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From Drawing to Figure and Theories of Didactic Situations and Instrumentation

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Abstract: ***This research addresses the analysis of instrumental genesis process of displacement in the CabriGéomètre II Plus Software which was implemented in classrooms from two different schools,in the courseof eight sessions. Regarding Dynamic Geometry of such computational environment we have verified that the use of the displacement as an instrument could promote significant changes in geometry teaching and learning. In this context, we will go in search of possible contributions or changes the displacement could bring when used as an instrument in flat geometry teaching and learning process. In particular the construction of learning situations by using displacement of Cabri-géomètre computational environment regarding primitive properties of classical geometric objects (quadrilaterals, triangles, circles, etc.) and the relationship among them. The work related to this communication was developed in collaboration with the Laboratory Leibniz and basic education schools, particularly in the transition from Elementary School I to Elementary School II, and has been an ally to the work of Mathematics teachers.***

Keywords: displacement;*Cabri* software; geometric properties learning

1. Introduction

In elementary school, the geometry works requested of the students consist essentially of construction of geometric figures, using paper and pencil, in where the emphasis is given to the reading of properties and description of figures thus constructed. The transition from Elementary School I to Elementary School II is accompanied by abrupt changes in students' conceptions. In this new phase, the geometrical properties of a figure should not be read only from a particular drawing of the geometrical figure. Such figure must be analyzed regarding general properties of the figure class which it belongs to. The properties of a particular drawing are seen as hypotheses of features of the geometric figure. On the one hand, the control that a student should have on these hypotheses to obtain an overall concept of the figure is, a priori, hardly carried out in the paper / pencil environment, where the drawings are seen in a fixed and isolated position. On the other hand, contemporary information technologies suggest to the education professional diverse computational environments capable of providing new forms of learning. This is the particular case of the *Cabri-Géomètre II Plus* environment. In this environment, such control is carried out by the student in a dynamic way, sothat the student can visualize characteristics and/or properties of the figure in all possible directions in real time, from the direct manipulation of base objects by using the mouse or other automatic animation tools available in that environment. Among several possibilities offered by the *Cabri-Géomètre*, these can lead students to acquire knowledge in this field. Therefore, this environment seems to be an adequate tool for flat geometry teaching and learning, especially in elementary school classes.

1.1 Some experiments in this research line

On the one hand, referring to traditional teaching, several researches indicate that the representation systems of mathematical knowledge have a static character. This is evident when we analyze the textbooks or watch a 'classical' class. This static character often hinders the construction of meaning, and the signifier becomes a set of symbols and words or drawings to be memorized. Thus, as Kaput (1992) emphasizes, it should not be surprising when students fail to transfer a concept or theorem to a situation that does not coincide with the prototypical recorded from the presentation of the book or teacher.

The physical instance of a representation system substantially affects the construction of concepts and theorems. New technologies offer physical instances in which the representation happens to have a dynamic character, and it has consequences in cognitive processes, particularly related to the mental realizations. The same mathematical object has a changeable representation, unlike the static representation of physical instances such as "pencil and paper" or "chalk and chalkboard". Dynamism is achieved through direct manipulation of the representations that appear on the computer screen. For example: in geometry are the elements of a drawing that are manipulable; in the study of functions are manipulable objects that describe growth / decay relationship among variables.

An important aspect of mathematical thinking is the abstraction of invariance. For recognition and understanding of it, nothing is more proper than variation. The dynamism of representation highlights the invariants. In this context Kaput (1992) emphasizes:

> "The transition between intermediary states is an important resource of dynamic representation programs, from a cognitive point of view" (*op. cit*., p. 74).

We can observe, for example, that after a static presentation of height of a triangle concept, the students learn that "*the height of a triangle is always from the base to the top "regardless the orthogonality property"* or "*height is the vertical line that connects the base side of triangle to the opposite vertex*" (Gravina, 1996), showing the students' inadequate mental realization.

On the other hand, the experiences already carried out by several researchers with students in computer labs show how important, meaningful, facilitative, motivating and among other cognitive reasons is the positive use of technological tools in students' learning, particularly in geometry. Taking back the above observation and considering the *CabriGéomètre* computational environment, the problem of the triangle height can become evident to the student. In this environment, a triangle can be manipulated together with corresponding heights. By keeping one side fixed and the opposite vertex being moved in a straight parallel line to the side, a family of triangles and heights (segments perpendicular to the side) are obtained in several positions, which may enable the mental embodiment in harmony with the mathematical concept of triangle heights.

What makes the consideration of dynamic geometry interesting, when representing its geometric situations properly constructed on computer screen, is the facility to control elaboration process and knowledge acquisition in geometry. Properties of geometrical figures under analysis can be visualized in all possible dimensions and directions of the associated drawings. According to Duval (1988):

> "The drawing has the faculty of organizing graphically formal data and also enhances the sequential character relationsamong the graphic elements and highlights their properties" (*op. cit*., p. 57).

The sequential character, citedby the author refers, is naturally present in the properties of geometric figures, and therefore, in literature. However, it is practically unexplored in traditional geometry teaching, due to limitations of the paper / pencil environment and the blackboard. From this perspective, the teacher can be based on the mathematical knowledge present in literature, taking advantage of the technological resources, since the advance, both in software and hardware, provides a variety of means and resources to the education professionals. It will give support to their workin classroom.It shows that a good development of teaching-learning process can be achieved through the exploitation of technological resources available. According to Valente (1993):

> "Computers can be used to teach. The number of educational programs and different modes of computer use shows that technology can be very useful in the teaching / learning process. Also, more, for the implementation of computer in education, four ingredients are needed: the computer, the educational software, the teacher who is capable to use the computer as an educational medium, and the student. Software is aningredient as important as others. Without it, the computer could never be used in education" (*op. cit.,* p. 45).

In our case, the software cited by Valente, the *Cabri-Géomètre II Plus*, is the computer environment, which is describedbelow.

