Applications of Double Laplace Transform Method to Coupled Systems of Fractional Telegraph Equations

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Abstract: ***In this study we consider the fractional telegraph equation system in one dimension and we solved the system with double Laplace transform with initial conditions. In this method, the fractional derivative is considering in Caputo sense. In addition, some illustrative examples to show authoritative results in the form of numerical approximation and comparison with other methods.***

Keywords: Double Laplace Transform, System Telegraph Equation and Caputo Fractional Derivative

1. Introduction

Fractional partial differential equations are very important subject nowadays. In recent years, all researchers in the field of partial differential equations (PDEs) will be focus and transform the classical models of PDEs such as fluid mechanics, electromagnetism, acoustic, analytical chemistry, signal processing, biology and many other areas of physical science and engineering to fractional partial differential equations (FPDEs).

There are many reasons to make the researchers orientation to fractional calculus (FC), firstly FC described successfully different phenomena in engineering and applied sciences models, secondly FPDEs eventually converges to the integer order PDEs at exact values, thirdly all most numerical methods can be used to find approximate solutions for these models and then interpret them and extract and judge the results.

Recently, fractional telegraph equation solved by many methods, for example [1] using Laplace transform method [2] homotopy perturbation technique, [3] obtained the approximate solution of the telegraph equation by using double Laplace transform method, [4] introduced the Mixture of a new integral transform and homotopy perturbation method (HPM), [5] radial basis functions and [6] suggested Laplace variational iteration method to obtained the approximate solution.

The main objective of the present work is to solve fractional telegraph equation coupled system by using the double Laplace transform and the noise terms phenomenon [7] to determine the solutions of fractional telegraph equation system.

Definitions

In this section, we define the definitions and properties of the fractional calculus which are help further in this paper.

Definition 1

[3]The Laplace transform of fractional order derivative, is defined
By

$$\mathcal{L}_x\mathcal{L}_t[{}_0D_x^{\alpha} f(x)] = p^{\alpha}\mathcal{L}_x\mathcal{L}_t[f(x)] - \sum_{k=0}^{n-1} p^{\alpha-k-1}[f^k(x)]_{x=0}, n-1 < \alpha \le n, n\epsilon\mathbb{R} \quad (2.1)$$

Definition 2: [8] The gamma function $\Gamma(z)$ is defined by the integral such as:

$$\Gamma(z) = \int_0^{\infty} e^{-t}t^{z-1}dt, \quad \mathbb{R}(z) > 0 \quad (2.2)$$

Definition 3: [9] The Laplace transform of a function $f(t)$ defined as:

$$\mathcal{L}[f(t)](s) = F(s) = \int_0^{\infty} e^{-st}f(t)dt, \quad (2.3)$$

Where > 0 .

Definition 4. [9] The inverse double Laplace transform is defined as in by the complex double integral formula such as:

$$\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}[F(p,s)] = f(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{px}dp \, \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{st}F(p,s)ds, \quad (2.4)$$

where $F(p,s)$ must be an analytic function for all p and s in the region defined
by the inequalities $\mathbb{R}(p) \ge c$ and $\mathbb{R}(s) \ge d$, where $c$and $d$are real constants to bechosen suitably.

2. The Solution Methods

In this section, we derive the main idea of fractional double Laplace decomposition method to solve coupled systems of fractional telegraph equations.

Theorem 1- We consider the coupled systems of fractional telegraph equations with initial condition as follows:

$$\begin{aligned}\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} &= \frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial u(x,t)}{\partial t} + u(x,t) + v(x,t) + f(x,t)\\ \frac{\partial^\alpha v(x,t)}{\partial x^\alpha} &= \frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial v(x,t)}{\partial t} + v(x,t) + u(x,t) + g(x,t)\\ &0 < \alpha \le 2 \text{ and } x,t \ge 0\end{aligned} \tag{3.1}$$

