eigen values
omegan=sqrt(EIG);%Natural frequencies rad/sec
fprintf('omgn%24.18f\n',omegan)
```