First Year Students' Understanding of Specific Concepts in Selected Mathematics Topics: The Case of the University of Zambia

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Abstract: ***This study investigated the understanding which University of Zambia (UNZA) first year students of mathematics had of specific concepts in selected mathematics topics. Procedural and conceptual understanding underpinned the investigation. It was also the intention of the study to determine whether there exist any relationship between the students' confidence levels and their procedural and conceptual understanding of particular concepts. A quantitative approach was followed and specifically a case study design was employed. Three hundred and seventy eight (378) randomly sampled first year students of mathematics wrote a test which was based on sets, functions, polynomials, trigonometry, and complex numbers as taught in the first year at UNZA. To enhance content and face validity, the administered test was preceded by document analysis, piloting, and expert judgement by UNZA lecturers of mathematics. Subsequently, a follow up questionnaire was administered to elicit the participating students' experiences as they solved the test items. The test data was analysed using standard indices while the data derived through questionnaires was analysed using multivariate techniques. The study revealed that the majority of the students had procedural, and not conceptual understanding of the concepts assessed.***

Keywords: Procedural understanding, Conceptual understanding, Confidence levels

1. Background

First year mathematics taught at the University of Zambia provides basics for subsequent admittance into several discipline such as medicine, agriculture, engineering to mention but a few. Some of the students of mathematics ultimately becomes teachers of the subject in secondary schools and colleges. Despite the significance of the subject, no study has been conducted to assess students' understanding of mathematics concepts at first year.

A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes (National Research Council, 2001:116). Nevertheless, the study is important to be researched on because it could help students understand mathematical concepts and procedures. Not only was the research worth doing at the University of Zambia but also would complement other research in the same field by contributing and extending existing knowledge on university understanding of mathematics. Two essential types of knowledge that students acquire are conceptual understanding and procedural skill (Alibali, Rittle-Johnson and Siegler, 2001:346). In many domains, students must learn both fundamental concepts and correct procedures for solving problems. For example, mathematical competence rests on students developing and connecting their knowledge of concepts and procedures (Alibali and Rittle-Johnson, 1999:175). Procedural and conceptual understanding of mathematics at universities across the globe and within Africa and the sub-region has been of special concern for mathematics lecturers. Engelbrecht Johann, HardingsAnsie and Potgieter Marietjie, (2005:701) argues that 'Mathematics pedagogy based on Vygotskian theory approaches mathematics as a conceptual system rather than a collection of discrete procedures' which is not in agreement with the suggestion by the same authors that mathematics understanding is procedural.

Even though, most students entering university education in Zambia have developed mathematical skills and competences very few students are able to demonstrate full knowledge of conceptual skills. Nevertheless, they seem to have difficulties also in procedural topics. It is against this background that this paper sought to highlight challenges faced by first year students at the University of Zambia in the understanding of specific concepts in selected mathematics topics. Henceforth, the researcher prepared questions consisting of procedural and conceptual aspects which he gave to students to test them on their procedural and conceptual competencies respectively.

The University of Zambia First Year Students' Performance in Mathematics (2014-2016)

The study sought to look at the performance before it looked at the understanding part. It could have been difficult or even impossible to look at all the streams of students doing mathematics at the University of Zambia. To add on, first year students were chosen because of the big numbers doing mathematics and that they come with good grades from ordinary level but performs badly when it comes to university mathematics. The researcher had intensions to look at the results of students since the change of orientation of learning from semester to termly. Unfortunately, the information for 2012-2014 was not availed to the researcher.

Henceforth, the researcher was only given 2014-2016 data which he analysed as shown in the table below:

MAT1100 Final Assessment (Analysis of Results)

Table 1.1: University of Zambia Final Examination Assessment from 2014-2016

Subject	Enter ED	SAT	Absent	Grades								Pass % B-A^+	Pass % C-A^+
				A^+	A	B^+	B	C^+	C	D^+	D	Qualitative	Quantitative
MAT 1100 2014-2015	1534	1383	151	5	20	61	116	185	526	1	449	14.6%	66%
MAT 1100 2015-2016	1411	1251	160	10	50	94	148	198	418	0	335	24.1%	73.4%

The performance is poor as shown above by first year students in mathematics at UNZA meaning also the understanding is equally bad.

