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Longevity Swaps: Derivatives for Hedging Longevity Risk Embedded in Variable Annuities (Pension Plans)

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Abstract: ***Longevity risk is becoming an increasing important risk facing individuals in developed economies and financial institutions are busy developing products to hedge this risk. It is a good social development but for pension plans it is a risk because it allows them to make loss as they pay more liabilities than they expected. For many years insurance companies has underestimated the longevity of individuals as survival probabilities increases across the developed world. This improvement has caused unsettlement in the future of the annuity and pension providers as they try to find ways to reduce this risk. This paper proposes the use of longevity swaps as a hedging tool on which a life insurer (annuity provider) can transfer the longevity risk to financial markets. The Lee-Carter model is calibrated on Australian male population to forecast mortality rates and the Monte-Carlo Simulation is used to pricing the swaps.For each sample path continue by calculating the payoff of the longevity index swap by looking at the change in mortality rate with respect to the fixed mortality rate. The fixed mortality rate is obtained from the Australian life tables. At maturity time a settlement is calculated and the risk is transferred to the hedge provider (financial market) through a longevity swaps and thus longevity risk is reduced. The index swap pays out in case the forecasted mortality rate becomes smaller than the fixed mortality rate. In this case the clients of an insurer live longer than expected, which means that the liabilities of the insurer are higher than anticipated on beforehand.***

Keywords: longevity risk : variable annuities ; hedging ; longevity index swaps

1. Introduction

Longevity is the risk of people living longer than expected. This is one of the main risk insurance companies encounter and it has become increasingly more important in the early 2000s. This has led to pension plans making loss as they are supposed to pay their valued customers regular payments until they die. Variable annuity (pension plans) market have grown drastic for the past years as a result of a growing demand for products that manage longevity risk by an ageing population. This has caused financial distress in the insurance industry as it becomes a matter of great urgency to hedge longevity. Since pension plans, life insurance and annuity contracts are often designed to last for decades, the uncertainty about the future survival rates can expose retirement programs and annuity providers to the threats of financial distress. Samuel Wills et.al[1] proposed the securitization of longevity risk focusing on the structuring and pricing of a longevity bond using techniques developed for the financial markets in particular mortgages and credit risk. The model was based on Australian mortality data. According to Andrew Ngai et.al[2], investigates the static hedging strategies for longevity risk and management using longevity bonds and derivatives (q forwards) for the retailed products: life annuity, deferred life annuity, indexed life annuity and variable annuity with guaranteed lifetime benefits. Two models were used that is the market model and the mortality model and Australian data was used. The basis risk between the annuitant portfolios and population mortality was based on UK experience. Hedging instruments used were q forwards and longevity bonds on different annuity products. The results were that the hedging process was more effective on the life annuities than to indexed annuities and variable annuities. This was due to the market and longevity risk in the variable annuities since it offers Guaranted Minimum Maturity Benefits (GMMB). Longevity risk was significant in deferred annuities as compared to immediate annuities, because they provide longevity insurance for the higher ages where mortality improvements are more likely to differ from expected. In 2010 also Sharon S et.al proposed the hedging to deal with longevity risk for a life insurer considering basis risk. A hedging strategy to capture longevity risk involves both internal natural and external hedging using longevity linked securities. Basis risk was taken into account in the hedging strategy and employ Yang and Wang multi-population mortality mode to capture the mortality dynamics and life insurance business. A unique set of data of annuity and life insurance policies was used to calibrate the multi-population dynamics for different lines of insurance policies and calculate their liabilities in the profit function. An optimal hedging strategy was also derived by applying Taylor expansion on the profit function. Tzuling Lin et.al[3] came up with a two risk hedge schemes in which the annuity provider transfers the mortality(longevity) risk of the portfolios of a life annuity exposures into a financial institution by paying the hedging premium of a mortality-linked security. Mortality data for USA males and females was used in the Lee-Carter model to calculate the optimal units with the closed form formulas for the life insurer and the annuity provider respectively. Lee-Carter and bootstrap models were used to evaluate hedging performances. The mortality linked security proposed in this paper highlights that in order to achieve a feasible hedge a life insurer should first calculate the total number of in-force life exposures units for each age over an age span, determines the optimal units for minimizing the variance of her loss function with hedge and enters an agreement with financial intermediary. Both a life insurer and a pension plan transfer their own risks; the financial intermediary can reduce the transferred risk with natural hedges. According to Cocco and Games, a defined benefit pension plan for a

65-year-old- US male would have needed 29 percent more wealth in 2007 than in 1970. In 1950 Australian men reached on average an age of 66 years and women 71 years. 63 years later in 2013 men became 80 years and women 84 years on average. For example, in 2006 people retired in the Australia at the age of 65 and become on average 81.04 years old, which means someone receives a pension for 16.04 years. In 2016 people became 82.50 years on average, which results in a pension of 17.5 years. For the insurer this implies an 8% increase in its liabilities.

