

# Variable Holding Cost & Time Dependent Quadratic Demand Model Optimization by Preservation Technology Investment

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**Abstract:** The purpose of this paper is to develop time dependent quadratic demand and variable holding cost inventory system, for instantaneous deteriorating items using the preservation technology (PT). A solution procedure is presented to find the optimal solution of cost function. Shortage are not allowed. Results have been validated with relevant examples. Sensitivity analysis is performed to show the effect of changes in the parameters on the optimum solution with and without using the preservation technology. The analysis of the model shows that the solution of the model is quite stable and can be applied for optimizing the total inventory cost of deteriorating items for the business enterprises.

**Keywords:** Inventory, Cost Optimization, Preservation Technology, Time dependent demand

## 1. Introduction

Inventory System which is essential in business enterprises and Industries is one of the main streams of the Operations Research. Interest in the subject is constantly increasing. Deterioration is applicable to many inventory products like radioactive substances, drugs, blood, fashion goods, electronic components, and high-tech products. The deteriorating items are subject to a continuous loss in their masses or utilities throughout their lifetime due to decay, damage, spoilage, and plenty of other reasons. Existing inventory models for deteriorating items are numerous, most of which consider a constant deterioration rate over time items in practice, like volatile liquids, agricultural products etc.

The inventory system for deteriorating items has been an object of study for a long time. Due to improved technology now-a day's authors have also start to study the effect in optimizing cost due to reduce rate of product deterioration. Or in other word the consideration of preservation technology (PT) is important due to rapid social changes and by the use of preservation technology one can not only study the effect on optimization but also reduce the deterioration rates significantly by which one can control the economic losses, improve the customer service level and overall increase business competitiveness.

Ajanta Roy [2008] developed an inventory model for deteriorating items with time varying holding constant, price dependent demand. Huang and Hsu [2008] presented a simple algebraic approach to find the exact optimal lead time and the optimal cycle time in the constant markets demand situations. Sarala Pareek and Vinoda Kumar [2009] developed a deterministic inventory model for deteriorating items with salvage value and shortage. P.H. Hsu et al. [2010] proposed a deteriorating inventory policy when the retailer invests on the preservation technology to reduce the rate of product deterioration. Mishra and Singh [2011] developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. D. Dutta and Pavan Kumar [2012] developed a fuzzy inventory model without shortages using trapezoidal fuzzy number.

R. Amutha and E. Chandrasekaran [2013] presented an inventory model for deteriorating items with quadratic demand and time dependent holding costs. A.M.P. Chandragiri [2016] developed a fuzzy inventory model without shortages using signed distance method. P. Mariappan, M. Kameswari and M. Antony Raj [2017] developed a lot-sizing model for deteriorating items with no shortages. Due to shortage of space works of so many authors are not possible to write in this part.

Motivated by all authors work and keeping the present market criteria in this paper, an inventory model is developed with time dependent quadratic demand and variable holding cost using with and without preservation technology investment to study the effect of optimization on cost.

## 2. Mathematical Model

A deterministic inventory model for deteriorating items under time dependent quadratic demand and variable holding cost using an optimization approach considering with and without preservation technology investment have been analysed as Model-I. The following notation have been used to develop the inventory model.

### Notations

$OC$  Ordering Cost

$C_d$  Deteriorating cost per unit per unit time

$h_1$  Holding cost per unit time in the time interval  $[0, \mu]$

$h_2$  Holding cost per unit time in the time interval  $[\mu, t_1]$

$\theta_0$  Constant deterioration rate

$\xi$  Preservation Technology (PT) cost

$m_1(\xi)$  Reduced deterioration rate due to preservation technology

$\theta(\xi)$  Resultant deterioration rate,  $\theta = \theta_0 e^{-\alpha\xi}$

$T$  Duration of cycle

$K_0$  Setup Cost

$I_0(t)$  Initial inventory level

$I_i(t)$  Inventory level in the  $i^{th}$  interval  $[t_i, t_{i-1}]$ , where  $i = 1, 2, 3$

