

Numerical Investigation of the River Flood Wave Propagation Rate and its Destruction Power Effects Downstream

Okoth C.B.¹, Sigey J.K.², Okelo A.J.³, Marigi E.⁴

^{1,2,3}Jomo Kenyatta University of Agriculture and Technology (JKUAT), Nairobi, Kenya

⁴Dedan Kimathi University of Science and Technology (DeKUT), Nyeri, Kenya

Abstract: *The study investigated the flood wave propagation rate and its destruction power effects downstream of the river. The two dimensional advection-diffusion (2D-ADE) governing equation describing the river flood wave propagation rate downstream with the boundary conditions was solved by Finite Difference Method (FDM) using the MATLAB computer generated program. Using the advection-diffusion equation model, the effects of varying the river slope, the river hydraulic radius and the river flow velocity on river flood wave propagation rate downstream were considered. It was determined that an increase in the river flow velocity and river slope leads to the increase in flood wave flow rate and hence maximum destruction power effects downstream. It was also determined that an increase in the river hydraulic radius leads to the decrease in flood wave flow rate and hence decreased destructive power effects downstream. The effects of changing these river channel flow parameters on the river flood wave propagation rate downstream are presented in tabular form and graphically.*

Keywords: ADE, River slope, Hydraulic radius, Celerity, Diffusivity, propagation rate

1. Introduction

1.1 Background of study

River flooding occurs when a river bursts its banks and water spills over or overflows onto the floodplain due to heavy precipitation such as rainfall upstream. A flood wave is a rapid rise of water in a river channel that occurs when a head of storm runoff develops and moves downstream in wave fronts. Most flash floods consist of several waves in succession each causing additional water level rise as runoff from each individual upstream tributary arrives. River flooding occurs during river flow which is an open channel flow. The open channel flows can be categorised as steady or unsteady, uniform or non-uniform, lamina or turbulent and subcritical, critical or supercritical. The river flood wave can be considered destructive or non-destructive depending on the impacts it has on the river basin downstream and its environs i.e, its erosive nature and the dangers it poses to the human life and property. This destructive nature is a factor of its propagation rate. The higher the propagation rate, the more likely it is to be destructive. Hence the factors that

affect the flood propagation rate also contribute to its destructive power. Flood prediction is of critical importance given that human life and property are vulnerable to the destructive power of flooding. One approach to advance flood management is to better determine the physical controls on flood wave propagation rate, i.e the processes controlling flood volume, flood peak amplitude and time. Prior studies on flood wave propagation can be classified into experimental and theoretical methods. Experimental methods can be further classified into small scale laboratory experiments and field observations. The mathematical model describing the transport and diffusion processes is the ADE. Mathematical modeling of heat transport, pollutants, and suspended matter in water and soil involves the numerical solution of a convection-diffusion equation i.e a parabolic PDE, which describes physical phenomena where energy is transformed inside a physical system due to two processes: convection and diffusion. When velocity field is complex, changing in time and transport process cannot be analytically calculated, and then numerical approximations to the convection equation are indispensable.

1.2 Geometry of the Model

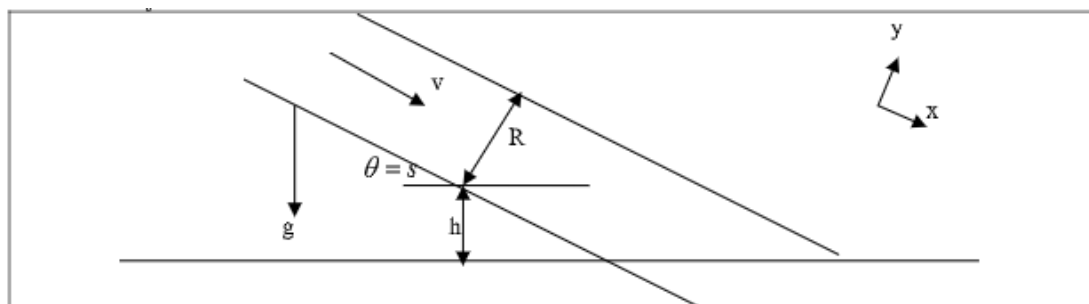


Figure 1: Schematic diagram of river flow

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The term of friction forces due to the drag of sides of the river is not considered. The theoretical solution of the model at the end point of the domain that guaranteed the accurate of the approximate solution is presented. The river has a simple two-space dimension as shown in Fig.1. Averaging the equation over the depth, discarding the term due to Coriolis force, it follows that the two dimensional river flow and advection-diffusion-reaction equations are applicable.