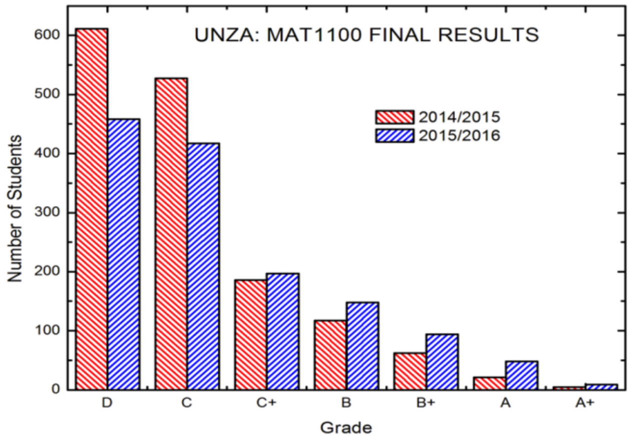


Figure 1.1: MAT1100 Final Assessment (Analysis of Results)

The results above shows that good grades in mathematics to enable a candidate enter into a career of choice had a very small number. The grade of B to A^+ for the two consecutive years where less than two hundred which may make a candidate be in the profession of choice. To add on, it was observed that about half of the number with grade of C had failed initially but due to moderation of results hence survived by the grace of God or the mercy of the department.

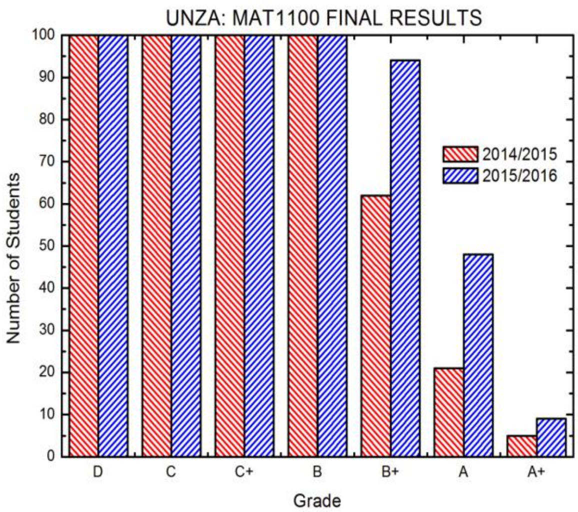


Figure 1.2: Analysis of Results

Rational for Providing Procedural and Conceptual Understanding of Mathematical Concepts in Mathematics

Howe (1999:882) argues that successful completion of college course work is not evidence of through understanding of elementary mathematics. Nevertheless, Howe (1999) in agreement with Deborah Ball and others (1990) stipulates that even though most university mathematicians see much of advanced mathematics as a deepening and broadening, a refinement and clarification and fulfilment of elementary mathematics it is not possible to take and pass advanced courses without understanding how they illuminate more elementary material, particularly if one's understanding of that material is superficial. However, Howe (1999:884) further argues that, traditional curriculum allowed millions of people to be taught procedures for finding correct answers to important problems, without either the teachers or the students having to understand why the procedures worked. Calculators may help students solve elementary calculations without understanding the procedures and the concepts in the question. Simply learning computational procedures without understanding them will not develop the ability to reason about what sort of calculations are needed.