1.2 Cabri-Géomètre II Plus

The *Cabri-Géomètre II plus* (see the picture) is a computational environment with microworld[1] characteristics, intended for Euclidean Flat Geometry teaching and learning. This software was developed by Jean-Marie Laborde and Franck Bellemain, at Leibniz Laboratory of IMAG (Informatics and Applied Mathematics Institutefrom Grenoble - France), in collaboration with CNRS (National Center of Scientific Research) and Texas Instruments. This environment has been previously prepared with the tools like pencil, rubber, compass, ruler, and other tools. Which are necessary for performing activities related to geometric constructions on paper. Thus, it allows exploration of elementary geometry universe and presents interface with construction menus in classical geometry language. The figures constructed with it can be modified from displacement of its base elements, keeping properties assigned initially.

It is an open[2] and interactive software that allows students to build their own knowledge. The various possibilities it offers, when they are well explored in teaching, can contribute significantly to students' learning. Henriques (2001), Regarding possible contributions that this microworld can bring in geometry teaching-learning process, Henriques (2001) stresses:

> It should be noted that the visualization and direct manipulation of mathematical objects in a computational environment is a primary task when one intends to use computer technology in teaching/learning process. In this understanding, it is worth noting that one of the main concerns when designing the *Cabri-Géomètre* software was to allow students to visualize, on the computer screen, different designs that correspond to the same description, *i. e*., belonging to the same configuration or class, from direct manipulation. This idea allows students to explore properties of a geometric configuration. (*op. cit*., p.45).
> He also adds:

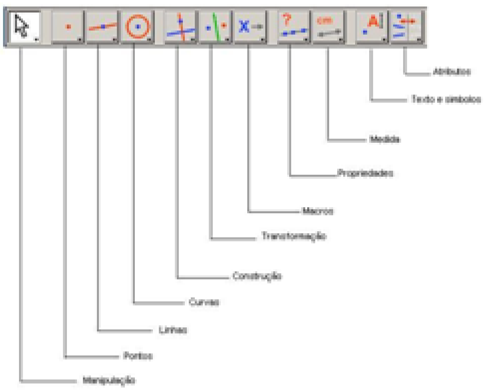
[1]Microworld is like "a sub-set of reality or constructed reality, whose structure houses the structure of a cognitive mechanism in order to provide an environment where it can operate effectively" (Papert, 1980, p.204).
[2] Open because users are free to manipulate and build new tools from the existing ones.

> The Cabri-Géomètre II is a [computational environment] that allows to construct and explore, interactively, objects of Elementary Geometry universe in a language very similar to the paper-and-pencil universe. The constructed figures can be deformed by the displacement of their base elements, keeping properties. This Cabri II characteristic enables to observe all possible "figure cases" for theset of figures with same properties. [...].

In this context, we understand that the drawings of geometric figures are made from the properties which define them. But it's not only this what the environment offers, as the author emphasizes, through displacements applied to the elements which make up the drawing, it is transformed, conserving the geometric relations that characterize the geometric figure. Thus, for a given object a collection of "moving drawings" is associated, and the appearing invariant features correspond to the geometric properties of the object.

Such environment means a lot from knowledge building perspective. If, from the displacement of base objects, the drawing doesn't correspond to what is intended, two possibilities may occur:

- or the object was poorly constructed, i. e., the properties which characterize it weren't correctly applied.
- or our visual image of the object isn't adequate,i. e., the construction was correctly performed, but our perception is mistaken.



In either case, the "moving drawing" feature leads to adjustment of our conception and visualization of the object. In this context, the classical configurations have multiple visual aspects, and with this they are identified in non-prototypical situations. "Moving drawings" naturally create a research environment. The invariants stand out and this influences on acquisition of new knowledge due to: (1) the presence of the dynamic and interactive interface ('moving drawings' that can be automated through use of the 'buttons'); (2) multiple representations (work with synthetic geometry and a little of analytical geometry); (3) capturing procedures (existence of commands allowing access to history of construction and commands for creating macros).

The figure above shows *Cabri-Géomètre* toolbar, and the names of different toolboxes. Each box provides the student with the tools needed to deal with problems. It is worth noting that being an open environment, it is possible to restrict or build new tools which are not available in those boxes. In addition, we can exclude the existing ones so that they can be built by the students.

1.3 Problems and justification

Considering the microworld characteristics of *Cabri-Géomètre*dynamic geometry, were built with the idea of leading the user to the discovery of geometric knowledge and a better construction of this knowledge. We can observe that Cabri-géomètre dynamism allows the passage of the representation of a geometric construction, a static drawing, to several representations of same geometric figure. There the displacement allows passing from drawing to the figure.
According to definition given by Laborde and Capponi (1994):

> "The geometric figure is the geometric object described by the text that defines it, an idea, a creation of the spirit and the drawing is a representation ... Drawing can be considered as a meaning of a theoretical reference (...). The geometric figure apparently consists of a reference to all drawings. It is then defined as the set of couples formed by two terms, the first term being the referent, the second being one of the drawings representing it. The second term is taken in the universe of all possible referent drawings. " [translation of authors]

The difference between drawing and figure in geometry is not clear to either teachers or students who are supposed to be aware of this difference. When the teacher observes a drawing he/she sees only the geometric figure - the represented object. The student, on other hand, just sees the drawing, considering that he/she doesn't have the same amount of knowledge the teacher has.

Drawing as a geometric figure representative can play the role of a reducer and/or producer. As a representation, it may not be interpreted correctly and the geometric properties can be missed in this interpretation. Besides, a drawing also providefake information.

To enable the appropriate reading of the theoretical properties desired tointerpret the drawingit's necessary to have enough knowledge to read and see what is being communicated. That's why every reader will give to the drawing his/her own interpretation and its meaning, according to his/her perception.

