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A Bayesian Approach for Estimating the Scale Parameter of Double Exponential Distribution under Symmetric and Asymmetric Loss Functions

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Abstract: ***The main objective of this paper is to study the Bayes estimators of the parameter of Double Exponential distribution under different loss functions and then compared among them as well as with the classical estimator named maximum likelihood estimator (MLE). In our real life, we always try to minimize the loss and we also want to gather some prior information (distribution) about the problem to solve it accurately. Here the conjugate (Gamma) prior is used as the prior distribution of Double Exponential distribution for finding the Bayes estimator. In our study, we used different symmetric (squared error and quadratic) and asymmetric (MLINEX and NLINEX) loss functions. Finally, mean square error (MSE) of the estimators are obtained and then presented graphically.***

Keywords: Bayes estimator, Maximum Likelihood Estimator (MLE), Squared Error (SE) Loss Function, Modified Linear Exponential (MLINEX) Loss Function, Non-Linear Exponential (NLINEX) Loss Function

1. Introduction

Double exponential distribution is a very popular continuous probability distribution. It has generally two parameters. One is location parameter θ and the other is scale parameter λ. Practically location parameter has limited use. Here only scale parameter is considered to estimate. A continuous random variable *X* is said to have Double exponential (λ, θ) distribution if its probability density function (pdf) is given by [1]

$$f(x;\lambda,\theta)=\begin{cases}\frac{1}{2\lambda}e^{-\frac{|x-\theta|}{\lambda}} & ;-\infty<x<\infty,-\infty<\theta<\infty,\lambda>0\\ 0 & ;otherwise\end{cases} \quad (1)$$

Where, θ is the location parameter and λ is the scale parameter.

Replacing $\frac{1}{\lambda}$ by *p* we get,

$$f(x;p,\theta)=\frac{p}{2}e^{-p|x-\theta|};-\infty<x<\infty,p>0 \quad (2)$$

Double exponential distribution is used in hydrology to extreme events such as annual maximum one- day rainfall, temperature and river discharges. This distribution has also been used in speech recognition to model priors on discrete Furrier transform (DFT) coefficients and in joint photographic experts group (JPEG) image compression to model AC coefficients generated by a discrete cosine transform (DCT) [2]. Here we are interested to find the Bayes estimator of scale parameter λ under different loss functions.

2. Literature Survey

Double exponential distribution plays an important role in data analysis. Many authors have developed inference procedures for exponential model. For example, Rahman *et al*. (2012) studied the Bayes estimators under conjugate prior for power function distribution [9]. Kulldorff (1961) devoted a large part of book to the estimation of the parameters of the exponential distribution based on completely or partially grouped data [11]. Sarhan (2003) obtained the empirical Bayes estimators of exponential model [12]. Janeen (2004) discussed the empirical Bayes estimators of the parameter of exponential distribution based on record values [14]. To more details the work of Balakrishnan *et al*. and Al-Hemyari of exponential distribution [13, 14] can be seen.

3. Prior and Posterior Density Function of Parameter p

Considering a gamma prior for the parameter *p* having density function [6]

$$\pi(p)=\frac{\beta^{\alpha}}{\Gamma\alpha}e^{-\beta p}p^{\alpha-1};\alpha,\beta,p>0 \quad (3)$$

Then the posterior density function of parameter *p* for the given random sample *x* is given by [5]

$$f(p/x)=\frac{\left[\prod_{i=1}^{n}f(x_i/p)\right]\pi(p)}{\int\left[\prod_{i=1}^{n}f(x_i/p)\right]\pi(p)dp}$$

$$=\frac{\left(\frac{p}{2}\right)^{n}e^{-p\sum|x_i-\theta|}e^{-\beta p}p^{\alpha-1}}{\int_0^{\infty}\left(\frac{p}{2}\right)^{n}e^{-p\sum|x_i-\theta|}e^{-\beta p}p^{\alpha-1}dp}$$

$$=\frac{e^{-(\sum|x_i-\theta|+\beta)p}p^{\alpha+n-1}}{\int_0^{\infty}e^{-(\sum|x_i-\theta|+\beta)p}p^{\alpha+n-1}dp}$$

$$f(p/x)=\frac{(\sum|x_i-\theta|+\beta)^{\alpha+n}e^{-(\sum|x_i-\theta|+\beta)p}p^{(\alpha+n)-1}}{\Gamma(\alpha+n)} \quad (4)$$

This implies that

$$f(p/x) \sim Gamma[a+n, \sum |x_i - \theta| + \beta]$$

4. Different Estimators of Parameter p

Here, Bayes estimator of the parameter p for different loss functions along with maximum likelihood estimator has been determined.