Petersen, Jeffery Naomi (2003) suggested that, "there is a profound difference between understanding mathematical concepts and procedures and using that knowledge to design effective instruction for a real group of students." Therefore, there is need to discuss mathematical procedures and concepts before introducing topics to learners and also the instruments used to measure the gaps in learning. There is still a lot of arguments, as seen by "The National Council of Teachers of Mathematics (1989) who postulates that there is a paradigm shift from teaching mathematics as skill-oriented to a more conceptual understanding of deep subject matter knowledge (AAAS, 1990). To add on, The Mathematics Teaching Profile (MTP) Petersen, Brown, Sage, and Zizzo (2003) postulates that, "a measure of teacher quality may serve the profession's continuing commitment to closing the gaps between theory and practice, between knowledge and teaching, and, indirectly, the achievement gap among students.

Furthermore, Hallett Darcy, Nunes Terezinha, Bryant Peter and Thorpe, M. Christina (2012:469) argues that "recent research on children's conceptual and procedural knowledge has suggested that there are individual differences in the ways that children combine these types of knowledge across a number of mathematical topics. Cluster analysis has demonstrated that some children have more conceptual knowledge, and some children have more procedural knowledge, and some children have an equal level of both".

1.1 Statement of the Problem

The University of Zambia has been training mathematics students for a long time. Notwithstanding, stakeholders usually focus on students final scores attained in tests and examinations. They also normally concentrate on generic aspects such as how many students passed or failed the assessments. This is done without in-depth consideration of the understanding students acquire of the mathematics concepts studied. Likewise, no research study has ever been conducted to establish the understanding students acquire as they study mathematics concepts. To redress the scenario, the current study investigate first year mathematics students' understanding of specific concepts in selected mathematics topics.

1.2 Purpose of the study

The purpose of this study is to investigate first year students understanding of specific concepts in selected mathematics topics at the University of Zambia.

1.3 Study Objectives

- To determine the kind of understanding possessed by first year UNZA mathematics students of concepts in mathematics.
- To determine the relationship between the confidence levels of students and their understanding of concepts in mathematics.
- To investigate the relationship between students' confidence and their actual performance in procedural and conceptual mathematical problem.

1.4 Research questions

- What kind of understanding do first year UNZA mathematics students have of concepts in first year mathematics?
- What is the relationship between students confidence levels and there understanding of concepts in mathematics topics?
- What relationship is there between students' confidence and their actual performance in procedural and conceptual mathematical problems?

1.5 Significance of the Study

It revealed the extent of the relation between students' conceptual and procedural understanding of specific concepts in selected mathematics topics.

The study also elaborated the significance of the relationship between students' confidence levels when handling procedural and conceptual problems
It created awareness on the relationship between students' confidence and their actual performance in procedural and conceptual mathematical problems.

Since no studies has been done to the researchers knowledge at the University of Zambia concerning first year students' understanding of specific concepts in mathematics: Therefore, the study would be used to provide information that might be used by other researchers who may deal with the related problem.

However, the study is useful to several stakeholder such as university lecturers of mathematics content and methodology and also curriculum planners and various examination bodies.

1.6 Theoretical and Conceptual Framework

Theoretical and Conceptual Framework establishes the theory and the concepts which the researcher used in the study. The development of the research model was based on the research questions and literature surveys.

1.7.1Theoretical Framework

In this study Auguste Comte's, Emile Durkheim and Logical Positivism theorems were used.

1.7.1.1 Auguste Comte

Auguste Comte (1798-1857) was the founder of sociology, as he is widely identified. He analysed the society based on social order. Most importantly, he argued that, 'the scientific method should be applied to the study of society. Henslin (1998:6) points out that, 'the idea of applying scientific method to social world is known as positivism and was apparently first proposed by Auguste Comte. To add on, Comte argued that, the right way is to apply scientific method to social life which is today called sociology. Nevertheless, Comte stipulated that we must observe and classify human activities in order to uncover society's fundamental laws.