One of the objectives of school is to teach students how to make a mathematical demonstration. The mathematical demonstration concept in geometry is related to the fact that a geometric object with certain properties of a given drawing can be found.

The *Cabri-géomètre* allows to preserve the primitive geometric properties of drawing by direct manipulation, which means: when an element is moved, the figure will be deform butwill saving the original geometric properties of drawing stroke. This enablesinvalidation of traces which were made carelessly, *i. e.*, without considering geometry rules, and also validation of figures which were constructed from geometric primitives.

Thus, the software drives the user to apply his/her mathematical knowledge in order to construct a figure that will oppose to the displacement. In a geometric construction work.Thedisplacement also allowsthat the user can come to realize and distinguish the "true"and "false" properties of the figure, that is:those that are apparent in the static design and are invalidated due to the displacement. A construction of an object is valid, or correct, if and only if, it opposes to the displacement test, that is, if this construction is not deformed duo to the displacement. Validation doesn't depend on the physical appearances of a drawing.

In elementary school, students work on geometry on the drawing, the transition to other school grade followed by a change in interpretation which is expected from students.

The geometric properties of figure should no longer be clearly read over a specific design but also established as hypotheses or deduced from the problem data when appropriate.

The displacement in Cabri-géomètre should allow students to visualize and distinguish the original geometrical properties of figure (the hypotheses and properties that might be deduced) from properties which are specific of a particular drawing. The original properties are preserved in the course of the displacementbut the specificonesaren't. In this context the Cabri-géomètre software was selected to be used in this research as it had been initiated in LeibnizLaboratory on the IMAGproject framework, of the IAM[3]groupregardinggeometry teaching and learning with support of computer science. Working with *Cabri-géomètre*which was designed to be used in the elementary and middle school is an important articulation among all school levels.

Much of Dynamic Geometry research has been focused on developing studies on higher education issues, by approaching its repercussion on teacher'steaching practice in this level education. It's also important to emphasize the relevance of studies focused on elementary education, as the only way to seek more uniform articulation and pay closer attention to the results of research performed to all education levels.

The lack of researches regarding elementary school level is reflected in lack of methods or strategies to teach mathematics using a computer support. This is one of the reasons which often intimidates teachers to accept challengesto use educational software, even recognizing that this technology is a reality not only in education environments, but in all sectors of society. A central question today concerns the increasing digitalization of services offered to population. Some schools are also being equipped with computers. It's fundamental teachers understand that the use of technological resources is necessary and irreversible in the current education contexts.

Also, teachers must understand that the computer will not replace thembut help them in their hole as mediators and formers of citizens. It is important to remember that new technologies are increasingly accessible to people. Therefore, it is necessary that teachers and students follow technological evolution in order to be prepared to use it in a critical way to better understand, interpret and transform reality.

With respect to this work the use of these technologies appears as relevant aspect in geometry teaching learning process through use of *Cabri-Géomètre.* As Soury-Lavergne (2004) says, the displacement in *Cabri-Gèomètre* should allow to visualize the geometric properties which are proper of the figure (the hypotheses and properties deduced) and distinguish them from those that are associated with a specific design. The figure geometric properties are preserved during the displacement and the geometric properties of a specific drawing aren't. *CabriGéomètre*, therefore, seems to be essential instrument to be used by the students from the very beginning of learning of deductive reasoning process,considering the displacement as a mean to distinguish status of the properties observed by students.

2. Objective

Such communication is based upon computer science in Mathematics Education in elementary and high school final grades, whose one of the objectives is to select a specific methodology for teaching and learning Mathematics in a computational environment. In this context, we will outline:

- Demonstrate how the *Cabri-Géomètre* II *Plus* computational environment can be used in construction of learning situations using displacement of this software around the primitive properties of classical geometric objects (quadrilaterals, triangles, circles, etc.) and the study of the relations among them properties.
- Furthermore, to present an analysis of the use of displacement in *Cabri-Géomètre*II Plus software, when this displacement is applied as an instrument capable of promoting significant changes in the geometry teaching and learning process with the support of new technologies in Education."

[3]Computer Science and Learning Mathematics

Thus, we will describe the possible contributions or changes that this use can bring to teaching and learning process of flat geometry in the mentioned school grades, being aware that, it is in those grades that construction of deductive reasoning starts.

On this horizon, with this communication, the greatest ambition is to contribute to the geometry teaching-learning process in elementary and secondary education levels, providing students and teachers, respectively, alternatives in acquisition of knowledge and pedagogical practices, with the help of computing environment, in particular *Cabri-géomètre II Plus*.

3. Methodology

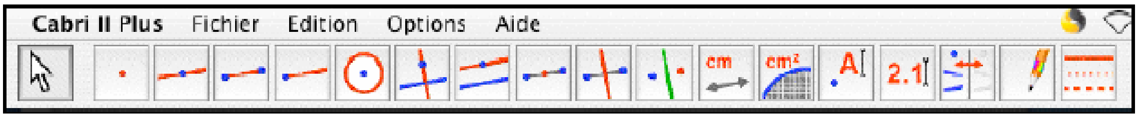
During the school year, 5th grade student classes from two French schoolshave used *Cabri-géomètre* during math classesin 2006. Our goal with this use was to help students to structure their geometric knowledge more solidly and to introduce the difference between figure and drawing and the notion of demonstration in mathematics.

Thus, during this phase the *Cabri* was used with such objectives and also makestudents learn how to use the primitives of this software as well as the displacement.

In the next phase the displacement was used to clarify invariants of a figure and establish conjectures, inorder to invalidate a false conjecture that may result from a misinterpretation of a static drawing.

The overall scenario used in the analysis consisted of a set of eight sessions during the classes in where the students used *Cabri*.

Instead of using the traditional *Cabri* toolbar, the *Cabri* Junior was specially designed to5th grades, as shown in the figure below.