4.1 Maximum Likelihood Estimator (MLE) of Parameter λ

Suppose, $X = (X_1, X_2, \ldots\ldots, X_n)$ is a random sample of size n drawn from Double exponential distribution. Let $(x_1, x_2, \ldots\ldots, x_n)$ is the observe value of $(X_1, X_2, \ldots\ldots, X_n)$. Then the likelihood function of the parameter λ for the random sample $(x_1, x_2, \ldots\ldots, x_n)$, is given by [5]

$$L(\lambda / x) = \prod_{i=1}^{n} f(x_i; \lambda) = \left(\frac{1}{2\lambda}\right)^n e^{-\frac{\sum_{i=1}^{n}|x_i - \theta|}{\lambda}}$$

The natural logarithm of likelihood function is given by

$$\log L(\lambda / x) = -n \log(2\lambda) - \frac{\sum_{i=1}^{n}|x_i - \theta|}{\lambda}$$

$$\log L(\lambda / x) = -n \log 2 - n \log \lambda - \frac{\sum_{i=1}^{n}|x_i - \theta|}{\lambda} \qquad (5)$$

Now the MLE of λ is obtained by solving [5] the following equation

$$\frac{\partial \log L(\lambda / x)}{\partial \lambda} = 0 \Rightarrow -\frac{n}{\lambda} + \frac{\sum_{i=1}^{n}|x_i - \theta|}{\lambda^2} = 0$$

$$\Rightarrow n\lambda = \sum_{i=1}^{n}|x_i - \theta|$$

Hence, $\hat{\lambda}_{MLE} = \frac{\sum_{i=1}^{n}|x_i - \theta|}{n}$ is the MLE of parameter λ where, θ is known.

4.2. Bayes Estimator of Parameter λ for Squared Error (SE) Loss Function

Here we have determined Bayes estimator of $\lambda = \frac{1}{p}$ for squared error loss function [6] defined by

$$L(\hat{p}, p) = (\hat{p} - p)^2 \qquad (5)$$

For squared error loss function Bayes estimator is the mean of posterior density function, so from (4) the Bayes estimator of p is given by $\hat{p}_{BSE} = \frac{\alpha + n}{\sum |x_i - \theta| + \beta}$. Since we have $p = \frac{1}{\lambda}$ hence, $\hat{\lambda}_{BSE} = \frac{1}{\hat{p}_{BSE}} = \frac{\sum |x_i - \theta| + \beta}{\alpha + n}$ is the Bayes estimator of λ under squared error loss function.

4.3. Bayes Estimator of Parameter λ for Quadratic Loss (QL) Function

Let, the quadratic loss function is defined as [7]

$$L(\hat{p}; p) = \left(\frac{\hat{p} - p}{p}\right)^2 \qquad (6)$$

Under this loss function the Bayes estimator of p is obtained by solving the equation

$$\frac{\partial}{\partial \hat{p}} \int L(\hat{p}; p) f(p/x) dp = 0$$

$$\Rightarrow \frac{\partial}{\partial \hat{p}} \int \left(\frac{\hat{p} - p}{p}\right)^2 \frac{(\sum |x_i - \theta| + \beta)^{\alpha+n}}{\Gamma(\alpha+n)}$$

$$e^{-(\sum |x_i - \theta| + \beta) p} p^{(\alpha+n)-1} dp = 0$$

$$\Rightarrow \frac{(\sum |x_i - \theta| + \beta)^{\alpha+n}}{\Gamma(\alpha+n)} \hat{p} \int e^{-(\sum |x_i - \theta| + \beta) p} p^{(\alpha+n-2)-1} dp$$

$$= \frac{(\sum |x_i - \theta| + \beta)^{\alpha+n}}{\Gamma(\alpha+n)} \int e^{-(\sum |x_i - \theta| + \beta) p} p^{(\alpha+n-1)-1} dp$$

$$\Rightarrow \frac{(\sum |x_i - \theta| + \beta)^{\alpha+n}}{\Gamma(\alpha+n)} \hat{p} \frac{\Gamma(\alpha+n-2)}{(\sum |x_i - \theta| + \beta)^{\alpha+n-2}}$$

$$= \frac{(\sum |x_i - \theta| + \beta)^{\alpha+n}}{\Gamma(\alpha+n)} \frac{\Gamma(\alpha+n-1)}{(\sum |x_i - \theta| + \beta)^{\alpha+n-1}}$$

$$\Rightarrow \hat{p} = \frac{\Gamma(\alpha+n-1)}{(\sum |x_i - \theta| + \beta)^{\alpha+n-1}} \frac{(\sum |x_i - \theta| + \beta)^{\alpha+n-2}}{\Gamma(\alpha+n-2)}$$

$$\Rightarrow \hat{p}_{BQL} = \frac{(\alpha+n-2)}{(\sum |x_i - \theta| + \beta)}$$