Subject to the initial condition,

$$\begin{aligned}u(0,t) &= f_1(t) \text{ and } u_x(0,t) = f_2(t),\\ v(0,t) &= g_1(t) \text{ and } v_x(0,t) = g_2(t),\end{aligned} \tag{3.2}$$

Proof-

We use the fractional double Laplace transform of partial derivatives for obtained the general solution of Eq.(3.1) in this case we get

$$\begin{aligned}&p^\alpha U(x,t) - p^{\alpha-1}U(0,s) - p^{\alpha-2}U_x(0,s)\\ &= \mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial u(x,t)}{\partial t} + u(x,t) + v(x,t) + f(x,t)\right]\\ &p^\alpha V(x,t) - p^{\alpha-1}V(0,s) - p^{\alpha-2}V_x(0,s)\\ &= \mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial v(x,t)}{\partial t} + v(x,t) + u(x,t) + g(x,t)\right]\end{aligned} \tag{3.3}$$

using the single Laplace transform for initial condition Eq.(3.2), we have

$$\begin{aligned}\mathcal{L}_t[u(0,s)] &= \mathcal{L}_t[f_1(t)] = \widehat{f_1}(s)\\ \mathcal{L}_t[u_x(0,s)] &= \mathcal{L}_t[f_2(t)] = \widehat{f_2}(s)\\ \mathcal{L}_t[v(0,s)] &= \mathcal{L}_t[g_1(t)] = \widehat{g_1}(s)\\ \mathcal{L}_t[v(0,s)] &= \mathcal{L}_t[g_2(t)] = \widehat{g_2}(s)\end{aligned} \tag{3.4}$$

By substituting Eq.(3.4) in Eq.(3.3), use the property of the double Laplace transform and simplifying, we obtain

$$\begin{aligned}U(x,t) &= \tfrac{1}{p^\alpha}\hat f(x,t) + \tfrac{1}{p}\widehat{f_1}(s) + \tfrac{1}{p^2}\widehat{f_1}(s) + \frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial u(x,t)}{\partial t} + u(x,t) + v(x,t)\right]\\ V(x,t) &= \tfrac{1}{p^\alpha}\hat g(x,t) + \tfrac{1}{p}\widehat{g_1}(s) + \tfrac{1}{p^2}\hat g(s) + \frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial v(x,t)}{\partial t} + v(x,t) + u(x,t)\right]\end{aligned} \tag{3.5}$$

where $\hat f(x,t)$, $\hat g(x,t)$are double Laplace of $f(x,t)$and $g(x,t)$respectively.

Taking inverse double Laplace transform to Eq.(3.5), we get,

$$\begin{aligned}u(x,t) &= \psi(x,t) + \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial u(x,t)}{\partial t} + u(x,t) + v(x,t)\right]\right]\\ v(x,t) &= \zeta(x,t) + \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial v(x,t)}{\partial t} + v(x,t) + u(x,t)\right]\right]\end{aligned} \tag{3.6}$$

Where

$$\begin{aligned}\psi(x,t) &= \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}f(x,t) + \frac{1}{p}\widehat{f_1}(s) + \frac{1}{p^2}\widehat{f_1}(s)\right]\\ \zeta(x,t) &= \left[\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\frac{1}{p^\alpha}\hat g(x,t) + \frac{1}{p}\widehat{g_1}(s) + \frac{1}{p^2}\hat g(s)\right]\end{aligned} \tag{3.7}$$

The solution of Eq.(3.6) can be written as infinite series terms such as

$$\begin{aligned}u(x,t) &= \sum_{n=0}^{\infty} u_n(x,t)\\ u(x,t) &= \sum_{n=0}^{\infty} u_n(x,t)\end{aligned} \tag{3.8}$$