The following criteria has been fulfilled to formulate the mathematical model

- 1) The replenishment rate is finite.
- 2) Shortages are not allowed.
- 3) The inventory system deals with single item.
- 4) Time horizon is finite
- 5) Lead time is zero
- 6) The demand function  $D(t)$  is considered as  $D(t) = a + bt + ct^2$  where a, b and c are constant.
- 7) Holding cost  $HC$  is variable function that is  $HC = HC_1 + HC_2$  where  $HC_1 = h_1$ ;  $0 \leq t \leq \mu$
- 8) and  $HC_2 = h_2$ ;  $\mu \leq t \leq t_1$
- 9) Preservation technology is used for controlling the deteriorating rate  $\theta$

### 3. Model Formulation & Solution

#### 3.1 Model-I

The level of the inventory at time  $t$  is  $I(t)$  after time  $t + \Delta t$  the level becomes  $I(t + \Delta t)$ . During the period  $[0, \mu]$  the inventory depletes due to demand only but during the period  $[\mu, t_1]$  the inventory depletes due to deterioration as well as demand as shown in figure-1

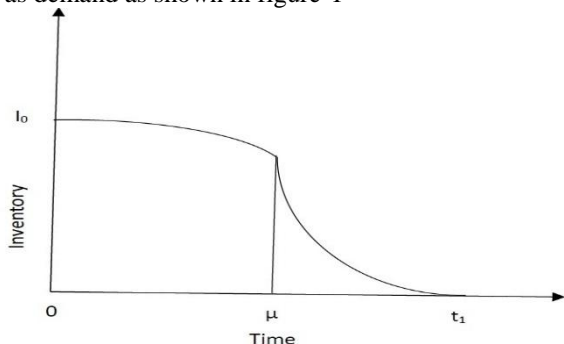


Figure 1: Graphical representation of inventory through time

If  $I_1(t)$  be the on hand inventory level at any time  $t \geq 0$  in the time interval  $0 \leq t \leq \mu$  and  $I_2(t)$  be the on hand inventory level at any time  $t \geq 0$  in the time interval  $\mu \leq t \leq t_1$  then at time  $t + \Delta t$ , the inventory in the interval  $[0, \mu]$  and  $[\mu, t_1]$  will be respectively

$$I_1(t + \Delta t) = I_1(t) - (a + bt + ct^2)\Delta t; 0 \leq t \leq \mu \quad (1)$$

$$I_2(t + \Delta t) = I_2(t) - \theta I_2(t)\Delta t - (a + bt + ct^2)\Delta t; \mu \leq t \leq t_1 \quad (2)$$

Dividing equation (1) and (2) by  $\Delta t$  and then taking limit as  $\Delta t \rightarrow 0$

$$\frac{dI_1(t)}{dt} = -(a + bt + ct^2); 0 \leq t \leq \mu \quad (3)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(a + bt + ct^2); \mu \leq t \leq t_1 \quad (4)$$

Since initially the level of inventory is  $I_0$  and the level reduces to zero at time  $t = t_1$  hence the boundary conditions are

$$I_1(t = 0) = I_0 \quad (5)$$

$$I_2(t = t_1) = 0 \quad (6)$$

Solution of equation (3) & (4) using (5) and (6) respectively as follows:

$$I_1(t) = I_0 - \left( at + \frac{bt^2}{2} + \frac{ct^3}{3} \right) \quad (7)$$

$$I_2(t) = k - (a + k\theta)t + (a\theta - b)\frac{t^2}{2} + (b\theta - 2c)\frac{t^3}{6} + (c\theta)\frac{t^4}{12} \quad (8)$$

where  $k = at_1 + (a\theta + b)\frac{t_1^2}{2} + (b\theta + c)\frac{t_1^3}{3} + (c\theta)\frac{t_1^4}{4}$  (9)