1.3 Statement of the problem

Little progress has been made so far to solve the two-dimensional Advection-Diffusion Equation using analytical and numerical methods when the kinematic wave celerity (c) and diffusion coefficient (D) are constant. Yang *et al.* (2016) investigated the propagated river flood wave originating at the upstream cross section estimated by the flow velocity, celerity and diffusivity of flood waves. They demonstrated with advection-diffusion theory that for small differences in watershed drainage area between the two river cross sections, inflow along the reach mainly contributes to the downstream hydrograph's rising limb and not to the falling limb. There is insufficient information on investigations done on how varying the river flow velocity (v), slope (s) and hydraulic radius (R) affect flood wave flow propagation rate on the river downstream. In the present study therefore, investigation of the effect of these quantities on flood wave propagation rate downstream has been done. A finite difference method has been used to solve the two-dimensional Advection-Diffusion Equation with a view to investigating the effects of varying of these parameters on flood wave flow propagation rate on river downstream.

1.4 The Specific objectives of the study

- i) To determine the effect of varying river slope on flood wave propagation rate downstream
- ii) To determine the effect of varying river hydraulic radius on flood wave propagation rate downstream
- iii) To determine the effect of varying river flow velocity on flood wave propagation rate downstream.

1.5 Significance of the study

- i) The study provides improved understanding of physical controls which support river management, dam design, flow regulation, and flood preparedness. It will therefore be important to civil engineers designing dams for flood management and river flood flow regulation and also for state departments for flood disaster management.
- ii) The study of the general properties of the Advection-Diffusion Equation gives some insights in scientific community due to its applications in the various fields such as gas dynamics, heat conduction, elasticity and fluid flow.
- iii) The study would form a basis of reference on the future researches in other scientific disciplines like Geology, Hydrology and Environmental science.

1.6 Assumptions

- i) The river water is assumed to have constant density.

- ii) The river is assumed to be having tributaries upstream and distributaries downstream which contribute to the changes of volume of river water.
- iii) The river flow is assumed to be unsteady and non-uniform.
- iv) The river channel flow is considered to be two dimensional.

2. Literature Review

Rishu S. (2012) developed a finite difference schemes based on weighted average for solving the one dimensional advection-diffusion equation with constant coefficients. By changing only the weighting parameter in the proposed scheme, four numerical schemes were obtained; explicit, Crank Nicolson Scheme (CNS), implicit and Lax-Wendroff. In this study, we develop and use the Central Difference Scheme (CDS) which is an explicit scheme. Oduor *et al.* (2013) presented a Lie symmetry approach in solving 1-D ADE, a nonlinear PDE called Burgers Equation which arises in model studies of turbulence and shock wave theory. In their study, they managed to give a global solution to the Burgers equation with no restriction on viscosity, λ i.e. for $\lambda \in (-\infty, \infty)$. Yang *et al.* (2016) investigated on the ADE routing model and demonstrated with advection-diffusion theory that for small differences in watershed drainage area between the two river cross sections, inflow along the reach mainly contributes to the downstream hydrograph's rising limb and not to the falling limb. The falling limb is primarily determined by the propagated flood wave originating at the upstream cross section estimated by the flow velocity, celerity and diffusivity of flood waves. Yen and Tsai (2001) demonstrated that the ADE can be formulated from different levels of wave approximations of the dynamic wave model equations under the assumption that the wave celerity and diffusivity are step-wise constants. The ADE has been used to simulate the transport of a water wave along the runoff pathway (Yang and Endreny, (2013); Kumar *et al.*, (2010); Kirchner *et al.*, (2001); Singh, (1995); Gillham *et al.*, (1984). The numerical techniques for solving the uniform flow of stream water quality model, especially the 1-D ADE, are presented by Chen *et al.* (2001), Li and Jackson (2007), Loughlin and Bowmer (1975), Dehghan (2004) and Stamon (1992). A 2-D model for natural convection in shallow water that reduces to a degenerated elliptic equation for the pressure, an explicit formula for horizontal components of the velocity and a vertical diffusion for the vertical component, is derived by Marusic (2007). Dube and Jarayaman (2008) used a rigorous nonlinear mathematical model to explain the seasonal variability of plankton in previous shallow coastal lagoons. The particle trajectories in a constant vorticity shallow water term flow over a flat bed as periodic waves propagate on the water's free surface are investigated by Ionescu-Kruse (2009). Tabuenca *et al.* (1997), Chen *et al.* (2001) and Pochai *et al.* (2008) used the hydrodynamics model and convection-diffusion equation to approximate the velocity of the water current in a bay, a uniform reservoir, and a channel, respectively. Tan *et al.* (1996) and Hu *et al.* (1996, 2002) developed a numerical model for the middle Yangtze River-Dongting Lake flood control system. Wu *et al.* (2003) constructed a model for simulating flow and sediment transport in the Jingjiang River-Dongting Lake network. Wu *et al.* (2004) proposed a