Then Eq.(3.6) become

$$\begin{aligned}\sum_{n=0}^{\infty} u_n(x,t) &= \psi(x,t) + \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\sum_{n=0}^{\infty}\frac{\partial^2 u_n(x,t)}{\partial t^2} + \sum_{n=0}^{\infty}\frac{\partial u_n(x,t)}{\partial t} + \sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} v_n(x,t)\right]\right]\\ \sum_{n=0}^{\infty} v_n(x,t) &= \zeta(x,t) + \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\sum_{n=0}^{\infty}\frac{\partial^2 v_n(x,t)}{\partial t^2} + \sum_{n=0}^{\infty}\frac{\partial v_n(x,t)}{\partial t} + \sum_{n=0}^{\infty} v_n(x,t) + \sum_{n=0}^{\infty} u_n(x,t)\right]\right], n \ge 0\end{aligned} \tag{3.9}$$

We define the general formula solution of Eq.(3.1) as follows

$$\begin{aligned}u_0(x,t) &= \psi(x,t) = \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\hat f(x,t) + \frac{1}{p}\widehat{f_1}(s) + \frac{1}{p^2}\widehat{f_1}(s)\right]\\ v_0(x,t) &= \zeta(x,t) = \left[\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\frac{1}{p^\alpha}\hat g(x,t) + \frac{1}{p}\widehat{g_1}(s) + \frac{1}{p^2}\hat g(s)\right]\end{aligned} \tag{3.10}$$

And

$$\begin{aligned}u_{n+1}(x,t) &= \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u_n(x,t)}{\partial t^2} + \frac{\partial u_n(x,t)}{\partial t} + u_n(x,t) + v_n(x,t)\right]\right]\\ v_{n+1}(x,t) &= \mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u_n(x,t)}{\partial t^2} + \frac{\partial u_n(x,t)}{\partial t} + u_n(x,t) + v_n(x,t)\right]\right], n \ge 0\end{aligned} \tag{3.11}$$

Note that the inverse double Laplace transform of each terms in the right side of Eq.(3.11) exists.

3. Numerical Examples

In this section, we demonstrate the applicability and stability of our method by applying numerical examples

Example 1: We consider the following system of fractional telegraph equation:

$$\begin{aligned}\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} &= \frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial u(x,t)}{\partial t} + u(x,t) + v(x,t) - 3xe^t - xt\\ \frac{\partial^\alpha v(x,t)}{\partial x^\alpha} &= \frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial v(x,t)}{\partial t} + v(x,t) + u(x,t) - x - xt - xe^t,\\ &0 < \alpha \le 2 \text{ and } x,t \ge 0\end{aligned} \tag{4.1}$$

subject to the initial condition,

$$u(0,t)=0 \text{ and } u_x(0,t)=e^t, \quad v(0,t)=0 \text{ and } v_x(0,t)=t, \qquad (4.2)$$

Solution

By using the Eq.(3.10) and Eq.(3.11) as general solution of coupled systems of fractional telegraph equations, we find a few terms of the series of $u(x,t)$and $v(x,t)$as

$$u_0(x,t)=xe^t-3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t$$
$$u_{n+1}(x,t)=\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u_n(x,t)}{\partial t^2}+\frac{\partial u_n(x,t)}{\partial t}+u_n(x,t)+v_n(x,t)\right]\right] \qquad (4.2)$$
$$v_0(x,t)=xt-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t$$
$$v_{n+1}(x,t)=\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u_n(x,t)}{\partial t^2}+\frac{\partial u_n(x,t)}{\partial t}+u_n(x,t)+v_n(x,t)\right]\right], n\geq 0$$

Then

$$u_1(x,t)=3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t \qquad (4.4)$$
$$v_1(x,t)=\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t$$
$$u_2(x,t)=-36\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t+10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t-4\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}t-6\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}+2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}+2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t \qquad (4.5)$$
$$v_2(x,t)=2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}-6\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}-4\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}t-28\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t+2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t+6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t$$

Therefore, the solution of Eq.(4.1) as series given by

$$u(x,t)=3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t$$
$$+3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t$$
$$+3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t \qquad (4.6)$$

$$v(x,t)=\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t$$
$$+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}t-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t$$
$$-6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}-6\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}-4\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}t$$
$$-28\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t+2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}t+6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t$$