Using the continuity at time  $t = \mu$ , that is  $I_1(\mu) = I_2(\mu)$  which gives

$$I_0 - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) = \left( k - (a + k\theta)\mu + (a\theta - b)\frac{\mu^2}{2} + (b\theta - 2c)\frac{\mu^3}{6} + (c\theta)\frac{\mu^4}{12} \right) \quad (10)$$

After simplification and neglecting higher power of  $\theta$  the initial level of inventory is given as:

$$I_0 = k - k\theta\mu + a\theta\frac{\mu^2}{2} + b\theta\frac{\mu^3}{6} + c\theta\frac{\mu^4}{12}$$

or  $I_0 = A + B\theta$  (11)

where  $A = a(t_1 - \mu) + \frac{b(t_1^2 - \mu^2)}{2} + \frac{c(t_1^3 - \mu^3)}{3}$  and

$$B = \frac{aL}{2} + \frac{bM}{6} + \frac{cN}{12} \text{ with } L = (t_1 - \mu)^2,$$

$$M = (t_1 - \mu)^2(2t_1 + \mu) \text{ and}$$

$$N = (t_1 - \mu)^2(3t_1^2 + 2t_1\mu + \mu^2)$$

Total demand fulfilled in the time interval  $0 \leq t \leq t_1$  is as

$$\text{follows: } \int_0^{t_1} (a + bt + ct^2) dt = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \quad (12)$$

#### 3.2 Cost Calculation

##### Case-1: With preservation technology investment

The cost of the model which is only the combination of setup cost, deteriorating cost, holding cost, purchase cost and preservation technology cost, are as follows:

$$\text{Setup Cost } OC \text{ is: } OC = K_0 \quad (13)$$

Deterioration Cost  $DC$  is:  $DC = C_d \{ \text{Initial level of inventory} - \text{total demand fulfilled} \}$

$$DC = C_d \left\{ I_0 - \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) \right\} = (14)$$

$$C_d \left\{ B\theta - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) \right\}$$

Holding Cost  $HC$  is:  $HC = HC_1 + HC_2$

Where,  $HC_1 = h_1 \int_0^\mu I_1(t) dt$  &  $HC_2 = h_2 \int_\mu^{t_1} I_2(t) dt$

Hence,

$$HC = \left[ \begin{array}{l} h_1 \left\{ \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) \mu + \right. \\ \left. B\theta\mu - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) \right\} + \\ h_2 \left\{ B + \theta \left( \frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) \right\} \end{array} \right] \quad (15)$$

where  $X = (t_1 - \mu)^3, Y = (t_1 - \mu)^3 (3t_1 + \mu)$  and  $Z = (t_1 - \mu)^3 (6t_1^2 + 3\mu t_1 + \mu^2)$ . (16)

Purchase Cost  $PC = C_p * I_0 = C_p (A + B\theta)$  (17)

Preservation Technology Cost  $PTC = \xi t_1$  (18)

Cost of the Model  $TC = OC + DC + HC + PC + PTC$  is given as follow:

$$TC = \left[ \begin{array}{l} K_0 + C_d B\theta - C_d \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) + \\ \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) h_1 \mu - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) h_1 \\ + h_1 B\theta\mu + B h_2 + \theta h_2 \left( \frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) \\ + C_p (A + B\theta) + \xi t_1 \end{array} \right] \quad (19)$$

**Case-2: Without preservation technology investment**

The cost of the model without preservation technology investment is also only the combination of setup cost, deteriorating cost, holding cost and purchase cost as follows: Setup Cost  $OC$  is:  $OC = K_0$

Deterioration Cost  $DC$  is:  $DC = C_d$  {Initial level of inventory – total demand fulfilled}

$$DC = C_d \left\{ I_0 - \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) \right\} = \quad (20)$$

$$C_d \left\{ B\theta_0 - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) \right\}$$

Holding Cost  $HC$  is:  $HC = HC_1 + HC_2$

Hence,

$$HC = \left[ \begin{array}{l} h_1 \left\{ \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) \mu + \right. \\ \left. B\theta_0\mu - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) \right\} + \\ h_2 \left\{ B + \theta_0 \left( \frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) \right\} \end{array} \right] \quad (21)$$