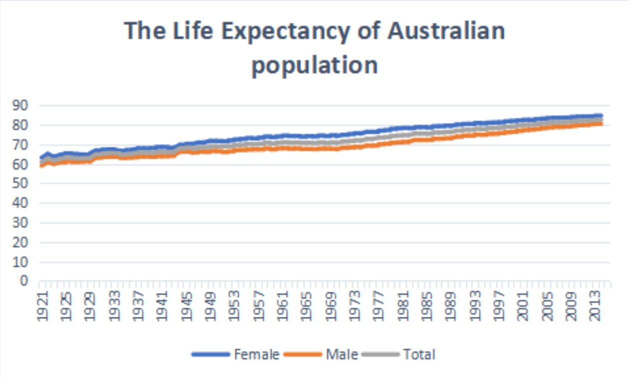

Figure 1: The life expectancy of Australian male and female population

Longevity and the risk of an ageing population will be one of the world's major social issues in the future. The gain in life expectancy is certainly an important social achievement, but it also poses a threat to the population's retirement income security. The increase in survival probabilities which is posing a great threat on insurance companies is due to the great improvements in the medical field, treatment of cancers and good health practiced by people. Many alternatives to hedge longevity risks have been studied and applied in practice. In this paper we are going to look at one of the toolto hedge longevity risk among ageing population by using longevity index swaps. The remainder of this paper is organized as follows: Section 2-Longevity Index Swaps background; Section 3-The Lee-Carter model;Section 4_ Results; Section 5-Forecasting and Simulations; Section 6-Hedging longevity using the longevity swap; Section 7-Conclusion; Section 8-Recommendations; Section 9-Acknowledgements.

2. Longevity Index Swap Background

An index swap is a deal in which one party pays the second party a predetermined fixed cashflow and receives a floating cashflow. The floating cashflow is based on some index. In our case, the index swaps are calibrated on the Australian male population and are designed to hedge longevity risk. A well-known problem however is that people live longer than anticipated on, with the result that future liabilities are higher than expected. To get more certainty insurers can buy index swaps from a certain counterparty. Index swaps act a lot like the more common interest rate swap. One party pays a fixed amount and receives a floating amount and the counterparty vice versa. These amounts are usually not actually paid to one another. Instead, only the net cash flow is paid to the rightful party. The amounts are determined by multiplying a predetermined notional with the fixed or realized mortality rate, depending on cash flow direction. The fixed rate is the best estimate of the mortality rate. The floating rate is determined using the forecasted mortality rate, for example the forecasts made with the Lee-Carter model. The insurer can buy longevity index swap, which means that the company receives the floating amount and pays the fixed. This results in the fact that the insurer has less uncertainty concerning his future liabilities, since he pays a fixed amount. If people live longer and thus have a lower mortality rate, the insurer takes a loss on the longevity swap, but it will be hedged against increasing future liabilities.

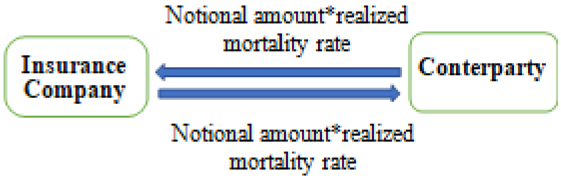

Figure 2: Payment of an index swap to hedge longevity risk

3. The Lee-Carter model

Brouhns et al. (2002) have implemented the Lee-Carter model assuming a Poisson distribution of the number of deaths and using the log link function with respect to the force of mortality $\mu_{x,t}$. The predictor structure proposed by Lee and Carter (1992) assumes that there is a static age function,α_x, a unique non-parametric age-period term (N = 1), and no cohort effect. Thus, the predictor is given by:

$$\ln\eta_{x,t} = \alpha_x + \beta_x^{(1)} K_t^{(1)} \tag{1}$$

In order to project mortality, the time index $K_t^{(1)}$ is modeled and forecasted using ARIMA processes. Typically, a random walk with drift has been shown to provide a reasonable fit, that is,

$$K_t^{(1)} = \delta + K_{t-1}^{(1)} + \xi_t \xi_t \sim (0, \sigma_K^2)\text{i.i.d} \tag{2}$$

where δ is the drift parameter and $\xi_t$is a Gaussian white noise process with variance σ_K^2.The Lee-Carter model is only identifiable up to a transformation, as for arbitrary real constants $c_1$and $c_2 \neq 0$ the parameters in Equation 1 can be transformed in the following way

$$(\alpha_x, \beta_x^{(1)}, K_t^{(1)}) \rightarrow (\alpha_x c_1 \beta_x^{(1)}, \frac{1}{c_2}\beta_x^{(1)}, c_2(K_t^{(1)} - c_1)) \tag{3}$$

leaving $\eta_{x,t}$ unchanged. To ensure identifiability of the model, Lee and Carter (1992) suggest the following set of parameter constraints

$$\sum_x \beta_x^{(1)} = 1, \sum_t K_t^{(1)} = 0 \tag{4}$$

which can be imposed by choosing

$$c_1 = \frac{1}{n}\sum_t K_t^{(1)}, c_2 = \sum_x \beta_x^{(1)} \tag{5}$$

in transformation (2).

4. Results

In this section we present the results of the estimation and forecasting of the Lee-Carter model using Australian male mortality date ranging from 1921 until 2014. The data cover the ages from 0 year until 99 year. The data are provided by the Human Mortality Database and can be downloaded from their website, (http://www.mortality.org).

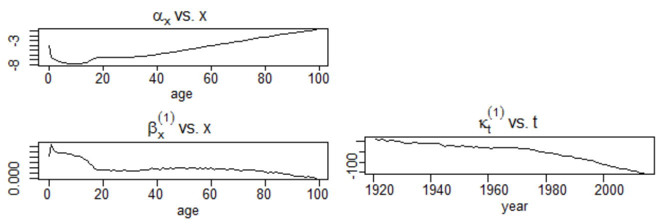

Figure 3: Parameters for the Lee-Carter (LC) model fitted to the Australian male population for ages 0-99 in the period 1921-2014.

Figure 3 shows estimates of α_x, $\beta_x^{(1)}$ and $K_t^{(1)}$. We want to solve the linear regression model in (1) and find the solution, given the restriction in equation (4&5), using singular value decomposition. From figure 3 we see that the parameter α_x rises quite linear, which means that the average logarithmic mortality rate increases linear with respect to age. The parameter $\beta_x^{(1)}$ shows at which ages mortality has improved faster. At ages 40-45 mortality is higher than around ages 80-85. $\beta_x^{(1)}$ also shows that the sensitivity of age-effects is rather constant, given some fluctuations, until the age of 60. From this point $\beta_x^{(1)}$ decreases almost linear with respect to age. From the age of 60 mortality rates become every year less sensitive to mortality improvements than the year before. A mortality improvement for someone of e.g. the age of 80 years, implies perhaps a few months extra to live, for which for someone of 40 years old this could imply a few years extra to live. The parameter $K_t^{(1)}$ describes the declining mortality and is quite linear and is very useful for forecasting as it shows a linear trend.

5. Forecasting and Simulations

From 2014 we forecast mortality rates for 50 years ahead and using the Lee Carter model to do so, we use the regression model in equation (1) and simulate K_t with equation (2). The forecasted mortality rates below are obtained by applying the Lee Carter model using the above mentioned equations.

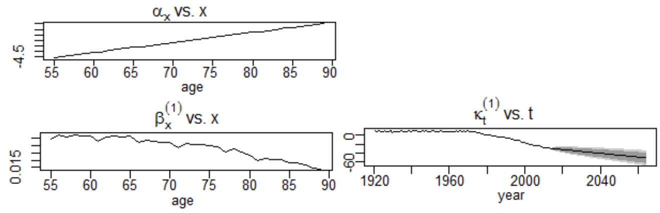

Figure 4: The forecasted mortality rates of Australian male population using Lee Carter model for 50 years

The last diagram above shows the forecasted mortality for 50 years from 2014 for Australian male population. The forecasted results show that mortality rates continue to decrease with time for the ageing population for the next 50 years which puts the pension plans in a risk of paying more liabilities to the ageing population.