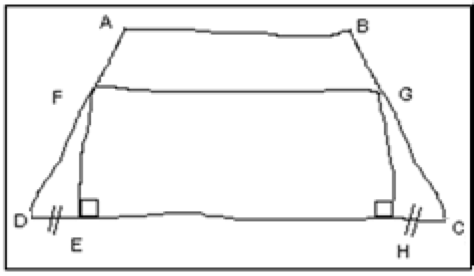


The goal of using this new toolbar is to assemble only the elements we consider necessary for students' activities as it is shown next. The idea was to facilitate students work by using such new toolbarwith a less amounts of tools.

During these sessions, different functions of the displacement were shown and requested from the students. We asked students to move a point, either by a direct displacement, picking up the point and moving it directly, or from a dependent relation, in which the displacement of a point is linked to other elements, that is: the point may or may not be moved. While moving the point the students were also asked to observe what was going on, and also move to validate and invalidate a figure from certain properties given by a statement.

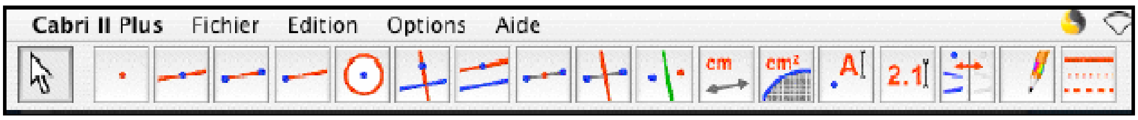
However, the appropriation of the displacement requires more time since beginning of its use until the moment the students will realize the meaning. So, before thinking about using the displacement during a session, students will think about doing other things. Therefore, it was necessary to invest in the meaning of the displacement, so that the students could then use it. For example, during a session, students received a paper sheet with a statement and a hand drawing figure, showing signs representing perpendicularity and equality of lengths between segments, and two questions, as we can see below.

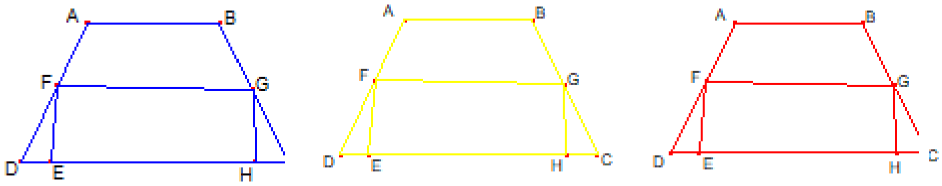
ABCD is a trapezoid



- The points points E and H belong to the CD side
- Segment DE = Segment HC
- F is the point of intersection between the AD side and the line perpendicular to CD passing through E
- G is the point of intersection between the side BC and the line perpendicular to CD passing through H

1 - Is the EF segment parallel to the HG segment?
2 - Is the DC segment parallel to the FG segment?
Then, as it is shown below, there are three trapezoids constructed in Cabri and the students were asked to choose the one that corresponds to statement of hand drawing figure.





Blue figure   Yellow figure   Red figure

As soon as two of those figures are eliminated, students should take the sheet and answer the two questions.

The points E and H of three figures are fixed. The perpendicularity of the segments EF and HG to the segment DC was respected in the construction of blue figure, but not the equality of distance values between segments DE and HC. The displacement of point D and point C allows to see such fact, because when moving the point D or point C the segments DE and HC do not keep of the same measurements which consequently invalidates this figure. Segments DE HC of yellow figure have same measure, but when moving the points F, A, B and G, it can be seen that segments EF and HG don't remain perpendicular to segment DC. Therefore, the red figure is the searched figure and will allow to answer the questions that were initially made, since the segments EF and HG are perpendicular to the segment DC allowing tosee the parallelism between the segments EF and HG answering question 1. The segments FG and DC are not necessarily parallel, and this fact can be verified by moving the points F and G of the figure, answering question 2.

During one of the sessions the teacher conducted a discussion to recapitulate how to invalidate the figures that didn't have the properties sought. When the displacement of a figure is done, there must be an objective, to know what being sought, what is to be moved and what is to be observed. Analyzing following discussions:

Teacher: "When do you hold which point?" (...) Whatdoes it moves?"
Student 1: "F"
Teacher: "The F and what do you notice?"
Student 1: "The centimeters"
Teacher: 'You notice the centimeters, the distances! You observe the distances between D and E and between H and C".

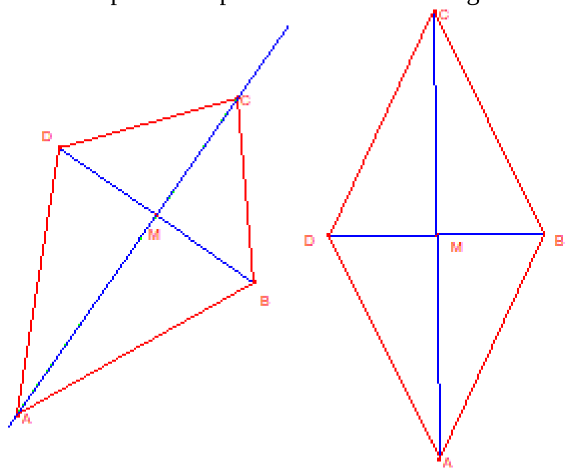
Another student:
Teacher: "What do you hold to make your figure move?"
Student 2: "Point A or B or C or D or E or F or G or H!"
Teacher: "Well, when you hold point F or point G, what do you observe?"
Student 2: "The segments DE and HC"

It's clear to see that the students have difficulty in verifying and using the geometric properties of questions.

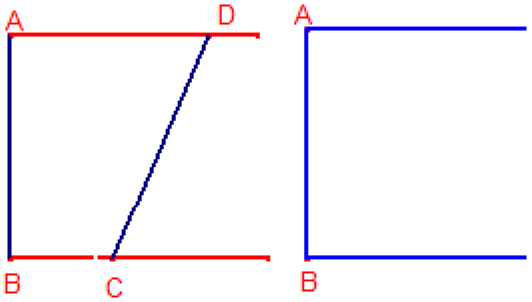
In the second session of the experiment, we created a wording situation to see if students had learned how to use the displacement to invalidate a drawing.

