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An EOQ Model with Backorder for Two Defective Quality Products

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Abstract: ***The EOQ model will have a period of its disclosure in two years, as of late still, many experts have been utilizing elective ways to deal with model and explain stock frameworks. The EOQ models have been created utilizing diverse enhancement strategies. Nonetheless, in huge numbers of the works that arrangement with the EOQ with delay purchases for one defective quality product for direct expense is considered. This paper proposes another simple technique which utilizes fundamental ideas of logical backorder for two defective quality products. The proposed technique finds the ideal part size and backordering level considering both straight and settled delay purchases costs. Furthermore, this paper exhibits a survey of the distinctive enhancement techniques used in inventory hypothesis.***

Keywords: Economic order quantity; Economic production quantity; Backorders

1. Introduction

In general EOQ model can be solved based on the formulation of various assumptions. In general model of EOQ the issues of quality is unnoticed. Through the production, it can be perceived that there are defective items are to be produced. These can be rejected or reworked and try to sell it off. It may be returned by the customers and it causes to extra cost. So, it is necessary to consider the quality related cost for model construction. Many researchers extended the classical EOQ model to include defective quality production process; and try to construct new model.

Porteus is the first researcher who implemented the model for defective product. He discussed about a system to producing only good products, the probability can increased for defective products, at which time all other units were found to be defective, he studied and developed a model to study the optimal setup deal to moderate the probability.

Rosen Blatl& Lee studied that the production of defective items raise smaller lost sizes. Theysupposed that the lead time between the production in sorting of production can be goes out of control and it causes the defective production. They used to implement the process, assessment during the construction process to avoid defective production.

Zhaug and Gerdrak studied about the lot sizing and study policy of model for defective items are to be produced. They studied about deviation from the optimum quality of production and replacement of non-defective products.

Urban formulated model which incorporated about the learning effects and possibilities ofdefective items in production.

Chan, Ibrahim and Luchert have discussed for 100% inspection in order to classify the quantity of decent products towards the defective item in a lot. They assume that defective production could not be used or sold to a purchaser at lower cost.

Ouyang and Chang assess the quality improvement model on the change lot models included lead time and partial backorders. They recommended reducing the production lost size, reordering point, process quality level and leading time.

Ben Daya presented multi-stage lot sizing models for defective production in a lot. The study is based on defective lot sizing decision and error in process.

Chen and Chang developed model for defective production process when partial back order is construct the model for quality improvement and reduction set up cost. The discussion stated clearly that classical model are developed and considered regarding quality issues and defective production.

This study is extension of Salameh and Jaber's model; it considered two defective quality products.

2. Theoretical Framework

The study presents an extension of models given by Salmeh and Jaber's. When a batch of items arrives from supplier, imperfect quality items do exist in the batch. Therefore, screening processes are required before the items are delivered to customers. Salameh & Jaber (2000) first extended the classical economic order quantity (EOQ) model by integrating a screening process for imperfect quality items into the model. We also extend necessary change in notations employed by them they are as follows:

$$y_1 = Order\ Size\ of\ product\ one$$
$$y_2 = Order\ Size\ of\ product\ two$$
$$C_1 = Unit\ Variable\ Cost\ of\ Product\ one$$
$$C_2 = Unit\ Variable\ Cost\ of\ Product\ two$$
$$K = fixed\ cost\ of\ placing\ an\ order$$
$$P_1 = Percentage\ of\ defective\ items\ in\ lot\ one$$
$$P_2 = Percentage\ of\ defective\ items\ in\ lot\ two$$
$$f(P_1 + P_2) = probabilitydensity\ function\ of\ P_1 and\ P_2$$

$S_1 = Selling\ prince\ per\ unit\ for\ product\ one$
$S_2 = Selling\ prince\ per\ unit\ for\ product\ two$
$V_1 = Selling\ prince\ of\ defective\ item\ per\ unit\ for\ product\ one$
$V_2 = Selling\ prince\ of\ defective\ item\ per\ unit\ for\ product\ two$
$x = screening\ rate$
$d = screening\ cost$
$T = Cycle$
$b = shortage\ cost\ per\ unit$
$B = Expected\ shorting\ level$
$M = Maximum\ inventory\ level$
$t = palnning\ time$

3. Assumptions

1) The produced products are not totally perfect, it can be used for another process
2) A lot size delivered immediately and the purchasing price is taken for each unit
3) The probability, density function is known when each lot is received for defective products
4) Defective production is selling at discount rate
5) Shortage is permitted

Considering the assumptions we may derive the model as follows:

$$f(y_1, y_2, B) = K + C_1(y_1+y_2) + C_2(y_1+y_2) + h\left[\frac{(y_1+y_2)\{1-(P_1+P_2)\} - B}{2} t_1 + \frac{(P_1+P_2)(y_1+y_2)^2}{x} + \frac{Bb}{2} t_2\right] \quad \ldots(1)$$

The profits for process of defective product can be written:

$$g(y_1, y_2, B) = (S_1+S_2)(y_1+y_2)[1-(P_1+P_2)] + (V_1+V_2)(y_1+y_2)(P_1+P_2) \quad \ldots(2)$$

The profit π can be determine as follows by using (1) and (2)

$$\pi(y_1, y_2, B) = g(y_1, y_2, B) - f(y_1, y_2, B)$$

$$\pi(y_1, y_2, B) = [(S_1+S_2)(y_1+y_2)[1-(P_1+P_2)] + (V_1+V_2)(y_1+y_2)(P_1+P_2)] - \left[K + C_1(y_1+y_2) + C_2(y_1+y_2) + h\left[\frac{(y_1+y_2)\{1-(P_1+P_2)\} - B}{2} t_1 + \frac{(P_1+P_2)(y_1+y_2)^2}{x} + \frac{Bb}{2} t_2\right]\right] \quad \ldots(3)$$