Since we have $p = \frac{1}{\lambda}$ hence, $\hat{\lambda}_{BQL} = \frac{(\sum |x_i - \theta| + \beta)}{(\alpha+n-2)}$ is the Bayes estimator of λ under quadratic loss function.

4.4. Bayes Estimator of Parameter λ for MLINEX Loss Function

Let, the MLINEX loss function [7] is defined as

$$L(\hat{p};p)=\omega\left[\left(\frac{\hat{p}}{p}\right)^{c}-c\log\left(\frac{\hat{p}}{p}\right)-1\right],\omega>0,c\neq 0 \quad (7)$$

For MLINEX loss function the Bayes estimator of $\lambda=\frac{1}{p}$ is obtained by $\hat{p}_{BML}=\left[E(p^{-c}/x)\right]^{-\frac{1}{c}}$ (8)

Here, $E(p^{-c}/x)=\int_0^\infty p^{-c}f(p/x)dp$

$$=\frac{(\sum|x_i-\theta|+\beta)^{\alpha+n}}{\Gamma(\alpha+n)}\int_0^\infty e^{-(\sum|x_i-\theta|+\beta)p}p^{(\alpha+n-c)-1}dp$$

$$=\frac{(\sum|x_i-\theta|+\beta)^{\alpha+n}}{\Gamma(\alpha+n)}\frac{\Gamma(\alpha+n-c)}{(\sum|x_i-\theta|+\beta)^{\alpha+n-c}}$$

$$\therefore E(p^{-c}/x)=\frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}(\sum|x_i-\theta|+\beta)^{c}$$

Therefore from (8) we get,

$$\hat{p}_{BML}=\left[\frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}\right]^{-\frac{1}{c}}(\sum|x_i-\theta|+\beta)^{-1}$$

Since we have $p=\frac{1}{\lambda}$ hence,

$\hat{\lambda}_{BML}=\left[\frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}\right]^{\frac{1}{c}}(\sum|x_i-\theta|+\beta)$ is the Bayes estimator of λ under MLINEX loss function.

4.5. Bayes Estimator of Parameter λ for NLINEX Loss Function

Let, the NLINEX loss function [8] of the form

$$L(D)=k\left[\exp(cD)+cD^2-cD-1\right],k>0,c>0 \quad (9)$$

Here, D represents the estimation error i.e., $D=\hat{p}-p$.

For NLINEX loss function Bayes estimator of $\lambda=\frac{1}{p}$ is [6] given by

$$\hat{p}_{BNL}=-[\ln E_p\{\exp(-cp)\}-2E_p(p)]/(c+2) \quad (10)$$

Where, $E_p(.)$ stands for posterior expectation

Now, $E_p\{\exp(-cp)\}=\int_0^\infty e^{-cp}f(p/x)dp$

$$=\frac{(\sum|x_i-\theta|+\beta)^{\alpha+n}}{\Gamma(\alpha+n)}\int_0^\infty e^{-(c+\sum|x_i-\theta|+\beta)p}p^{(\alpha+n)-1}dp$$

$$=\frac{(\sum|x_i-\theta|+\beta)^{\alpha+n}}{\Gamma(\alpha+n)}\frac{\Gamma(\alpha+n)}{(c+\sum|x_i-\theta|+\beta)^{\alpha+n}}$$

$$=\left(\frac{c+\sum|x_i-\theta|+\beta}{\sum|x_i-\theta|+\beta}\right)^{-(\alpha+n)}$$

$$\therefore E_p\{\exp(-cp)\}=\left(1+\frac{c}{\sum|x_i-\theta|+\beta}\right)^{-(\alpha+n)}$$