1-D implicit discrete numerical model for non-uniform sediment transport in river networks. Nakayama and Watanabe (2008) developed an integrated catchment-based eco-hydrology model to study the role of the flood storage capacity of lakes in the Yangtze River Basin. These research achievements have great significance to the flood control system of the middle Yangtze River and Dongting Lake. Zuo-tao *et al.* (2012) developed a 1-D model for flood routing in the river network of the Jingjiang River and Dongting Lake using the explicit finite volume method. The 1-D model for the river network and the horizontal 2-D model for the Jingjiang flood diversion area were coupled to simulate the flood process in the Jingjiang River, Dongting Lake, and the Jingjiang flood diversion area. Mukharamova *et al.* (2018) dealt with the construction of mathematical models, which describe the dependence of the river runoff modulus in the territory of the European part of Russia on landscape-geographical conditions and anthropogenic load using modern regression-type statistical models. With increasing steepness of the slopes the speed of water movement increases and evaporation losses decrease.

3. Methodology

In this section, the governing equations and the methods of solution are presented. The governing equations are discussed and presented in their finite difference forms.

3.1 Governing Equation

In nature, transport occurs in fluids through the combination of advection and diffusion. This chapter incorporates advection into our diffusion equation (deriving the advective diffusion equation) and presents various methods to solve the resulting partial differential equation for different geometries.

The generalized advection-diffusion equation defined by Yen and Tsai (2001) describe the two-dimensional channel flow is;

$$\frac{\partial q}{\partial t} + c \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} = D \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} \right) \quad (1)$$

The function q [L^3/t] is the flow rate at a distance x , $v = 1$, $c = u$, [L] downstream from the point $x=0$ where the wave perturbation happens, c (L/t) is the kinematic wave celerity, and D [L^2/t] is the diffusivity which reflects the tendency of the water wave to disperse longitudinally as it travels downstream.

This nonlinear partial differential equation is a homogenous quasi-linear parabolic partial differential equation which encounters in the theory of shock waves, mathematical modeling of turbulent fluid and in continuous stochastic processes. Such type of partial differential equation is

$$\frac{q_{i,j}^{n+1} - q_{i,j}^n}{\Delta t} + \frac{5v}{3} \left(\frac{q_{i+1,j}^n - q_{i-1,j}^n}{2\Delta x} \right) + \frac{q_{i,j+1}^n - q_{i,j-1}^n}{2\Delta y} = \frac{vR}{2s} \left[\frac{q_{i-1,j}^n - 2q_{i,j}^n + q_{i+1,j}^n}{(\Delta x)^2} + \frac{q_{i,j+1}^n - 2q_{i,j}^n + q_{i,j-1}^n}{(\Delta y)^2} \right] \quad (5)$$