Purchase Cost  $PC = C_p * I_0 = C_p (A + B\theta_0)$  (22)

Cost of the model  $TC = OC + DC + HC + PC$  is given as follows:

$$TC = \left[ \begin{array}{l} K_0 + C_d B\theta_0 - C_d \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) + \\ \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) h_1 \mu - \left( a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) h_1 \\ + h_1 B\theta_0\mu + B h_2 + \theta_0 h_2 \left( \frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) \\ + C_p (A + B\theta_0) \end{array} \right] \quad (23)$$

**4. Objective**

The objective of the study is to determine the optimal value of the preservation cost  $\xi$  for the model when preservation is allowed. The optimal value of  $\xi$  which minimizes the total cost  $TC$  that is  $\xi^*$  will be calculated from equation (24) using Mathematical-software as given by

$$\frac{dTC}{d\xi} = \left[ -\alpha\theta_0 e^{-\alpha\xi} \left( C_d B + h_1 B\mu + h_2 \left( \frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) + C_p B \right) + t_1 \right] = 0 \quad (24)$$

If and only if the second derivative with respect to  $\xi$  of the cost  $TC$  is positive in nature that is:

$$\frac{d^2TC}{d\xi^2} = \left[ \alpha^2\theta_0 e^{-\alpha\xi} \left( C_d B + h_1 B\mu + h_2 \left( \frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) + C_p B \right) \right] > 0 \quad (25)$$

The second objective of the study is to determine the optimal value of the time  $\mu$  when the deterioration start which minimizes  $TC$ . Now differentiating  $TC$  with respect to  $\mu$ , where  $\mu$  is a discrete variable twice as follows:

$$\frac{\partial TC}{\partial \mu} = \left[ \begin{array}{l} C_d \theta \frac{\partial B}{\partial \mu} - C_d (a + b\mu + c\mu^2) + \\ \left( at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) h_1 - (a + b\mu + c\mu^2) h_1 + h_1 B\theta \\ + (h_1 \theta \mu + h_2) \frac{\partial B}{\partial \mu} + \theta h_2 \left( \frac{a}{6} \frac{\partial X}{\partial \mu} + \frac{b}{24} \frac{\partial Y}{\partial \mu} + \frac{c}{60} \frac{\partial Z}{\partial \mu} \right) \\ + C_p \left( \frac{\partial A}{\partial \mu} + \theta \frac{\partial B}{\partial \mu} \right) \end{array} \right] \quad (26)$$

$$\frac{\partial^2 TC}{\partial \mu^2} = \left[ \begin{array}{l} C_d \theta \frac{\partial^2 B}{\partial \mu^2} - C_d (b + 2c\mu) \\ - (b + 2c\mu) h_1 + 2h_1 \theta \frac{\partial B}{\partial \mu} + (h_1 \theta \mu + h_2) \frac{\partial^2 B}{\partial \mu^2} + \\ \theta h_2 \left( \frac{a}{6} \frac{\partial^2 X}{\partial \mu^2} + \frac{b}{24} \frac{\partial^2 Y}{\partial \mu^2} + \frac{c}{60} \frac{\partial^2 Z}{\partial \mu^2} \right) \\ + C_p \left( \frac{\partial^2 A}{\partial \mu^2} + \theta \frac{\partial^2 B}{\partial \mu^2} \right) \end{array} \right] \quad (27)$$

If and only if  $\frac{\partial^2 TC}{\partial \mu^2} > 0$  then the optimal value of  $\mu$  will be calculated from equation (26) using Mathematica-software.