We observed that the noise terms appear in a part of $u_0(x,t)$ then appears in $u_1(x,t),u_2(x,t),\ldots$ and $v_0(x,t)$,$v_1(x,t),v_2(x,t),\ldots$ respectively. By cancelling the noise term we find the exact solution of Eq.(4.1) and given by

$$u_0(x,t)=xe^t, \quad v_0(x,t)=xt \qquad (4.7)$$

Example 2: We consider the following system of fractional telegraph equation:

$$\frac{\partial^\alpha u(x,t)}{\partial x^\alpha}=\frac{\partial^2 u(x,t)}{\partial t^2}+\frac{\partial u(x,t)}{\partial t}+u(x,t)+v(x,t)-3xe^t-x\sin t,$$
$$\frac{\partial^\alpha v(x,t)}{\partial x^\alpha}=\frac{\partial^2 v(x,t)}{\partial t^2}+\frac{\partial v(x,t)}{\partial t}+v(x,t)+u(x,t)-xe^t-x\cos t, \qquad (4.8)$$
$$0<\alpha\leq 2 \text{ and } x,t\geq 0$$

subject to the initial condition,

$$u(0,t)=0 \text{ and } u_x(0,t)=e^t, \quad v(0,t)=0 \text{ and } v_x(0,t)=\sin t, \qquad (4.9)$$

Solution

By using the Eq.(3.10) and Eq.(3.11) as general solution of coupled systems of fractional telegraph equationsof Eq.(4.8), we find a few terms of the series of $u(x,t)$and $v(x,t)$as

$$u_0(x,t)=xe^t-3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}\sin t$$
$$u_{n+1}(x,t)=\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u_n(x,t)}{\partial t^2}+\frac{\partial u_n(x,t)}{\partial t}+u_n(x,t)+v_n(x,t)\right]\right] \qquad (4.10)$$
$$v_0(x,t)=x\sin t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}\cos t$$
$$v_{n+1}(x,t)=\mathcal{L}_x^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{p^\alpha}\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 u_n(x,t)}{\partial t^2}+\frac{\partial u_n(x,t)}{\partial t}+u_n(x,t)+v_n(x,t)\right]\right], n\geq 0$$

Then

$$u_1(x,t)=3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}sint-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}cost \quad (4.11)$$

$$v_1(x,t)=\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}cost$$

$$u_2(x,t)=10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t-36\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t+2\frac{x^{\alpha 3+1}}{\Gamma(3\alpha+2)}sint-2\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}cost \quad (4.12)$$

$$v_2(x,t)=6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t-28\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t-\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}cost$$

Therefore, the solution of Eq.(4.8) as series given by

$$u(x,t)=xe^t-3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}sint+3\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}sint$$
$$-2\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}cost+10\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t-36\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t+2\frac{x^{\alpha 3+1}}{\Gamma(3\alpha+2)}sint-2\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}cost \quad (4.13)$$

$$v(x,t)=xsint-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}cost+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}e^t-6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t+\frac{x^{\alpha+1}}{\Gamma(\alpha+2)}cost$$
$$+6\frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}e^t-28\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}e^t-\frac{x^{3\alpha+1}}{\Gamma(3\alpha+2)}cost$$

We observed that the noise terms appear in a part of $u_0(x,t)$ then appears in $u_1(x,t), u_2(x,t), \ldots$ and$v_0(x,t)$,$v_1(x,t), v_2(x,t), \ldots$ respectively. By cancelling the noise term we find the exact solution of Eq.(4.8) and given by

$$u_0(x,t)=xe^t$$
$$v_0(x,t)=xsint \quad (4.14)$$

4. Conclusion

The solutions of coupled systems of fractional telegraph equations is derivedusing the fractional double Laplace transform. The general solution in section (3) is give directly the solution of the system in terms of a convergent series and noise terms as in example (1 and 2).Finally, the fractional double Laplace transform is faster and high accuracy than other methods.

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