Dividing total profit (π) by per cycle, $= \frac{(y_1+y_2)[1-E(P_1+P_2)-B]}{D}$, replacing t_1 by $\frac{(y_1+y_2)[1-E(P_1+P_2)-B]}{D}$ and t_2 by $\frac{B}{D}$ in (3) the total profit per unit time can be written as:

$$\pi(y_1, y_2, B) = D\left[(S_1+S_2) - (V_1+V_2) + \frac{h(y_1+y_2)}{x}\right] + D\left[(V_1+V_2) - \frac{h(y_1+y_2)}{x} - (C_1+C_2) - d - \frac{K}{(y_1+y_2)}\right] . E\left[\frac{1}{1-(P_1+P_2)}\right] - \frac{h(y_1+y_2)\{1-E(P_1+P_2)\} - B}{2} + \frac{B^2(b+h)}{2(y_1+y_2)[1-E(P_1+P_2)]} \quad \ldots(5)$$

The profit function for two products can be solved by taking derivative of π. w.r.t. (y_1, y_2, B) respectively.

$$\frac{\partial \pi}{\partial y_1} = \frac{Dh}{x} + D\left[\frac{-h}{x} - K(y_1+y_2)^{-1}\right] . E\left[\frac{1}{1-(P_1+P_2)}\right] - \frac{h\{1-E(P_1+P_2)\}}{2} + \frac{B^2(b+h)}{2(y_1+y_2)[1-E(P_1+P_2)]}$$

$$= \frac{Dh}{x} + D\left[-\frac{h(y_1+y_2)^{-1}}{x(y_1+y_2)^2} + xK\right] . E\left[\frac{1}{1-(P_1+P_2)}\right] - \frac{h\{1-E(P_1+P_2)\}}{2} + \frac{B^2(b+h)}{2(y_1+y_2)^2}$$

$$= \frac{Dh}{x} + D\left[-\frac{(y_1+y_2)^2}{x(y_1+y_2)^2}(-h+xK)\right] + E[2(y_1+y_2)^2 - 2h(1-E(P_1+P_2)).1 - \frac{E(P_1+P_2)(y_1+y_2)^2 + 2B^2(b+h)(1-(P_1+P_2))}{2(1-(P_1+P_2))(y_1+y_2)^2}$$

Same way final values for optimum of y_1^* *and* y_2^*

$$= \left[\frac{2DK\left(\frac{1}{1-(P_1+P_2)}\right)}{h(1-E(P_1+P_2)).\frac{b}{b+h} - \left(\frac{2D}{x}\right)\left[1 - E\left[\frac{1}{1-E(P_1+P_2)}\right]\right]}\right]^{1/2}$$

It can be determine for multiple product during process and

$$\frac{\partial \pi}{\partial B} = h - \frac{B(h+b)}{(y_1+y_2)[1-E(P_1+P_2)]}$$

$$\therefore B^* = \frac{(y_1+y_2)^*[1-E(P_1+P_2)h]}{(h+b)}$$

For maximum inventory level of defective products we have,

$$M = (y_1+y_2) - B$$

by using optimum solution from derived equation

$$M^* = \frac{(y_1+y_2)^*[1-E(P_1+P_2)b]}{h+b}$$

to study the sufficient condition, the Hessian matrix can be determine as;

$$H = \begin{bmatrix} \frac{\partial^2(y_1+y_2, B)}{x(y_1+y_2)^2} & \frac{\partial^2 \pi(y_1+y_2, B)}{\partial(y_1+y_2)\partial B} \\ \frac{\partial^2 \pi(y_1+y_2, B)}{\partial(y_1+y_2)\partial B} & \frac{\partial^2 \pi(y_1+y_2, B)}{\partial B^2} \end{bmatrix}$$

H here,

$$\frac{\partial^2 \pi(y_1+y_2,B)}{\partial(y_1+y_2)} = \frac{-2\left[DK\,.E\left(\frac{1}{1-(P_1+P_2)}\right)\left(1-E(P_1+P_2)\right)+B^2(h+S_1+S_2)\right.}{\{1-E(P_1+P_2)](y_1+y_2)^3\}}$$

$$\frac{\partial^2 \pi(y_1+y_2,B)}{\partial B\,\partial(y_1+y_2)} = \frac{B\;(h+S_1+S_2)}{\{1-E(P_1+P_2)](y_1+y_2)^2\}}$$

$$\frac{\partial^2 \pi(y_1+y_2,B)}{\partial B^2} = \frac{-\,(h+S_1+S_2)}{\{1-E(P_1+P_2)](y_1+y_2)\}}$$

Based on Hessian Matrix,

$$|H_{11}| = \frac{-2\left[DK\,.E\left(\frac{1}{1-(P_1+P_2)}\right)\left(1-E(P_1+P_2)\right)+B^{*2}(h+S_1+S_2)\right.}{\{1-E(P_1+P_2)](y_1+y_2)^{*3}\}} < 0$$

$$|H_n| = \frac{2\,DK\,.E\left(\frac{1}{1-(P_1+P_2)}\right)\left(1-E(P_1+P_2)\right)(h+S_1+S_2)+B^{*2}(h+S_1+S_2)^2}{\{1-E(P_1+P_2)]^2(y_1+y_2)^{*4}\}} < 0$$

Here, Hessian Matrix is defined negative for $[(y_1+y_2*, B*]$ shows, unique values for $y1+y2$ and B.

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