Multiply both sides by $6s(\Delta t)$ and let $\phi = \Delta t / \Delta x = \Delta t / \Delta y$ and $\mu = \Delta t / (\Delta x)^2 = \Delta t / (\Delta y)^2$, (5) becomes;

$$(5v\phi - 6vR\mu)q_{i,j}^{n+1} + (24vR\mu - 6s)q_{i,j}^n - (5s\phi - 6vR\mu)q_{i-1,j}^n = (6vR\mu - 6s - 3s\phi)q_{i,j+1}^n + (6vR\mu + 3s\phi)q_{i,j-1}^n \quad (6)$$

introduced by Bateman (1915) proposes the steady-state solution of the problem.

3.2 Specific Governing Equation

The flow celerity c and diffusivity D were estimated as (Liu *et al.* (2003));

$$c = \frac{5}{3}v, \quad D = \frac{vR}{2s} \quad (2)$$

with initial and boundary conditions;

$$q(x,y,0) = f(x), \quad 0 < x \leq L, \quad q(0,y,t) = g(t), \quad 0 < t \leq T \quad (3)$$

where f , g and h are known functions. Substituting (2) into (1), we get;

$$\frac{\partial q}{\partial t} + \frac{5v}{3} \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} = \frac{vR}{2s} \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} \right) \quad (4)$$

3.3 Numerical Methods

The numerical techniques to solve the non-uniform flow of streamwater quality model, one-dimensional advection-diffusion-reaction equation, are presented by Pochai (2009) using the fully implicit schemes: Crank-Nicolson (CNS) method system of hydrodynamic model and backward time central space (BTCS) for dispersion model. The finite difference methods, including both explicit and implicit schemes, are mostly used for one-dimensional problems such as in longitudinal river systems by Chapra (1997). Researches on finite difference schemes have considered on numerical accuracy and stability. They are requirements for advection-dominated systems. Although these schemes need boundary and initial conditions that make them difficult to use. They need more computing effort since iterations for more grids are involved in each computation step. The simple finite difference schemes become more inviting for general model use. The simple explicit schemes include Forward-Time/Centered-Space (FTCS) scheme and the Saul'yev scheme. These schemes are either first-order or second-order accurate (Li and Jackson (2007)), and have the advantages of simplicity in coding and time effectiveness in computing without losing too much accuracy and thus are proceeding for several model applications. In this research, we develop and use the Central Difference scheme (CDS) for Advection-Diffusion equation that govern flood wave flow propagation rate downstream.

3.4. Discretization of Advection-Diffusion Equation

We consider Advection-Diffusion equation (4) and use it to investigate the flood flow rate of the river downstream. For the central scheme (CDS), the values q_x , q_y , q_{xx} and q_{yy} are replaced by central difference approximation while q_t is forward difference. When these values are substituted into Equation (4), we get;

Taking $\Delta x = \Delta y = 0.02$ on a square mesh and $\Delta t = 0.01$, $\Rightarrow \phi = \Delta t / \Delta x = \Delta t / \Delta y = 0.5$ and $\mu = \Delta t / (\Delta x)^2 = \Delta t / (\Delta y)^2 = 25$ into equation (3.22), we get the scheme;

$$(2.5s - 150vR)q_{i+1,j}^n + (600vR - 6s)q_{i,j}^n - (2.5s - 150vR)q_{i-1,j}^n = (150vR - 1.5s)q_{i,j+1}^n + (150vR + 1.5s)q_{i,j-1}^n - 6sq_{i,j}^{n+1} \tag{7}$$