1.7.1.2 Emile Durkheim

Durkheim identified social integration as the degree to which people are tied to their social group, as a key social factor. To add on, the primary professional goal of Emile Durkheim (1858-1917) was to get sociology recognized as a separate academic discipline. However, another goal for Durkheim was to show how social forces affects people's behaviour. The modern academic discipline of sociology began with the work of Émile Durkheim (1858–1917). While Durkheim rejected much of the details of Comte's philosophy, he retained and refined its method, maintaining that the social sciences are a logical continuation of the natural ones into the realm of human activity, and insisting that they may retain the same objectivity, rationalism, and approach to causality. Thibodeaux (2016) in agreement with Schunk (2008) argues that, 'Comte was the only major sociological thinker to postulate that the social realm may be subject to scientific analysis in exactly the same way as natural science, whereas Durkheim saw a far greater need for a distinctly sociological scientific methodology'. Ferrante (2011:14) argues that,' Emile Durkheim suggested that the system of social ties acts as a cement binding people to each other and to the society.

1.7.1.3 Conceptual Framework

Conceptual framework consists of concepts that are positioned within a coherent and orderly design, indicatingless formal structures and used in studies in which existing theory is inappropriate or inadequate based on specific concepts. A conceptual framework is usually a diagram that shows how a program or research fits into a wider context. However, the conceptual framework clarifies assumptions about causal relationships and also shows how research components will operate to influence outcomes. Furthermore, a conceptual framework is used as a guide indicator identification to check if all the causal pathways are impacting the final analysis of the research report.

1.7.1.4 Conceptual Framework of the Research

The study was guided by two conceptual frameworks. The conceptual frameworks were used to help the researcher focus on variables in the study. Firstly,the researcher used operationalizing the concept of learning model then trans-contextual model before he developed his model as here-under exemplified.

1.7.1.5: Operationalizing the Concept of Learning Model (Figure 1.3)

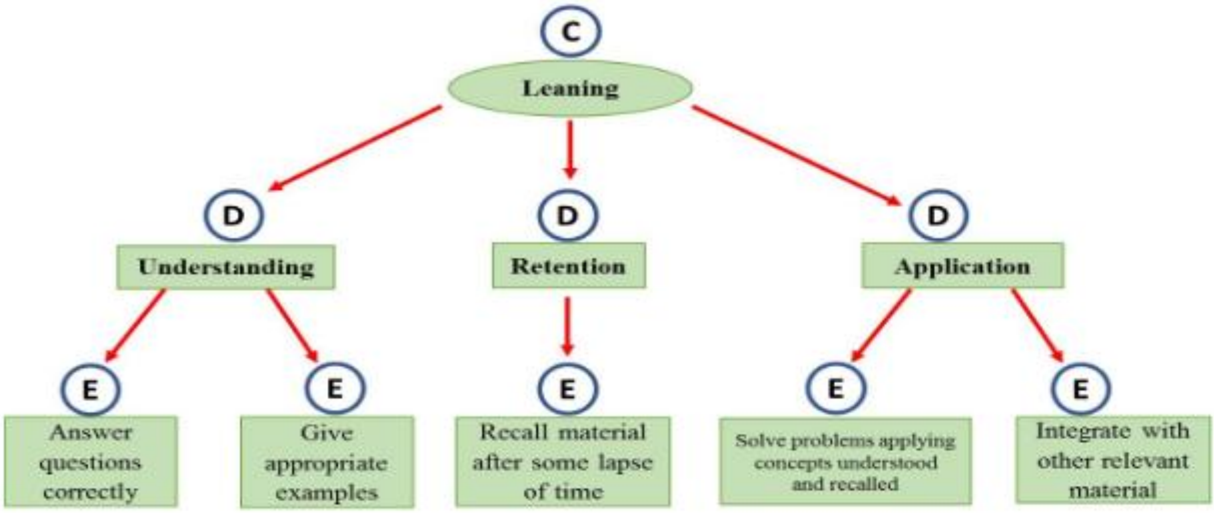


Figure 2.1: Operationalizing the Concept of Learning Model
Source: (Sekaran, 2003: 183) Research Methods for Business. A Skill Building Approach

In this model the researcher picked the component of understanding and married it with answering the questions correctly and also give appropriate examples in order to measure learning. This model helped the researcher to measure understanding by formulating a test which was used to measure understanding by employing standard indices, factor analysis and also multidimensional scaling.