**Proposition 1:** The remark (25) that is double derivative of the cost  $TC$  with respect to  $\xi$  is convex in nature if  $B > 0, X > 0, Y > 0$  and  $Z > 0$

**Proof:** The value of  $B$  will be positive since  $t_1 > \mu$ . By using equation (12) that is  $B = \frac{aL}{2} + \frac{bM}{6} + \frac{cN}{12}$  where  $L = (t_1 - \mu)^2 > 0, M = (t_1 - \mu)^2 (2t_1 + \mu) > 0$  and  $N = (t_1 - \mu)^2 (3t_1^2 + 2t_1\mu + \mu^2) > 0$  hence  $B > 0$ . Moreover  $X > 0, Y > 0$  and  $Z > 0$ , can be shown directly since  $t_1 > \mu$  by using equation (16) where  $X = (t_1 - \mu)^3, Y = (t_1 - \mu)^3 (3t_1 + \mu)$  and  $Z = (t_1 - \mu)^3 (6t_1^2 + 3\mu t_1 + \mu^2)$

**Proposition 2:** The remark (27)  $\frac{\partial^2 TC}{\partial \mu^2} > 0$  that is double

derivative of the total cost TC of the model with respect to  $\mu$  is convex in nature if  $h_2 > 2h_1$  and  $a > b > 2c$  in such a way that overall  $ah_2 > b(h_1 + C_d + C_p)$  and also  $bh_2 > 2c(h_1 + C_d + C_p)$

**Proof:** Equation (27) has been rewritten after substituting the derivative of  $B, X, Y$  and  $Z$  with respect to  $\mu$  as given by:

$$\frac{\partial^2 TC}{\partial \mu^2} = \theta \left[ \begin{array}{l} -(C_d + h_1 + C_p)(b + 2c\mu) + (a + b\mu + c\mu^2)h_2 \\ \left[ \begin{array}{l} 2h_1 \left\{ \frac{a}{2}(-2t_1 + 2\mu) + \frac{b}{6}(-3t_1^2 + 3\mu^2) \right\} \\ + \frac{c}{12}(-4t_1^3 + 4\mu^3) \end{array} \right] \\ (C_d + h_1\mu + C_p)(a + b\mu + c\mu^2) \\ + h_2 \left[ \begin{array}{l} \frac{a}{6}6(t_1 - \mu) + \frac{b}{24}12(t_1^2 - \mu^2) \\ + \frac{c}{60}20(t_1^3 - \mu^3) \end{array} \right] \end{array} \right]$$

Rearranging as follows:

$$\frac{\partial^2 TC}{\partial \mu^2} = \left[ \begin{array}{l} \left( \frac{a + C_d + C_p}{b\mu + C_p} \right) h_2 - \left( \frac{C_d + C_p}{h_1 + C_p} \right) \left( \frac{b + 2c\mu}{2c\mu} \right) \\ + \theta \left[ \begin{array}{l} \left( \frac{h_2}{2h_1} \right) \left\{ \begin{array}{l} a(t_1 - \mu) + \frac{b}{2}(t_1^2 - \mu^2) \\ + \frac{c}{3}(t_1^3 - \mu^3) \end{array} \right\} \\ + \left( \frac{a + C_d + C_p}{b\mu + C_p} \right) \left[ \begin{array}{l} C_d \\ + h_1\mu \end{array} \right] \end{array} \right] \end{array} \right] \quad (28)$$

Second part will be positive if and only if  $h_2 > 2h_1$ . Moreover the first part after properly rearranging gives  $[ah_2 - b(C_d + h_1 + C_p)] + \mu[bh_2 - 2c(C_d + h_1 + C_p)] + c\mu^2h_2$ . This part is positive if  $a > b > 2c$  in such a way that overall  $ah_2 > b(h_1 + C_d + C_p)$  and  $bh_2 > 2c(h_1 + C_d + C_p)$ . Hence the proposition-2 is also proved.