Taking and $i = 1, 2, 3, \dots, 6$ and $j = 1$ we form the following systems of linear algebraic equations;

$$\left. \begin{aligned} (2.5s - 150vR)q_{2,1}^0 + (600vR - 6s)q_{1,1}^0 - (2.5s - 150vR)q_{0,1}^0 &= (150vR - 1.5s)q_{1,2}^0 + (150vR + 1.5s)q_{1,0}^0 - 6sq_{1,1}^1 \\ (2.5s - 150vR)q_{3,1}^0 + (600vR - 6s)q_{2,1}^0 - (2.5s - 150vR)q_{1,1}^0 &= (150vR - 1.5s)q_{2,2}^0 + (150vR + 1.5s)q_{2,0}^0 - 6sq_{2,1}^1 \\ (2.5s - 150vR)q_{4,1}^0 + (600vR - 6s)q_{3,1}^0 - (2.5s - 150vR)q_{2,1}^0 &= (150vR - 1.5s)q_{3,2}^0 + (150vR + 1.5s)q_{3,0}^0 - 6sq_{3,1}^1 \\ (2.5s - 150vR)q_{5,1}^0 + (600vR - 6s)q_{4,1}^0 - (2.5s - 150vR)q_{3,1}^0 &= (150vR - 1.5s)q_{4,2}^0 + (150vR + 1.5s)q_{4,0}^0 - 6sq_{4,1}^1 \\ (2.5s - 150vR)q_{6,1}^0 + (600vR - 6s)q_{5,1}^0 - (2.5s - 150vR)q_{4,1}^0 &= (150vR - 1.5s)q_{5,2}^0 + (150vR + 1.5s)q_{5,0}^0 - 6sq_{5,1}^1 \\ (2.5s - 150vR)q_{7,1}^0 + (600vR - 6s)q_{6,1}^0 - (2.5s - 150vR)q_{5,1}^0 &= (150vR - 1.5s)q_{6,2}^0 + (150vR + 1.5s)q_{6,0}^0 - 6sq_{6,1}^1 \end{aligned} \right\} \tag{8}$$

With initial and boundary conditions $q_{i,0}^0 = q_{i,j}^n = 0$ and $q_{i,j}^0 = 100m^3/s$ respectively, the above algebraic equations (8) can be written in matrix form as;

$$\begin{bmatrix} (600vR - 6s) & (2.5s - 150vR) & 0 & 0 & 0 & 0 \\ (2.5s - 150vR) & (600vR - 6s) & (2.5s - 150vR) & 0 & 0 & 0 \\ 0 & (2.5s - 150vR) & (600vR - 6s) & (2.5s - 150vR) & 0 & 0 \\ 0 & 0 & (2.5s - 150vR) & (600vR - 6s) & (2.5s - 150vR) & 0 \\ 0 & 0 & 0 & (2.5s - 150vR) & (600vR - 6s) & (2.5s - 150vR) \\ 0 & 0 & 0 & 0 & (2.5s - 150vR) & (600vR - 6s) \end{bmatrix} \begin{bmatrix} q_{1,1}^0 \\ q_{2,1}^0 \\ q_{3,1}^0 \\ q_{4,1}^0 \\ q_{5,1}^0 \\ q_{6,1}^0 \end{bmatrix} = \begin{bmatrix} (150vR - 1.5s) \\ (75vR - 2.5v) \\ (75vR - 2.5v) \\ (75vR - 2.5v) \\ (75vR - 2.5v) \\ (75vR - 2.5v) \end{bmatrix} \tag{9}$$

We use equation (9) to investigate the effects of v, s and R on the flood flow rate of the river downstream.

graphical forms. Further, physical observations are made on the data and discussed by giving scientific explanation.

4. Results and Discussion

Introduction

In this chapter, the results of the numerical investigation based on the equation (3.25) of the effects of flood velocity, river slope and river hydraulic radius on the flood wave flow rate of the river downstream are presented in tabular and

4.1 Effects of river velocity on flood wave flow rate of the river downstream

We hold constant the values of $R = 10m$, $s = 2$ and solve equation (9) for values of $v = 0.5m/s, 0.7m/s$ and $0.8m/s$ in equation (9), we obtain solutions as presented in the fig 1.