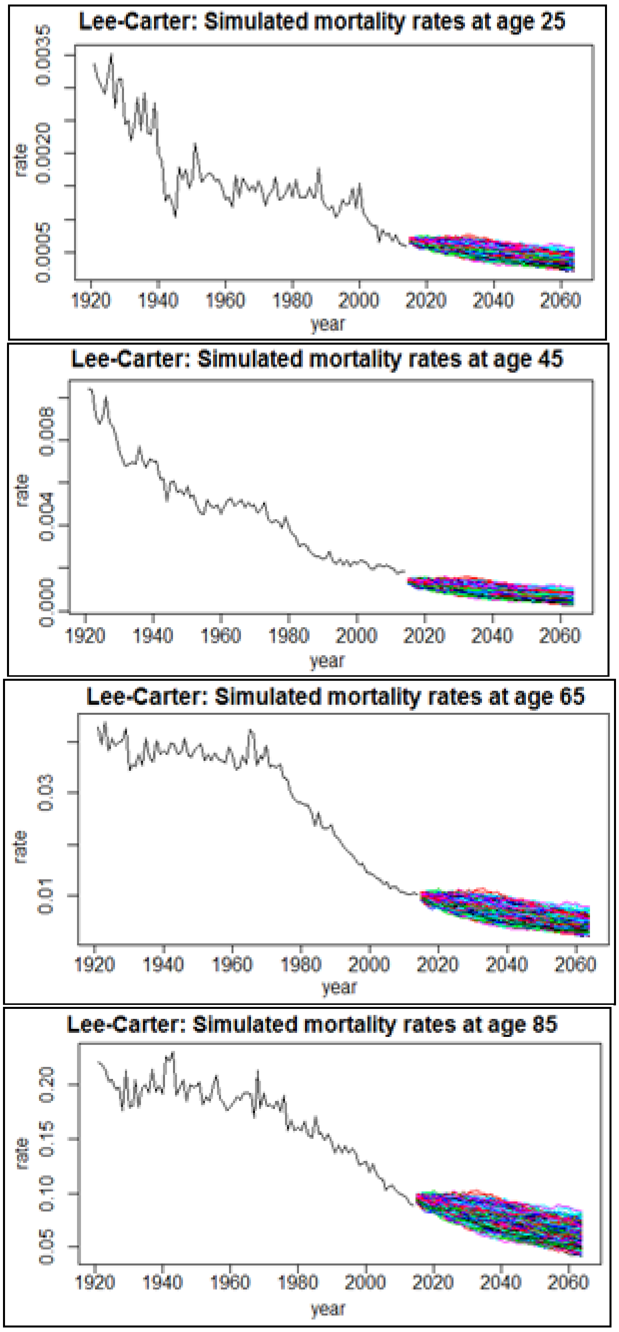

Figure 5: The simulated mortality rates of different ages of Australian male population

Assuming the estimates of β and to be constant we forecast mortality rates using the regression model in equation (2). For K we simulate 1,000 sample paths using equation (1). If we take the average over all 1,000 sample paths we obtain the forecasts. Figure 4 shows the forecasted mortality rates for the ages 25,45,55 and 85 years with 95% confidence intervals. We see that mortality rates keep decreasing over the years. This means for insurance companies that liabilities will increase in time and that extra capital needs to be held to deal with these increased liabilities. Based on the forecasted mortality rates we can price mortality-linked derivatives. In section 5 we discuss pricing of longevity index swaps using these forecasted mortality rates.

6. Hedging longevity using the longevity swap

We first calculate the forecasts of the mortality rates with the Lee-Carter model as described in section 3 and determine the best estimate by taking the x-year-old cohorts from mortality forecasts done by the Australian Bureau of Statistics. This seems logical given the fact that the longevity index swaps are standardized and need to be transparent. Using the best estimate of mortality rate as the fixed rate and the forecasted mortality rates obtained by Monte Carlo Simulation as the floating rate, we calculate the payoff for each age cohort as described above. The payoff of longevity index swaps depends on the realizations of the morality rates. The predetermined notional sets the magnitude of the payoff:

$$\text{Settlement}=\text{notional}*\{E[m(x,t)]–m(x,\text{fixed})\} \qquad (6)$$

Where $E[m(x, t)]$ is the expected value, or forecasted value, of the mortality rate and $m(x, \text{fixed})$ the predetermined fixed mortality rate, that is the best estimate. Using equation (6) to fit on Australian male data we obtain the following graph.