### 5. Numerical Analysis

Keeping in view of the proposition 1 and proposition 2 for validating the proposed model, following related data is considered and discussed as an illustration. Ordering cost  $OC$  is 300, deteriorating cost per unit per unit time  $C_d$  is 5, holding cost per unit time  $h_1$  in the time interval  $0 \leq t \leq \mu$  is 1, holding cost per unit time  $h_2$  in the time interval  $\mu \leq t \leq t_1$  is 3, constant deterioration rate  $\theta_0$  is 0.2, the constant  $a$  is 10,  $b$  is 8 and  $c$  is 5 in the demand function  $D(t)$ , preservation parameter  $\alpha$  is 2, the purchasing cost per unit  $C_p$  is 15, duration of a cycle without shortage  $t_1 (= T)$  is 10. The optimal value of the preservation technology cost  $\xi^*$  for reducing the deterioration rate in order to preserve the product, the optimal time when the deterioration starts  $\mu^*$  and the optimal total cost  $TC^*$  with and without preservation technology investment respectively

has been calculated using equation (24),(26),(19),(23) as follows in table-1.

**Table 1:** Optimal values of  $\xi, \mu$  and TC

With preservation technology		Without preservation technology		
$\xi^*$	$\mu^*$	Without Shortage TC*	$\mu^*$	Without Shortage TC*
3.92639	1.67518	3132.20	3.43187	14369.0

### 6. Sensitivity Analysis

The effect of changes in various parameters of the proposed model, which may happen due to uncertainties in any decisive situation, that is the sensitivity analysis is carried out by changing the specified parameter  $a, b, c, h_1, h_2$  and  $\theta_0$  by -20%, -10%, +10% and +20% keeping the remaining other parameter at their standard value. Table-2 shows the sensitiveness of the various parameters on optimal value of  $\xi^*, \mu^*,$  total cost  $TC^*$  with and without preservation technology investment respectively as follows:

**Table 2:** Sensitivity analysis of  $\xi, \mu$  and TC

Parameter	% change	With preservation technology investment			Without preservation technology investment	
		$\xi^*$	$\mu^*$	Total cost Without shortage TC*	$\mu^*$	Total cost Without shortage TC*
a	+20	-0.02	4.21	-3.50	0.42	-1.34
	+10	-0.01	2.11	-1.75	0.21	-0.66
	-10	0.01	-2.14	1.75	-0.21	0.66
	-20	0.02	-4.32	3.50	-0.41	1.33
b	+20	-0.10	0.23	-5.89	16.00	-4.94
	+10	-0.05	0.12	-2.94	16.13	-2.66
	-10	0.05	-0.13	2.94	16.42	1.90
	-20	0.10	-0.27	5.89	16.56	4.18
c	+20	-0.30	-4.08	-8.53	-0.95	-7.35
	+10	-0.19	-2.14	-4.26	-0.51	-5.24
	-10	0.19	2.39	4.26	0.59	0.98
	-20	0.30	5.07	8.53	2.26	1.15
$h_1$	+20	-0.47	9.53	-1.92	2.54	11.03
	+10	-0.23	4.66	-0.96	1.27	12.38
	-10	0.24	-4.82	0.96	-1.27	15.20
	-20	0.49	-9.01	1.92	-2.55	16.67
$h_2$	+20	-0.50	-23.50	0.00	-3.78	-3.44
	+10	-0.25	-12.55	0.00	-2.00	-1.72
	-10	0.26	14.63	0.00	2.27	1.72
	-20	0.52	32.07	0.00	4.90	3.44
$\theta_0$	+20	-2.32	-0.04	-0.02	-2.18	-16.65
	+10	-1.21	-0.02	-0.01	-1.15	-8.31
	-10	1.34	0.02	0.01	1.32	8.28
	-20	2.84	0.04	0.02	2.86	16.52

The study manifested the following facts:

- Optimal value of  $\xi^*$  changes slightly with the change in the value of parameters  $a, b, c$ , moderately with  $h_1, h_2$  and highly with  $\theta_0$
- Optimal value of  $\mu^*$  (with preservation technology) changes slightly with  $b$  and  $\theta_0$ , moderately with  $a$  and  $c$ , highly with  $h_1, h_2$ . Whereas, optimal value of  $\mu^*$

(without preservation technology) changes slightly with  $h_1$  and  $\theta_0$ , moderately with  $a$  and  $h_2$ , highly with  $b$  and  $c$ .