1.7.1.6 Trans-Contextual Model (Figure 1.4)

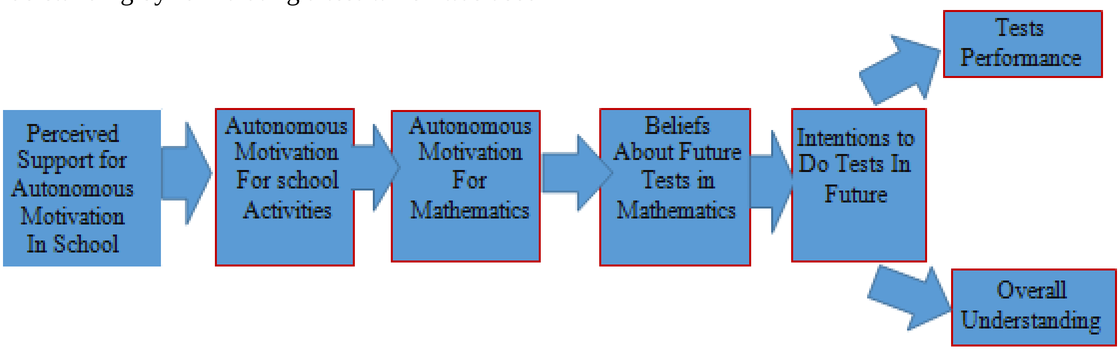


Figure 2.2: Trans – Contextual Model

An important question for any school educator is whether his or her instruction will affect pupils outside the school environment. Educators are interested whether pupils apply the knowledge and skills they have learned in educational contexts to out-of-school contexts. The trans-contextual model is a multi-theory approach to understanding the processes by which forms of motivation toward educational activities in an educational context lead to motivation toward similar activities and behaviours outside school in extramural contexts (Fortus&Vedder-Weiss, 2014). Specifically, a trans-contextual model of motivation was developed that specified the processes by which motivation in an educational context is transferred to motivation for activities performed outside of school and, importantly, to future intended behaviours to achieve key adaptive outcomes.

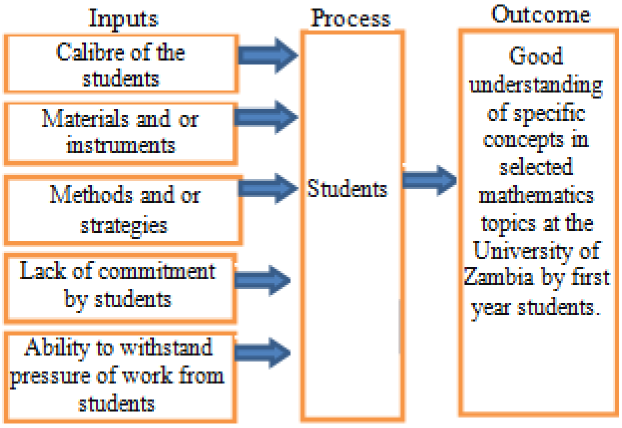


Figure 1.5: Conceptual Framework of the Research
Source: Authors elaboration (2019)

1.7 Scope of the study

The study sought to focus on first year students of mathematics at the University of Zambia. This study focused on the constraints of procedural and conceptual performance in mathematics and whereby the researcher investigated the extent of success in confidence in solving either procedural or conceptual aspects of mathematics.