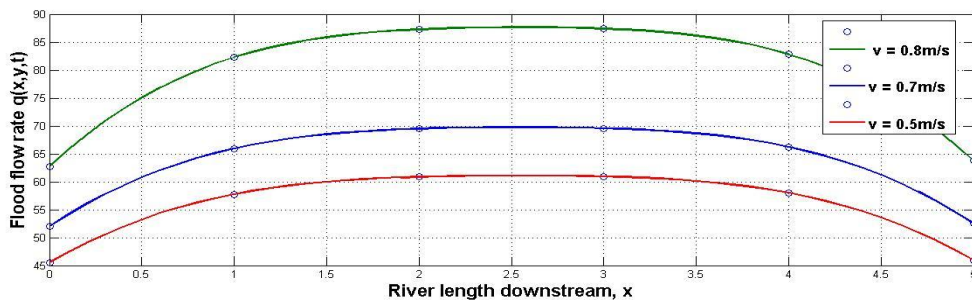


Figure 3: Flood wave flow rate against river length at varying river velocity

Fig.3 shows the variations of flood wave flow rate with river lengths at varying river velocity. As the river velocity increases ($v=0.5ms^{-1}$, $v=0.7ms^{-1}$ and $v=0.8ms^{-1}$), the flood wave flow rate also increases until the critical point is reached. Beyond the critical point, there is drop in flow rate. As the velocity of the flow increases, the flood wave flow rate increases according to the law of conservation of mass (continuity equation) given by the mass flow rate $\rho AV = constant$, since cross section area is assumed to be constant. This increase in flood wave flow rate with velocity continues until the critical point is reached

where there occurs maximum destructive power of the flood wave due to maximum kinetic energy of the flow.

4.2 Effects of river slope on flood wave flow rate of the river downstream

We hold constant the values of $R = 10m$, $v = 0.5m/s$ and varying $s = 2, 4$ and 6 in equation (3.25). Solving equation (9) for varying values of s , we obtain solutions as represented graphically as in fig. 4

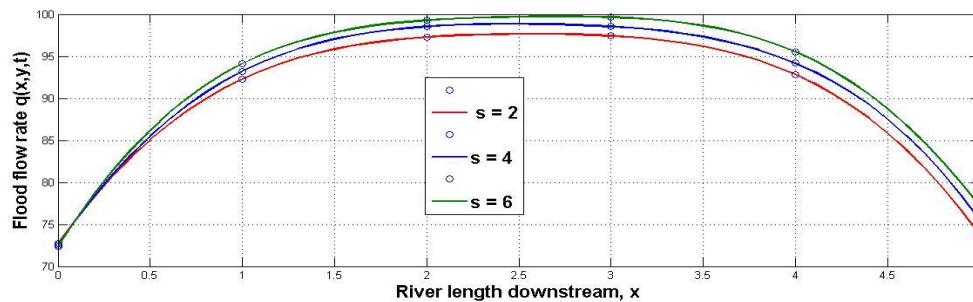


Figure 4: Flood flow rate against river length at varying river slope

Fig. 4 shows the variations of flood wave flow rate with river lengths at varying river slopes. In the figure, the flood wave flow rate increases with the increase in length of the river from the point of perturbation $x=0$, until a critical length ($x=2.5$) is reached when the flow rate is maximum. As the river slope increases ($s=2$, $s=4$ and $s=6$), the flood wave flow rate also increases until the critical point is reached. Beyond the critical point, the river flow rate decreases hence the destructive power also decreases. The flood flow rate increases with the increase in slope because

the potential energy is rapidly converted into the kinetic energy of the flood wave due to increasing flow velocity hence increase in the destructive power effect.

4.3 Effects of river hydraulic radius on flood wave flow rate of the river downstream

Taking $v = 0.5\text{m/s}$, $s = 2$ and solving equation (10) for values of R , and we get solutions as;

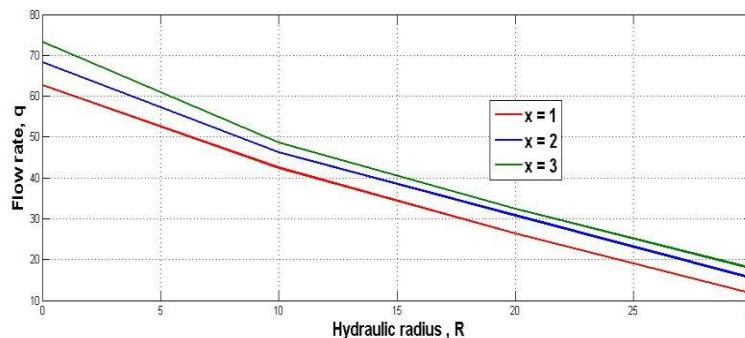


Figure 3: Flood wave flow rate against the river hydraulic radius at varying lengths