The objectives of this experiment were to observe: a) if students would anticipate the use of displacement in a formulation situation; b) how they had used the displacement; c) what they observed while using displacement, d) how they interpreted and if interpretation helped to clarify a question or a statement.

In each class, the students would start working with quadrilaterals. It was assumedthat this new phase of the eight sequences, would be a good way to introduce formally the quadrilaterals properties. For this, to constructions were introduced in a specific configuration: a "deer-wheel" that was "put into position" of diamond and a rectangular trapezoid, placed in a position of rectangle. The two figures were constructed to function in the same way; they shouldn't havea great difference between them for students' eyes. The fake lozenge, which was actually a "deer-flyer" was constructed as it follows: we plotted segment BD, midpoint M of segment BD, then we drew the perpendicular bisector of segment BD where points A and Cwere randomly plotted, forminga convex quadrilateral ABCD. Points A and C were plotted so that AB = BC = CD = AD, and students were asked if the represented picture was a truelozenge.



The false rectangle was constructed as it follows: traced segment AB, then line d1 perpendicular to segment AB passing through point A, then traced line d2 perpendicular to segment AB passing through point B. Next, we drew any two points, C belonging the straight line d1 and D belonging to the line d2. Both on the same side with respect to the segment AB, so that ABCD is a convex quadrilateral. Segments AB and CD were put in the position having the same measure. Same action was done with segments AD and BC.

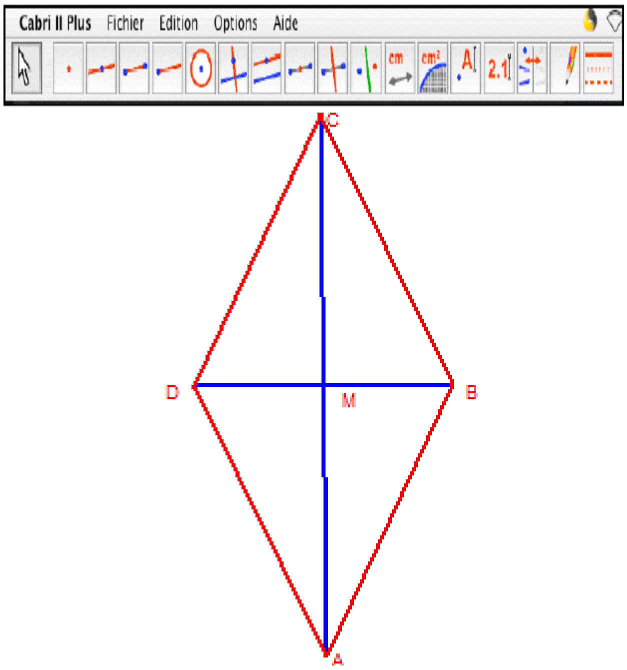


The students were asked if represented picture was really a rectangle. Each class was divided into student pairs and each pair of the same class was associated with another pairin the same class. Thus, two pairs were formed, an emitter pairand a receiver pairin a communication situation. We fed the emitter with a picture of *Cabri*figure and a question to be answered by this pair without manipulating the *Cabri*. However, the emitter should make questions to the receiver pair, through a token, whose answers will allow them to answer the question that was initially asked.

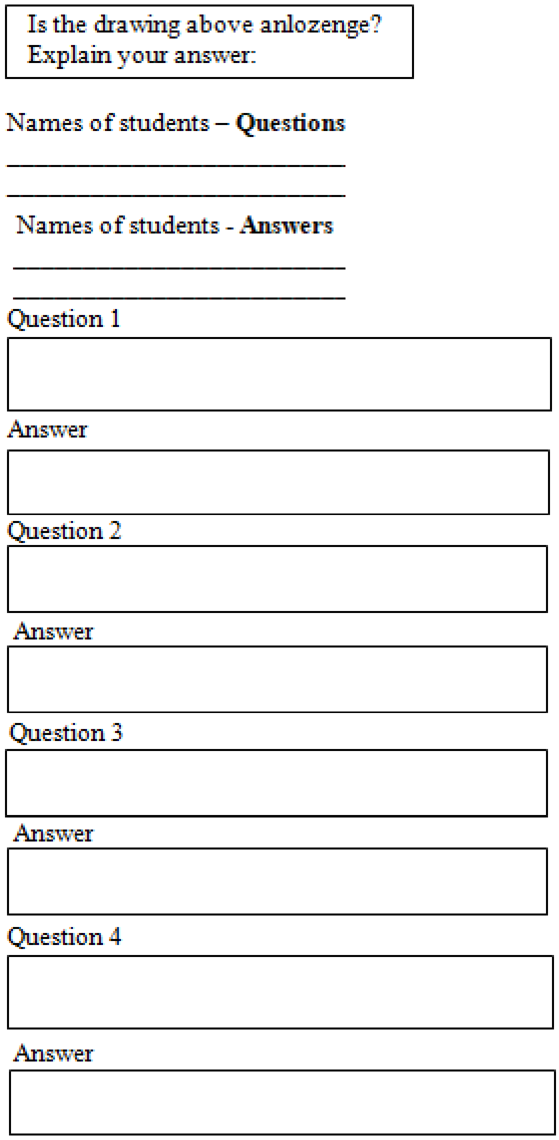
The receiver pairshould manipulate *Cabri* figure to be able to answer the questions that were asked by the emitter. After having received the responses of the receiver the emitter should answer the initial question. The two figures were proposed so that each paircould play both roles: the emitter, or acting as the receiver. So, the pairs play the role of emitters by using one of the figures and play the role of receivers by using the other figure. In this way, when answering the initial question, each pairhad only the answers obtained by their associated pair.