1.8 Operational Definitions

- **Procedural performance:** This is the performance attained by pupils when they solve problems by using procedures.
- **Conceptual performance:** This is the performance attained by pupils when they solve problems by using concepts.
- **Procedural confidence performance:** This is the ability to achieve solving a problem through manipulation of mathematical skills, such as procedures, rules, formulae and symbols used in mathematics.
- **Conceptual confidence performance:** This is the ability to achieve solving a problem through conceptual understanding by being able to interpret and apply mathematical concepts in relation to the level of confidence to perform that task.

1.9 Ethical Considerations

Letter of permission from the University of Zambia Ethical Committee was sought. The names of the participants were not revealed. Participants had the right to withdraw from the research at any time.

2. Literature Review

2.1 Conceptual and Procedural Understanding of Mathematics at International Level

In order to solve any problem correctly, students need both applications of understanding of conceptual and procedural knowledge (Cracolice et al 2008). Surif et al (2012:419) in agreement with Cracolice et al (2008) argues that, most students are weak in conceptual knowledge and that they continue to rely on algorithm problem solving techniques. This shows that students are only able to memorize and remember the formula and the process involved without understanding the concepts. Rittle-Johnson Bethany et al (2001) in agreement with (Gelman & Willliams, 1998; Siegler, 1991; Siegler & Crowley, 1994; Sophian, 1997) argues that conceptual and procedural knowledge influence one another.

2.2 Conceptual and Procedural Understanding of Mathematics in Sweden

Hiebert and Lefevre (1986) defined conceptual knowledge in mathematics as a network of knowledge in which relationships are prominent discrete of information. To add on, procedural knowledge is seen to constitute step-by-step procedures for solving mathematical task. To be competent in mathematics it involves not only the knowledge of concepts and knowledge of procedures but also of relations between these two types of knowledge.

Research suggest that there is a complex interplay between the two constructs regarding their interdependence and development (Rittle-Johnson & Alibali, 1999).Bergsten et al (2013: 2) in agreement with (Alpers 2010; Cardella 2008; Gnedeneko & Khali 1979; Kent & Noss 2003) points it out that, ‘ mathematical activities occur as the contextual embedding of mathematical models, as well as concepts and procedurals, that use objects drawn on un understanding of mathematical notations and graphics. Engelbrecht, Bersgsten and Kagesten (2012) found that first year engineering students tended to proceduralise tasks having a conceptual focus.

2.3 Conceptual and Procedural Understanding of Mathematics in South Africa

Engelbrecht et al (2009:3) suggests that even though students at universities in South Africa improve performance in mathematics over a semester there is need to make the transition from secondary to university mathematics somewhat soother. Furthermore, Engelbrecht et al 2009 in agreement with Hourigan and O' Donoghue (2007) argues that, there is a big difference between the nature of first – year students' mathematics experience at pre-university level and that which they experience at university in mathematics courses. It has been observed with dismay that, the unpreparedness of students causes permanent damage to students' further mathematics careers at university. As a consequence, students beginning their university studies have less training in deeper conceptual thinking. University teachers often complain that first-year students have little understanding of basic concepts of pre-calculus and even the high achieving students are only better in a procedural way of thinking (Engelbrecht, Harding, & Potgieter, 2005).

3. Methodology

Research Method: A quantitative approach was used.**Research Design:**A case study design wasused.

Population and Sample: The population involves first year students at the University of Zambia who takes mathematics in the School of Natural Sciences. The sample was of 380 students.

Sampling Techniques: Simple random sample.

Data Collection, Methods and Instruments: Datawas collected using a mathematics test accompanied by a questionnaire and also another questionnaire which lecturers of first year mathematics students filled in.

Data Analysis and Presentation: Data from test was analysed using standard indices and that from questionnaires was analysed using Factor analysis and Multidimensional Scaling using SPSS and XLSTAT respectively.

3.1 Research Sample

In this study to determine the sample size the researcher picked $\sigma = 22.7$, population mean$= 24$, sample mean $=$ 21 and the desired degree of precision was at 99 percent.