So, $\ln E_p\{\exp(-cp)\}$

$$=-(\alpha+n)\ln\left(1+\frac{c}{\sum|x_i-\theta|+\beta}\right) \quad (11)$$

Again, $E_p(p)=\int_0^\infty pf(p/x)dp$

$$=\frac{(\sum|x_i-\theta|+\beta)^{(\alpha+n)}}{\Gamma(\alpha+n)}\int_0^\infty e^{-(\sum|x_i-\theta|+\beta)p}p^{(\alpha+n+1)-1}dp$$

$$=\frac{(\sum|x_i-\theta|+\beta)^{\alpha+n}}{\Gamma(\alpha+n)}\frac{\Gamma(\alpha+n+1)}{(\sum|x_i-\theta|+\beta)^{\alpha+n+1}}$$

$$\therefore E_p(p)=\frac{(\alpha+n)}{(\sum|x_i-\theta|+\beta)} \quad (12)$$

Putting (11) and (12) in (10) we obtain

$$\hat{p}_{BNL}=-\left[-(\alpha+n)\ln\left(1+\frac{c}{(\sum|x_i-\theta|+\beta)}\right)-2\frac{(\alpha+n)}{(\sum|x_i-\theta|+\beta)}\right]/(c+2)$$

Since we have $p=\frac{1}{\lambda}$ hence,

$$\hat{\lambda}_{BNL}=\frac{(c+2)}{(\alpha+n)\left(\ln\left(1+\frac{c}{(\sum|x_i-\theta|+\beta)}\right)+\frac{2}{(\sum|x_i-\theta|+\beta)}\right)}$$ is the Bayes estimator of λ under NLINEX loss function.

5. Empirical Study

In order to compare estimators $\hat{\lambda}_{MLE}$, $\hat{\lambda}_{BSE}$, $\hat{\lambda}_{BQL}$, $\hat{\lambda}_{BML}$ and $\hat{\lambda}_{BNL}$ we have considered MSE of the estimators. The MSE of estimator $\hat{\lambda}$ is defined as $MSE(\hat{\lambda})=E[\hat{\lambda}-\lambda]^2=Var(\hat{\lambda})+[Bias(\hat{\lambda})]^2$. In this study 10,000 Samples have generated for each case. To obtain the variance of $\hat{\lambda}$, we have used the true (assume) value of the parameter λ under consideration. Again we have obtained the estimated value, bias and MSE of the

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estimators by using R- Code simulation from the Double exponential distribution. The results and their graphs using MS- Excel are presented below:

Table 1: Estimated value, Bias and MSE of different estimators of parameter λ of Double exponential distribution when $\alpha=1, \beta=2, \theta=1, \lambda=1$ and $c=1$

n	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
5	Estimated Value	0.504	0.807	0.953	1.045	1.023
	Bias	-0.301	-0.090	-0.366	-0.096	-0.063
	MSE	0.215	0.093	0.332	0.132	0.089
10	Estimated Value	0.769	0.592	1.159	0.889	1.059
	Bias	-0.309	-0.188	-0.009	-0.111	-0.176
	MSE	0.157	0.087	0.073	0.073	0.081
15	Estimated Value	0.450	0.694	0.785	0.574	1.324
	Bias	-0.304	-0.225	-0.117	-0.175	-0.215
	MSE	0.133	0.086	0.060	0.071	0.081
20	Estimated Value	0.558	0.721	0.877	0.965	0.904
	Bias	-0.305	-0.245	-0.165	-0.208	-0.239
	MSE	0.124	0.088	0.060	0.073	0.085
25	Estimated Value	0.440	0.850	0.777	0.664	0.789
	Bias	-0.307	-0.257	-0.197	-0.227	-0.249
	MSE	0.118	0.089	0.065	0.076	0.085
30	Estimated Value	0.858	0.916	0.924	0.791	0.657
	Bias	-0.306	-0.266	-0.215	-0.238	-0.259
	MSE	0.114	0.089	0.068	0.077	0.086

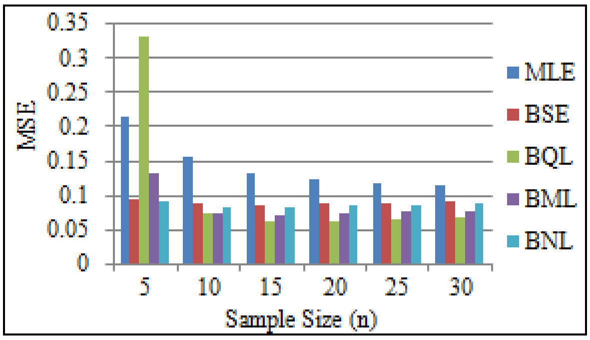

Figure 1: Graph of MSEs of different estimators of parameter λ for Double exponential distribution under different sample size.