In the fig 3, an increase in the hydraulic radius (R) leads to a proportionate increase in the wetted area (A) if the wetted perimeter (p) is kept constant according to the equation $R = A/p$. Further, the resulting increase in wetted area leads to the decrease in flow velocity according to the continuity equation, given by mass flow rate $\rho AV = \text{constant}$, hence the flood flow rate decreases. The decrease in the flow velocity leads to the decrease in kinetic energy of the flood wave hence less destructive power.

4.4 Discussion of Results

In the three results above, the flood wave flow rate increases with the increase in length of the river from the point of perturbation $x=0$, until a critical length ($x=2.5$) is reached when the flow rate is maximum. Beyond the critical point, the flood flow rate decreases. This increase in flood flow rate with length of the flow channel is attributed to the fact that, as the flood wave traverses from the point of perturbation $x=0$ i.e the point where the initial flood wavefront joins the flow channel from its tributary upstream, more and more wavefronts join the main channel from subsequent tributaries. This causes buildup of the flood wave as the subsequent wavefronts superpose (according to the principle of superposition of waves) onto the initial ones as the length of the flow channel increases. When no more

tributaries join the mainstream and hence no more wavefronts, maximum flow rate is attained at some critical length $x=2.5$. At this point, the flood wave has maximum destruction power due to maximum velocity and maximum flow rate and hence maximum kinetic energy as evidenced by erosion on the riverbed and dangers it poses to life and property. Beyond the critical point, the river flood flow rate decreases hence the destructive power also decreases as evidenced by deposition of silt and debris i.e sedimentation. The possible reason for the decrease in flow rate is that as the flood wave traverses downstream, there is decrease in slope, increased spill over and flow into distributaries downstream and hence loss of the wavefronts and the wave is also experiencing the damping effects as it traverses downstream hence less kinetic energy.

The critical points along the river channel are very important in the findings of this study. If the exact critical points for a given river channel can be determined, then measures can be taken at these points to minimize the propagation rate and the kinetic energy of the river flood waves and hence the decreased destructive power effects of the river flood downstream.

Such measures would include; increase of the hydraulic radius, construction of dykes to increase the wetted area

hence decreases in flood wave propagation rate and diversion of river channel (part of flow).

4.5 Validation of the Results

The earlier researchers who carried out similar studies either using the Advection Diffusion Equation (ADE) to model river channel flow or investigated similar river flow parameters obtained similar results; for instance:

- 1) Mukharamova S.S *et al.* (2018); "Modern approaches to Mathematical Modelling of River Runoff in the Territory of the European part of Russia". The purpose of the research was to study the statistical dependencies of river runoff characteristics on landscape-geographical conditions and anthropogenic load using regression type statistical model. In their findings, there was positive effect of average steepness of the slope, that is, with increasing steepness of the slope, the speed of water movement increases and evaporation losses decrease. Similarly, in this study, it was determined that an increase in slope leads to increase in flow velocity hence flood wave propagation rate resulting into increased destruction power effects of the flood wave downstream.
- 2) Yang Y. *et al.* (2016); "Application of Advection Diffusion Routing Model to Flood Wave Propagation: A case study on Big Piney River, Missouri, USA." The purpose of the study was to investigate how Advection diffusion Routing Model performed in flood wave propagation on 16Km long downstream section of the Big Piney River, MO. They demonstrated that small difference in watershed drainage area between the two river cross sections (upstream and downstream), inflow along the reach mainly contributes to the downstream hydrograph's rising limb and not to falling limb. The hydraulic properties of the river flood wave (velocity, celerity and diffusivity) can be calibrated by fitting the hydrographs at the two sections of the river.

Comparatively, in this study, the flow parameters like flow velocity hence propagation rate can be determined at any section of river flow.