Pair Sheets

A drawing made on *Cabri* appears on this page.
Colleagues belonging to the pair to the associated pair have the same *Cabri* drawing on their computer.



Is the drawing above anlozenge?
To get this answer you can ask questions to your associated pair.Pay attention, you can't use the name lozenge!


Is the drawing above anlozenge?
Explain your answer:

Names of students – **Questions**

Names of students - **Answers**

Question 1

Answer

Question 2

Answer

Question 3

Answer

Question 4

Answer

Comments:

4. Theoretical Framework

Some approaches of French Mathematics Didactics were applied to give us support in theoretical foundations. For example, the theory of didactic situations proposed by Brousseau (1986), with the intention of organizing and analyzing the students´different behaviors in problematic situations. The didactic transposition proposed by Yves Chevallard (1991) wasconsidered essential literature to better understand the study of the geometric properties of classical figures, without disregarding the effects of Computer Transposition. In addition to such examples other approaches were: cognitive approach of contemporary instruments according to Rabardel (1995), as well as the problematic drawing-figure developed by Laborde (1994) and Capponi (1994). Following pages briefly describe the approaches which are interesting to this context/article.

The instrumentation theory was arisen from cognitive ergonomic works, and concerns the learning of technological tools use. The starting point is the idea that a tool isn't automatically an effective and practical tool. In this context Rabardel (1995) proposed this theory as an approach to didactic modeling, where he distinguishes a tool (an artifact) from an instrument. Artifact is a material device used as a means of action. An instrument is constructed by the subject throughout a process in which an artifact is progressively transformed into an instrument. This genesis is a complex process allied to the characteristics of the artifact - its potentialities and its limitations, and to the activities of the subject - their knowledge, their previous experiences and their abilities. A hammer[4], for example, is a meaningless object, except when you have something to hammer into, making it a useful instrument. This idea also applies to any other object such as the computer or software (Cabri-Géomètre II Plus). In this context, the subject must develop competences to identify problems of which a given instrument is appropriate and then execute them through that instrument. This execution demands the subject to develop:

- use schemes - Corresponding to the activities related to the management of the characteristics and particular properties of the artifact;
- instrumented action schemes - Corresponding to the activities for which the artifact is a means of accomplishment.
- instrumented collective action schemes - Corresponding to the simultaneous or joint use of an instrument in a context of respectively shared or collective activities.

For analyzing the instrumented activities, Rabardel (1995) and Villon (1996) propose the SAI[5] model (shown below), evidencing the multiplicity of interactions between the essential elements in learning. Where, in addition to the usual subject-object interaction [S-O], other interactions are considered as interactions between: subject and instrument [S-i]; the instrument and the object [i-O]; the subject and the object mediated by the [S (i) -O] instrument. This system in turn is inserted in an environment constituted by a set of conditions (limitations, facilities, etc.) that intervene in activities.

Rabardel (1995) proposed a model that distinguishes instrumentation and instrumentalization. In this distinction Vérillon (1996) explains that the *instrumentation*[6] consists in the elaboration of the relation [S-i]: the subject must construct the schemes, the procedures, the operations necessary for the implementation of the artifact. He may, for example, consider in this relation [S-i] situations constructed in other contexts with other artifacts or, on the contrary, construct new relations in order to explore them or to elaborate them by imitation. Instrumentalization is interested in the construction of relations [i-O]. The subject attributes to the instrument a possibility to act on the object and builds the functional properties that allow the actualization of this possibility of action. This action may possibly be different from that originally envisaged by the artifact's author. Let us suppose that the object to which Rabardel and Verillon refer is a mathematical object, as primitive properties of classical geometric objects, that the subject **S** is a student of the Elementary School or High School of an educational institution in France, and **i** a software such as *Cabri-Géomètre II Plus*: instrumentation and instrumentation modeling describes the way in which the instrument influences the construction of the relationship [SO] through mediation. This relationship then noted [S (i) -O] will appear in all situations where *Cabri-Géomètre II Plus*will be available. It is notable that the instrumental genesis referred to by Rabardel and Verillon is present in the activities developed by each individual in construction and acquisition of individual or collective knowledge. The evolution of this construction doesn't depend on the interactions maintained by the subject with the object as a function of constructed instrument(s). Thus, when Rabardel (1995) tells us that artifact is a material device used as a means of action, and an instrument as something constructed by the individual throughout a process of instrumental genesis, we believe that this construction is a product of the institution where this individual is subject.

In displacement of Cabri-géomètre as an instrument, already mentioned, the *Cabri* is an artifact of dynamic geometry whose displacement is an essential part. In this direction Rabardel indicates that the instruments must be studied in relation to an activity and it is the instrumented activity that must be strictly observed, since it is from this observation that several instruments can be constructed.

Being the displacement a fraction of the *Cabri-Géomètre*, the subject can construct several instruments from the displacement, which will be elaborated according to types of activities carried out, so the displacement can be used in different types of activities. For example, to look up invariants of a figure, to validate or invalidate a figure knowing the properties of the figure that are provided or a drawing presented in a "good" position. Certainly,the displacement can be used for other purposes, which will vary according to the user needs, thereby enabling the construction of other instruments.

Due to the displacement, one can start from the construction of any design in *Cabri*, to explore the properties that are truly constituent of the figure, since a drawing constructed in Cabri can be placed in a position where it seems to present specific geometric properties for that figure.

The displacement can also be used in the opposite direction to that shown previously, it can be used to move certain objects of a figure in search of certain characteristics, for example: straight lines that are parallel, straight that are perpendiculars, angles of a certain measure, angles with the same length, as see certain properties.

These tasks that use displacement with the purpose of validating or invalidating a *Cabri* figure considering the properties given by a text or a statement can be used as an instrument in construction of geometry knowledge . That is, by using the displacement in the search for specific geometrical properties of a figure, one can then decide to validate or invalidate a construction.