Table 2: Estimated value, Bias and MSE of different estimators of parameter λ of Double exponential distribution when $n=20, \alpha=2, \beta=3, \theta=2$ and $c=2$

λ	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
1.0	Estimated Value	0.355	0.501	0.654	0.601	0.552
	Bias	-1.307	-1.235	-1.156	-1.178	-1.209
	MSE	1.739	1.550	1.368	1.417	1.489
1.5	Estimated Value	0.855	0.912	0.796	0.721	0.828
	Bias	-0.959	-0.917	-0.814	-0.840	-0.891
	MSE	0.989	0.896	0.729	0.772	0.852
2.0	Estimated Value	0.845	1.407	1.445	1.337	1.285
	Bias	-0.611	-0.600	-0.461	-0.499	-0.583
	MSE	0.492	0.463	0.334	0.365	0.439
2.5	Estimated Value	2.506	1.943	2.673	1.948	1.661
	Bias	-0.277	-0.295	-0.127	-0.159	-0.260
	MSE	0.277	0.245	0.203	0.209	0.228

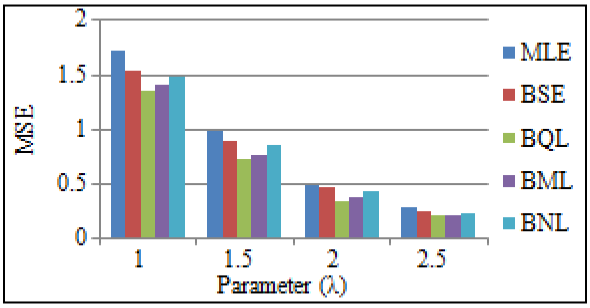

Figure 2: Graph of MSEs of different estimators of parameter λ of Double exponential distribution.

Table 3: Estimated value, Bias and MSE of different estimators of parameter λ of Double exponential distribution where $n=10, \alpha=1, \beta=1, \lambda=1$ and $c=1$

θ	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
-2.0	Estimated Value	0.799	0.776	0.791	0.859	0.561
	Bias	-0.309	-0.280	-0.118	-0.205	-0.262
	MSE	0.157	0.130	0.088	0.104	0.119
-1.5	Estimated Value	0.795	0.682	0.634	0.424	0.478
	Bias	-0.306	-0.279	-0.118	-0.203	-0.260
	MSE	0.155	0.129	0.088	0.103	0.119
-1.0	Estimated Value	0.601	0.670	0.797	0.765	0.578
	Bias	-0.304	-0.277	-0.118	-0.204	-0.261
	MSE	0.154	0.127	0.086	0.104	0.121
1.0	Estimated Value	0.720	0.631	0.837	0.890	0.627
	Bias	-0.311	-0.279	-0.116	-0.205	-0.262
	MSE	0.156	0.127	0.088	0.104	0.120
1.5	Estimated Value	0.676	0.583	0.653	0.728	0.668
	Bias	-0.303	-0.282	-0.114	-0.207	-0.265
	MSE	0.154	0.129	0.092	0.1031	0.120
2.0	Estimated Value	0.862	0.822	0.796	0.534	0.687
	Bias	-0.304	-0.280	-0.116	-0.206	-0.267
	MSE	0.153	0.129	0.089	0.102	0.121

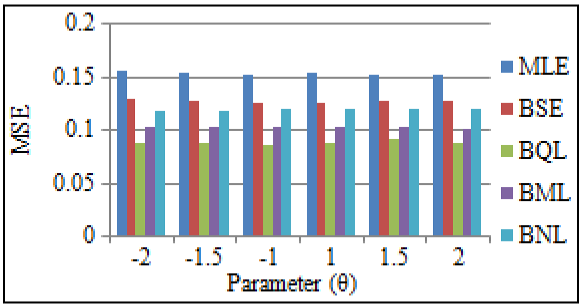

Figure 3: Graph of MSEs of different estimators of parameter θ for Double exponential distribution.

6. Real Study

For fitting Double exponential distribution, temperature data have been used in this paper. Monthly maximum temperature (in ^{0}C) data in 2012 in Sylhet have been chosen.