Solution

$$n = \left(\frac{z\sigma}{d}\right)^2$$

$\sigma = 22.7$, $d = 24 - 21, z = 2.576$ (at 1% level of significance, the value of z is 2.576).

Substituting the values yields:

$$n = \left(\frac{2.576 \times 22.7}{3}\right)^2$$

$$n = \left(\frac{58.4752}{3}\right)^2$$

$$n = (19.4917)^2$$

$$n \approx 379.927$$

$$\therefore n = 380$$

Hence the sample size of the study was 380 students. The number was generated using a lottery from 1 500 only 380students were picked for the study.

4. Findings and Discussion

Research Question one (1)

$$\text{PUI} = \frac{\sum \alpha^* U_{p^*d_p}}{1000 \sum U_{p^*d_p}} = 9.78$$

$$\text{CUI} = \frac{\sum \alpha^* U_{c^*d_c}}{1000 \sum U_{c^*d_c}} = 22.6$$

Procedural understanding was twice more than the conceptual understanding as shown from the calculation from the formula for research question one. It shows that the less the solution is the more understanding exhibited in that aspect.

Research Question two (2)

$$\text{PCUI} = \frac{\sum \beta^* U_p{}^* d_p}{1000 \sum 3^* U_p d_p} = 6.56$$

$$\text{CCUI} = \frac{\sum \beta^* U_c{}^* d_c}{1000 \sum 3^* U_c d_c} = 12.9$$

Procedural confidence of understanding from the indices above for research question two shows that students where more confident to handle procedural questions than conceptual. The less the solution is the more explained it is. Students had twice procedural confidence more than they had conceptual.

COLLERATION MATRIX

	Level of confidence to answer the question by students on set theory	Level of confidence to answer the questions by students on partial fractions and polynomial functions	Level of confidence to answer the questions by students on functions the domain and range	Level of confidence to answer the questions by students on transcendental functions trigonometric functions	Level of confidence to answer the questions by students on linear, quadratic functions and complex numbers	Level of confidence to answer the questions by students on binomial expansions and systems of equations
Level of confidence to answer the question by students on set theory	1.000	.335	.319	.286	.268	.269
Level of confidence to answer the questions by students on partial fractions and polynomial functions	.335	1.000	.391	.331	.346	.434
Level of confidence to answer the questions by students on functions the domain and range	.319	.391	1.000	.274	.323	.268
Level of confidence to answer the questions by students on transcendental functions trigonometric functions	.286	.331	.274	1.000	.273	.320

	Level of confidence to answer the questions by students on linear, quadratic functions and complex numbers	.268	.346	.323	.273	1.000	.385
	Level of confidence to answer the questions by students on binomial expansions and systems of equations	.269	.434	.268	.320	.385	1.000

Column Sums: 2.477 2.837 2.575 2.484 2.595 2.676

Sum of the column
Sums (T) = 15.644
$\therefore \quad \sqrt{T} = \mathbf{3.955}$

First Centroid factor

$$= \frac{2.477}{3.955}, \frac{2.837}{3.955}, \frac{2.575}{3.955}, \frac{2.484}{3.955}, \frac{2.595}{3.955}, \frac{2.676}{3.955}$$