[4] An example considered by Drijvers, 2000 (page 218) in his article entitled: algebra on the screen, on paper and algebraic thought (Trouche, 2000).
[5] SAI - Situations of Instrumented Activities.
[6] In the activities instrumented by instrumentation the subject adapts his problem to the artifact resources. However in the instrumentalization the subject modifies the properties of the artifact, to solve its problem.

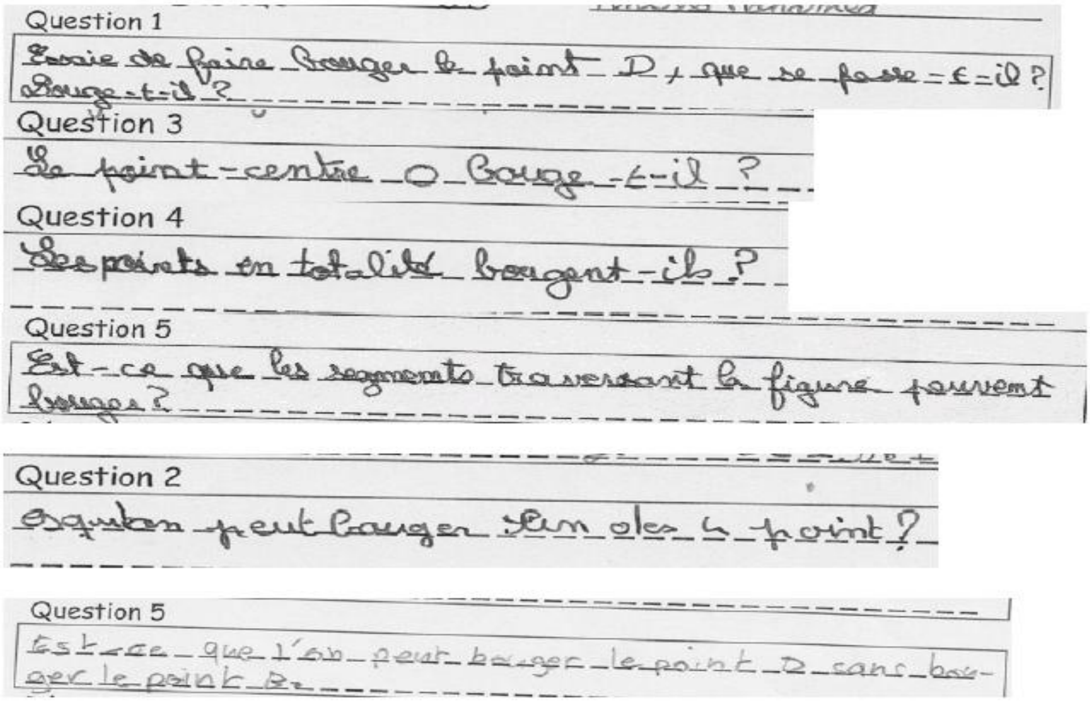
The use of displacement as an instrument continues to require accurate geometric knowledge from Cabri user. It is necessary to have sufficient knowledge to be able to recognize the geometrical properties of the figure as it moves. The displacement allows to find the geometric primitives of a figure and therefore contains, intrinsically, the idea of valid property underlying the demonstration. This may be the cause of the students'difficulties in appropriation of the displacement, especially when they are in the beginning of school grades, when students don't have this notion yet. In order to use the displacement, it is necessary not only to have geometric knowledge of represented object and of the software, but also it is necessary to know what is wanted to observe. In the same way, after construction of a figure in the *Cabri*, it is possible to see, thanks to the displacement, if all properties which are wanted to be assigned to the figure are present. Among these different tasks involving use of displacement, we chose to focus on the genesis of displacement aspect to observe how this instrumental genesis articulates among 5th grade school students. It is precisely at this learning level that construction of deductive reasoning begins. Up to a certain stage of teaching in elementary school notions of geometry are introduced by teachers through use and work on the figure: reading the figure properties, construction of figures, description of figures. The passage to the high school is followed by change, especially in relation to students' perceptions. In the high school the figure geometrical properties should no longer be only perceptibly read on a specific drawing, but also established as hypotheses (data of the figure) or deduced hypotheses. Therefore, it is important to analyze the problematic drawing-figure.

5. Development and analysis

We performed two levels of analysis; the *a priorianalysis* as an attempt to anticipatepossible students'answers for questions that were asked to them. And then, the later analysis regarding the true answers given by the students, which won't be presented in their entirety, but in some episodes which will described in the conclusion of this article.

6. Conclusion

Elementary geometry is considered an adequate space for didactic experiences, especially for its importance in knowledge acquisition process (Ferneda, 1993). However, it seems that this space hasn't been satisfactorily explored cause difficulties encountered by students to work adequately with geometrical properties of classic figures are often detected, as we can see in following pictures:


Question 1

Essaie de faire bouger le point D, que se passe-t-il? Bouge-t-il?

Question 3

Le point-centre O bouge-t-il ?

Question 4

Les points en totalité bougent-ils ?

Question 5

Est-ce que les segments traversant la figure peuvent bouger?