Table 4: Estimated value, Bias and MSE of different estimators of parameter λ of Double exponential distribution when $n=12, \alpha=2, \beta=3, \theta=2$ and $c=2$

λ	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
1.0	Estimated Value	29	25	29	28	25
	Bias	28	24	28	27	24
	MSE	784	579	798	735	581
1.5	Estimated Value	29	25	29	28	25
	Bias	27	23	27	26	23
	MSE	756	555	770	707	557
2.0	Estimated Value	29	25	29	28	25
	Bias	27	23	27	26	23
	MSE	729	532	742	681	533
2.5	Estimated Value	29	25	29	28	25
	Bias	26	22	26	25	22
	MSE	702	509	715	655	511

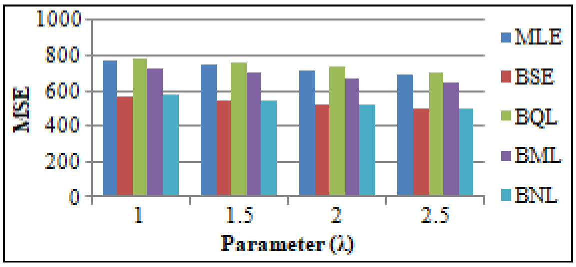

Figure 4: Graph of MSEs of different estimators of parameter λ of Double exponential distribution

Result and Discussion

Table 1 shows the variation in the performance of the estimators for varying sample size. It has been seen that the biases and MSEs of $\hat{\lambda}_{MLE}$ remain the largest for almost all cases (figure 1). Now we compare the rest of the estimators. The bias of all other estimators shows negatively increasing trend except for some cases. For sample size greater than 5 the bias of $\hat{\lambda}_{BQL}$ is smaller than all other estimators. But when sample size is 5 bias of $\hat{\lambda}_{BML}$ and bias of $\hat{\lambda}_{BNL}$ dominate the bias of $\hat{\lambda}_{BQL}$. Again the estimated bias for the two estimators $\hat{\lambda}_{BSE}$ and $\hat{\lambda}_{BNL}$ shows more or less similar pattern. When sample size increases the MSEs of estimators decreases. The MSEs of $\hat{\lambda}_{BQL}$ decreases sharply at the beginning and then it follows a parallel trend. For sample size greater than 5 the MSEs of all estimators are larger than that of $\hat{\lambda}_{BQL}$. Again the MSEs of $\hat{\lambda}_{BSE}$ and $\hat{\lambda}_{BNL}$ show approximately similar pattern (figure 1). Table 2 shows variation in the estimators, biases and MSEs with respect to scale parameter λ. Bias and MSEs of all estimators' decreases with increasing λ except for $\hat{\lambda}_{MLE}$ and these are least for $\hat{\lambda}_{BQL}$ (figure 2). Table 3 shows the variation in the estimators, biases and MSEs with respect to the location parameter θ. The values of θ has no effect on bias and MSE whether it is positive or negative. In this case the bias and MSE of $\hat{\lambda}_{BQL}$ is least than all other estimators (figure 3). Again for varying θ, $\hat{\lambda}_{BSE}$ and $\hat{\lambda}_{BNL}$ are close to one another. Table 4 shows variation in estimators; biases and MSEs with respect to scale parameter λ for real data. MSEs of all estimators' decreases with increasing λ and these are least for $\hat{\lambda}_{BSE}$ (figure 4). Also bias of all estimators' decreases and remain same for some cases with increasing λ and these are least for both $\hat{\lambda}_{BSE}$, $\hat{\lambda}_{BSE}$. Again for varying λ, bias of both $\hat{\lambda}_{BSE}$ and $\hat{\lambda}_{BNL}$ show similar pattern.

8. Conclusion

In this study, we have considered the Bayesian estimation approach to estimate the scale parameter of Double exponential distribution. In Bayesian approach, squared error, quadratic, MLINEX and NLINEX loss functions have been used. We conducted a comprehensive simulation and real data to judge the relative performance of the Bayes estimator under different loss functions at different sample sizes and varied parameters of prior distribution. From above analysis and graphical presentation we conclude that Bayes estimator under Quadratic loss function is better than all other estimators for simulated data. Also it is seen that Bayes estimator under NLINEX loss function is close to Bayes estimator under Squared Error (SE) loss function for real data. Finally conclude that, non-classical estimator (the class of Bayes estimator) is better than classical estimator (MLE). Therefore, Bayesian approach under quadratic loss function can be suggested to estimate the parameter of Double exponential distribution.

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