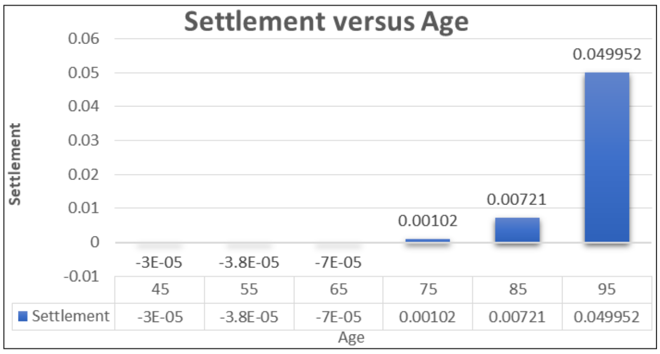

Figure 6: The figure above shows the settlement of different ages

From the figure above, for ages 45,55 and 65 fixed rate is almost equal to realized rate which implies settlement is zero. This shows that for these ages the probability of dying is almost equal to zero as the population are still in good health. Thus no payment has to paid either to the hedge provider or pension plan. The Australian male population with ages 75,85 and 95, the realized rate is greater than the fixed rate which implies settlement is positive. This shows that for the ageing population the risk of mortality is very high. Thus the pension plan pays the settlement payment to the hedge provider thereby offsetting the fall in the value of liabilities.

7. Conclusion

The main focus in this paper was to provide a complete framework of pricing and hedginglongevity risk with longevity index swaps. These index swaps are hardly traded in theearly 2010s and there is no liquid market whatsoever. Therefore, we work with a stylizedportfolio and simulated data. We start to model longevity with Australian male mortality rates from 1921 until 2014.With the Lee-Carter model we forecast the mortality rates and construct 1,000 sample paths by means of Monte Carlo simulations. For each sample path continue bycalculating the payoff of the longevity index swap by looking at the change in mortality rate with respect to a best estimate. This best estimate is the average prognosis of the mortality rates over a certain horizon specified for each age. At maturity a settlement is calculated using equation (6),if fixed rate is equal to realized rate which implies settlement is zero thus no payment has to paid either to the hedge provider or pension plan. If realized rate is less than fixed rate which implies that settlement is negative (low mortality), thus hedge provider pays the settlement payment to the pension plan so as to offset the longevity risk. If the realized rate is greater than the fixed rate which implies settlement is positive, thus the pension plan pays the settlement payment to the hedge provider thereby offsetting the fall in the value of liabilities. This shows that for the ageing population the risk of mortality is very high. The index swap pays out in case the forecasted mortality rate becomes smaller than the best estimate. In this case the clients of an insurer live longer than expected, which means that the liabilities of the insurer are higher than anticipated on beforehand.

8. Recommendations

This research can be extended in a number of directions and details to perfect. A disadvantage, for example, of index swaps is the arising basis risk. Basis risk arises due to differences in the calibration population and the reference population. An insurer could have a population with different characteristics from the national population, e.g. relatively less smokers. Smokers tend to live shorter on average than non-smokers, so an insurer with less smokers could be exposed to more longevity risk than anticipated on when she hedges against longevity risk with index swaps. In case of index swaps it is important for insurers and pension funds to be able to quantify and manage this basis risk and preferably reduce it to a minimum. Several studies indicate that the two most important drivers of variations in mortality experience over time are age and gender. Loeys, Panigirtzoglou and Ribeiro[4] say that a market in longevity must at least differentiate by these two drivers in order to be successful. Mortality rates are actually published separately for men and women and differentiate by age. Due to these differentiation differences in index and reference populations are already small and basis risk is already greatly reduced. On the other hand Coughlan et al.[5] discuss the significance of basis risk analysis. Basis risk is something that could be taken into account in future research. Another point of attention is the forecasting model. The standard Lee-Carter model was used, while there are several additions made to this model. The Lee-Carter model is however still the most widely used by far and therefore more suited to compare with other researches. To perfect this research a more sophisticated forecast model could be implemented. Furthermore, in this context, we could extrapolate the data on which the model was fitted. The data from the Human Mortality Database ranges from ages 1 to 99years old. It is not uncommon to extrapolate these data to the age of 120, since every year more people pass the 100 years of age.

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