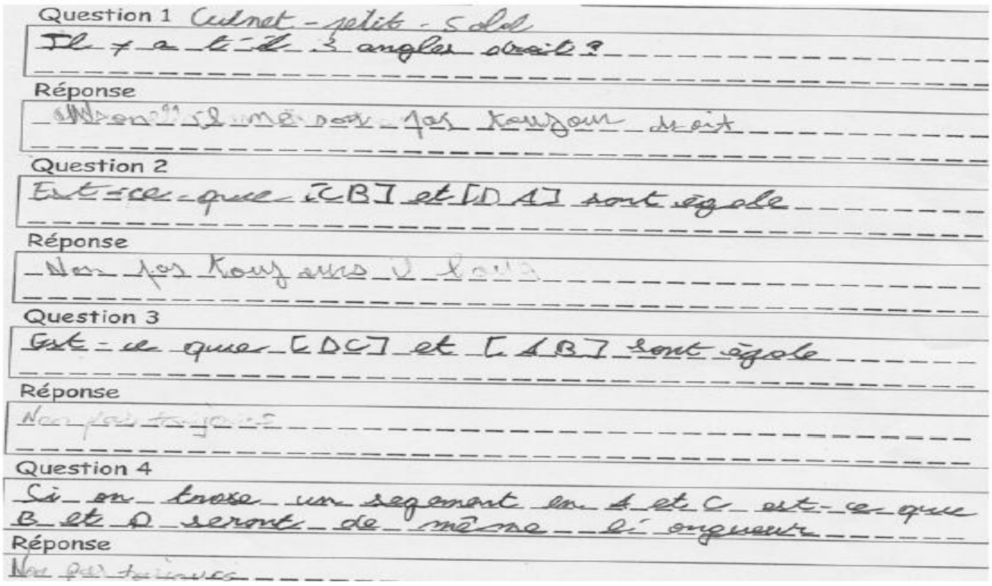
Question 2

Esqu'on peut bouger l'un des 4 point?

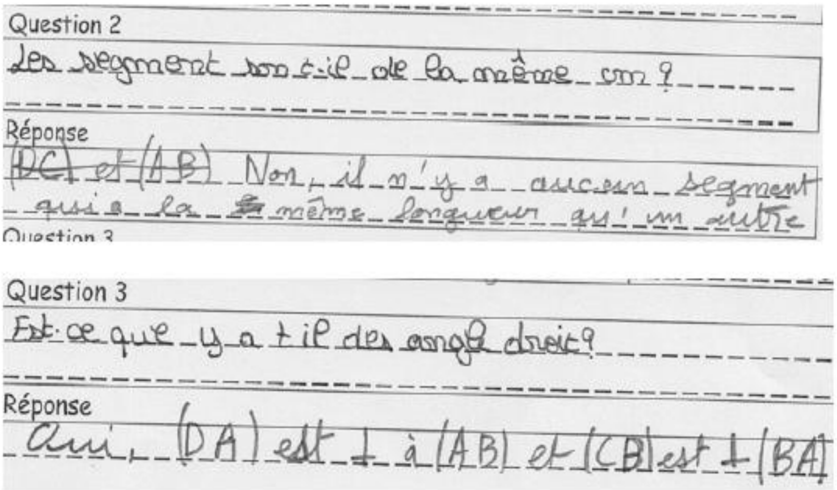
Question 5

Est-ce que l'on peut bouger le point D sans bouger le point B

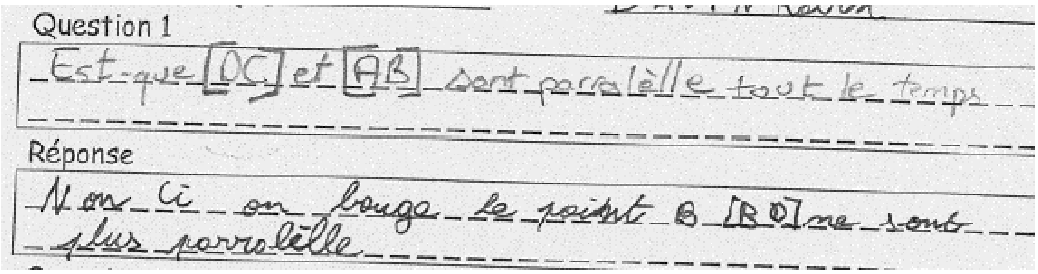
The *displacement* in *Cabri-géomètre*, during this experimentation phase, was introduced to students as an instrument for validation and invalidation of a figure geometric properties. We can verify its use not only in the *a priori* analysis, but also through the analysis of students' results where the use is evident,both in given questions and answers, as it is shown below:

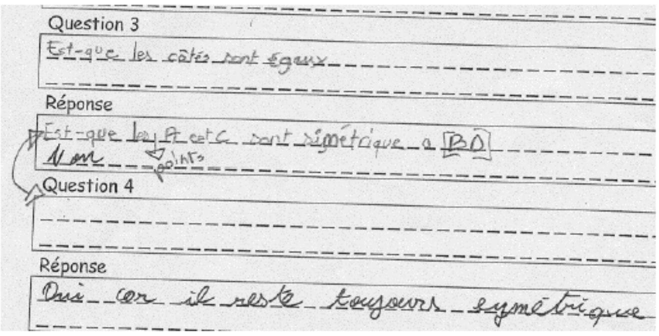


All data was collected from 34 pairs and 26 were selected to be analyzed. From the analysis of these 26 pairs, by observing strictly the sheets provided to students and also students' manipulation of the figure in the *Cabri*. Following picture gives an overview of displacement mobilized by the students in their actions in *Cabri*.



24 of 26 pairs made questions using the word *displacement*. In contrast, 21 of 24 pairsused the displacement of *Cabri* in the way it was predicted in *a priori* analysis, while exploring the figure. Of these 21 students who used the *displacement* 4 of them had anticipated the use without have been requested by the emitter pair. Consequently, the displacement effect is considered in the actions of almost all students. The students moved the figures points, observing what happened with other elements.





However, 8 pairs were unable to use the displacement efficiently, they weren't capable even to respond or make questions. That's why all data collected wasn't fully used in the analyzes. Thanks to a *Cabri* tool that records step-by-step of work done by the students, we can precisely analyze the type of manipulation made by pairs during experimentation. We noticed that students who used the displacement did so with partial understanding of function of that instrument, even when the manipulation occurred late. The use of displacement to validate or invalidate a construct could be called into question during these sequences, but it doesn't mean that these sequences are sufficient, especially considering difficulties which were presented by the students throughout their development process. Therefore, other works aiming to help in the process of instrumentalization of displacement can contribute to promote advances in this way.

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