= 0.626, 0.717, 0.651, 0.628, 0.656, 0.677

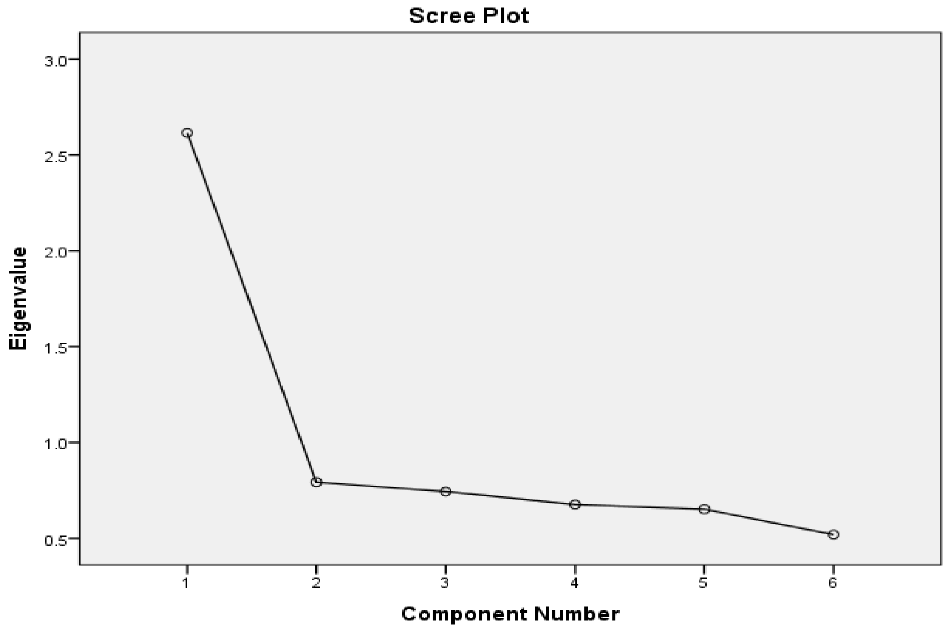
The second centroid factor B shall be found in the similar manner. Then
(First Centroid Factor)^2 + *(Second Centroid Factor)*^{2} = *Communality* (x^2)

The variance of one variable is accounted for by the centroid factor A and B and th remaining percent is a portion due to errors of measurement involved in assessing variables.

As from the correlation matrix it shows that the Level of confidence to answer the questions by students on Binomial expansions and systems of equation correlated highly tothe level of confidence to answer the questions by students on partial fractions and polynomials all the two questions (i.e. questions 1 and 3 in the test paper given to students were predomminantly procedural in nature. This shows that students were more confident toanswer procedural questions as compared to conceptual

Total Variance Explained						
Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.616	43.596	43.596	2.616	43.596	43.596
2	.793	13.212	56.808			
3	.744	12.403	69.211			
4	.676	11.270	80.481			
5	.652	10.864	91.345			
6	.519	8.655	100.000			
Extraction Method: Principal Component Analysis.						

It shows that out of the six eigen values only one is highly explained. The first eigen value explains up to 43.596 of the six variables. The five variables were dropped.



The scree plot shows that only one category out of the six was highly explained using Principal Component Analysis (PCA). It was discovered that only one component out of six were highly explained as shown in the scree plot.

Goodness of Fit Measure for Metric MDS

Eigen values measure variance associated with each dimension of the MDS solution Sum of first m eigen values relative to sum of all q eigen values (usually q=k):

$$\text{Fit} = \frac{\sum_{i=1}^{m} \varphi^2}{\sum_{i=1}^{q} \varphi^2_i}$$

Here, first eigen values is 2.616 and sum of the eigen values is 6.

$$\text{Fit} = \frac{2.616}{6} = 0.436$$

=43.6%

Research Question three (3)

Marks Obtained	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Procedural Understanding	61	68	34	51	55	24	24	19	17	25
Conceptual Understanding	69	72	72	40	45	21	26	21	9	3

The study for research question three displays more students having confidence in procedural problems and also performing well in procedural problems as compared to conceptual problems.

5. Conclusion

The study revealed that the majority of the students had procedural, and not conceptual understanding of the concepts assessed.

Furthermore, a significant relationship existed between the students' confidence levels and their understanding (procedural and conceptual).

6. Recommendations

Based on the findings, UNZA lecturers of mathematics should focus on the teaching methods which would enhance students' conceptual understanding of concepts in mathematics.

The results to this study might not be true if done to another university, because of the difference in teaching methodologies and strategies associated to various universities. It is against this background that it is important to do the study to other groups or even other disciplines.

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