5. Conclusion and Recommendations

5.1 Conclusion

The following conclusions were made on the findings of this study:

- 1) It was determined that an increase in the river flow velocity leads to the increase in flood wave flow rate and hence maximum destructive power effects downstream.
- 2) It was determined that an increase in the river slope leads to the increase in flood wave flow rate and hence maximum destructive power effects downstream.
- 3) It was determined that an increase in the river hydraulic radius leads to the decrease in flood wave flow rate and hence decreased destructive power effects downstream.

In all the three cases, the profiles depict increase in the flood flow rate as the flood wave propagates from the point of perturbation along the length of the river channel until a critical point when the flow rate is maximum hence likely

maximum destructive power effects. Beyond this point, the flood wave flow rate decreases hence less destructive.

5.2 Recommendations

The following areas are recommended for future study in relation to this study:

- i) This study has considered the effects of varying the river flow velocity, river slope and river hydraulic radius on the river flood wave propagation rate downstream. There is need for the study of the effects of varying other physical characteristics of the river channel such as river depth and channel roughness on the same.
- ii) Future studies to consider expanding the dimension of the ADE to 3 (i.e., x, y, z) since river discharge is the integration of both spatial and temporal information for the watershed, channel, and weather conditions.
- iii) Future studies to consider the effects of external force like friction force.

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Author Profile



Benedict Chrisphine Okoth: Mr. Okoth holds a Bachelor of Education Science degree, and specialized in Mathematics and Physics from University of Nairobi, Kenya. He is undertaking his final research project for the requirement of Msc. in Applied Mathematics of the Jomo Kenyatta University of Science and Technology (JKUAT), Kenya. He is a teacher at St. Peters Secondary School, Kenya. Phone number +254724516789.



Johana K. Sigey: Prof. Sigey holds a Bachelor of Science degree in mathematics and computer science First Class honours from Jomo Kenyatta University of Agriculture and Technology, Kenya, Master of Science degree in Applied Mathematics from Kenyatta University and a PhD in applied mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. Affiliation: Jomo Kenyatta University of Agriculture and Technology, (JKUAT), Kenya. He is currently the Director, JKuat, and Kisii CBD. He has been the substantive chairman - Department of Pure and Applied mathematics, JKUAT (2007-2012). He has published 40 papers on heat transfer, MHD and Traffic models in respected journals. Teaching experience: 2000 to date- postgraduate programme: (JKUAT); Supervised student in Doctor of philosophy: thesis (6 completed, 10 ongoing); Supervised student in Masters of Science in Applied Mathematics: (45 completed, 10 ongoing).



Jeconiah Okelo Abonyo: Prof. Okelo holds a PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology as well as a Master of Science degree in Applied Mathematics and first class honors in Bachelor of Education, Science; specialized in Mathematics with option in Physics, both from Kenyatta University. He has dependable background in Applied Mathematics in particular fluid dynamics, analyzing the interaction between velocity field, electric field and magnetic field. Affiliation: Jomo Kenyatta University of Agriculture and Technology (JKUAT). 2011 to date: Deputy Director, SODEL Examination, Admission & Records (JKUAT). He has published 44 papers on heat transfer in respected international Journals. He is a Professor in the Department of Pure and Applied Mathematics and Supervisor at JKUAT. Supervision of postgraduate students; PHD: thesis (3 completed and 3 ongoing); Msc in Applied Mathematics: (13 completed and 8 ongoing)



Emmah Marigi. Dr. Marigi holds PhD in Applied Mathematics (JKUAT-2013), Msc in Applied Mathematics (JKUAT-2007) and B.Ed (Science) from KU-1987. Her Research Interest is Modelling Mathematics and area of expertise is Applied Mathematics. Her responsibilities include: Senior Lecturer (DeKUT-2013 to date), Dean of Science (DeKUT-2012 to date), Head of Mathematics (2012), Examination Coordinator (2010-2012). She has done 5 Publications in the Applied Mathematics. She has done postgraduate Supervision of PhD thesis (1 ongoing) and Msc (1 completed and 5 on-going).