- Optimal value of  $TC^*$  (with preservation technology) changes slightly with  $h_1$  and  $\theta_0$ , moderately with  $a$  and  $b$ , and highly with  $c$ . Whereas, optimal value of  $TC^*$  (without preservation technology) changes slightly with  $a$ , moderately with  $b, c$  and  $h_2$ , highly with the change in  $h_1$  and  $\theta_0$ .

## 7. Graphical Analysis

The graphical representation of the optimal total cost with respect to the time  $\mu$  that is convexity of  $TC^*$  with respect to  $\mu$  and duration of cycle that is  $T$  has been shown with preservation technology in figure-2 and without preservation technology in figure-3. Also, the graphical representation of  $TC^*$  with respect to  $\mu$  and  $\xi$  has been shown in figure-4.

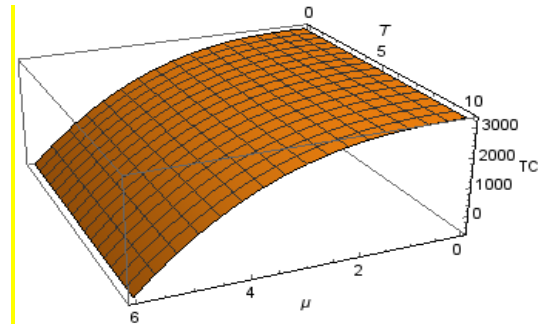


Figure 2: Graph of  $\mu$  vs  $T$  vs  $TC$  using preservation technology

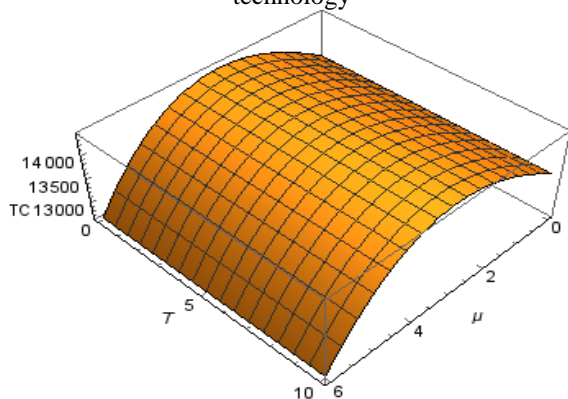


Figure 3: Graph of  $\mu$  vs  $T$  vs  $TC$  without using preservation technology

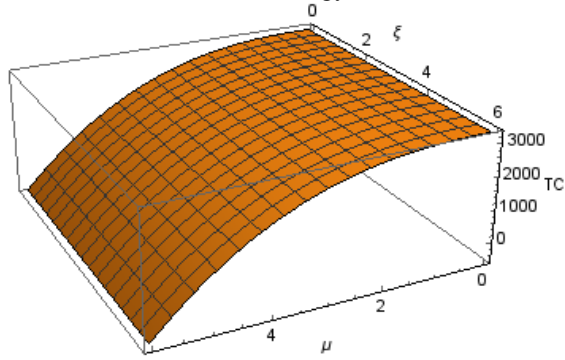


Figure 4: Graph of  $\mu$  vs  $\xi$  vs  $TC$

## 8. Conclusion

In this paper, a deterministic inventory model for deteriorating items under time dependent quadratic demand and variable holding cost has been studied. The basic assumption of the model is based on time dependent quadratic demand rate. The demand of seasonal and fashionable products can be described well with in this model, as the nature of demand of these products is increasing at the beginning of the season, steady in the mid of the season, and decreasing at the end of the season. Moreover, if the time past customer is not willing to purchase the back dated fashionable products or in summer no one is interested to get winter product hence this model stands good. As special case this model has been put in the following way i.e., sometime by changing a suitable attractive way of some fashionable products demand rate can be increase.

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