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Cascade and System Reliability for the New Rayleigh-Pareto Distribution

Rosaiah Kanaparthi[1], Jyothi Palakurthi[2], Lakshmi Narayana Ganapathi Dev Sanka[3]

[1]Department of Statistics, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur, A.P, India

[2]Department of Statistics, Univ. Arts and Science College (Autonomous), Kakatiya University, Warangal, T.S, India

[3]Department of Mathematics, Newton High School, GA, U.S.A

Abstract: ***In This paper we present the estimation of stress strength model by considering cascade stress strength model.. The system survives if and only if the strength is greater than stress, otherwise fails. The statistical model is derived to study the cascade and system reliability. Here we assume that all the components are independents and follow the New Rayleigh-Pareto strength stress distribution. The first four cascade reliabilities are obtained for various values of strength-stress parameters. The statistical variation in cascade and system reliability is shown graphically.***

Keywords: Cascade Reliability, New Rayleigh Pareto Distribution, Stress-Strength Model, Probability density function

1. Introduction

Cascade reliability model is a special type of Stress-Strength model. Reliability may be used as a measure of the system's success in providing its function properly. Mathematically, reliability $R(t)$ is that a system will be successful in the interval (0, *t*). If $R(t)=P(T>t)$ for $t \geq 0$, where T is a random variable denoting the failure time.

Cascade systems were first proposed and studied by [1] they have studied an n-cascade reliability for exponential distribution. They evaluate reliability values for a two-cascade system with gamma and normal stress and strength distribution. [2] Studied the reliability of a cascade system with normal stress and strength distribution. [3] Rekha and Shyam Sundar have derived an expression of the reliability of an n-cascade system by considering attenuation factor with the same parameter value. For their study, they considered exponential strength and gamma stress distribution. [4] Has done a case study of cascade reliability using Weibull distribution, [5] has done a case study of cascade reliability with Rayleigh distribution [6] have studied cascade reliability of stress-strength system considering strength follows mixed exponential distribution. [7] has done comparison of an n-cascade system by considering normal stress and exponential strength distribution. [8] studied the reliability analysis of a redundant cascade system by using Markovian approach assuming n-components which are arranged in the hierarchical order. The n-cascade system survive with loss of m components by k number of attacks. [9] studied about estimation of reliability for stress-strength cascade model by comparison between estimators made using data obtained through simulation experiment [9, 10, 11] [12] has done a cascade and system reliability for Exponential distribution.

In this paper we determined a statistical model of New Rayleigh-Pareto strength-stress distribution which are considered for computations of marginal and system reliabilities supported by reliability computations with numerical and graphical study along with various values.

2. Statistical Model

Let us consider the New Rayleigh Pareto distribution with probability density function (pdf)

$$g(x)=\frac{\lambda}{\alpha^{\lambda}}x^{\lambda-1}e^{-\left(\frac{x}{\alpha}\right)^{\lambda}} \qquad (1)$$

Where $0<x<\infty, \lambda>0, \alpha>0.$

The cumulative distribution function (cdf) of the NRPD is given by

$$G(x)=1-e^{-\left(\frac{x}{\alpha}\right)^{\lambda}} \qquad (2)$$

Where $0<x<\infty, \lambda>0, \alpha>0.$

Where $\alpha \succ 0$ is a scale parameter and $\lambda \succ 0$ is the shape parameter

The hazard function is

$$h(x)=\frac{\lambda}{\alpha^{\lambda}}x^{\lambda-1}$$

Time to failure is given by

$$F(t)=1-\exp\left[-\int_0^t h(x)dx\right]$$

$$=1-\exp\left[-\int_0^t \frac{\lambda}{\alpha^{\lambda}}x^{\lambda-1}dx\right]$$

By integrating we get

$$F(t)=1-e^{-\left(\frac{t}{\alpha}\right)^{\lambda}}$$

Let X denotes the strength and Y denotes the stress. The reliability of the component is given by

$$R=P(X>Y)=\int_0^{\infty}\left(\int_y^{\infty} g(x)dx\right)f(y)dy$$

$$=\int \overline{G}(y)f(y)dy \qquad (3)$$

Where $g(x)$ and $f(y)$ are probability density functions of X and Y respectively. $G(y)$ be the distribution function of Y and $\overline{G}(y) = 1 - G(y)$.

Let $X_1, X_2, X_3, \ldots. X_n$ be the strength of the n components arranged in order of activation respectively, having probability density function. Let another random variable Y is the stress imposed on the n components, having probability density function. The system survives up to failure of first $(n-1)$ components i.e., $X_i < Y$; $i = 1,2,3,\ldots(n-1)$ and $X_n > Y$, The system reliability $R_s(n)$ is given by

$$R_s(n) = R_1 + R_2 + R_3 + R_4 + \ldots. + R_n \quad (4)$$

The marginal reliability R_n is the reliability of the system for the n[th] component is given by

$$R_n = P(X_1 < Y, X_2 < Y, X_3 < Y, X_4 < Y, \ldots. X_{n-1} < Y, X_n > Y) \quad (5)$$

Hence,

$$R_n = \int_0^\infty \left[\int_0^y g_1(x_1)dx_1 \times \int_0^y g_2(x_2)dx_2 \times \int_0^y g_3(x_3)dx_3 \times \int_0^y g_4(x_4)dx_4 \ldots. \times \int_0^y g_{n-1}(x_{n-1})dx_{n-1} \times \int_y^\infty g_n(x_n)dx_n \right] f(y)dy$$

$$R_n = \int_0^\infty G_1(y) G_2(y) \ldots. G_n(y) \overline{G}(y) f(y) dy \quad (6)$$

Where,

$$G_i(y) = \int_0^y g_i(x_i)dx_i, i = 1,2,3,\ldots..n$$

and

$$\overline{G_i}(y) = 1 - G_i(y)$$

3. Reliability Computation

Let us consider the New Rayleigh Pareto distribution with probability density functions (pdf's) of X and Y.

Let X represents the strength of an component with density function

$$g(x) = \frac{\lambda_1}{\alpha^{\lambda_1}} x^{\lambda_1 - 1} e^{-\left(\frac{x}{\alpha}\right)^{\lambda_1}}, 0 < x < \infty, \lambda_1 > 0, \alpha > 0 \quad (7)$$

Where α is the scale parameter and λ_1 is the shape parameter and its distribution function is given by

$$G(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\lambda_1}} \quad (8)$$

Where $0 < x < \infty, \lambda_1 > 0, \alpha > 0$.

and the probability density function of stress Y is given by

$$f(y) = \frac{\lambda_2}{\alpha^{\lambda_2}} y^{\lambda_2 - 1} e^{-\left(\frac{y}{\alpha}\right)^{\lambda_2}}, \quad (9)$$

$$0 < y < \infty, \lambda_2 > 0, \alpha > 0$$

Where λ_2 and α are the shape parameter and scale parameter respectively. and its probability distribution function is given by

$$F(y) = 1 - e^{-\left(\frac{y}{\alpha}\right)^{\lambda_2}}, 0 < y < \infty, \lambda_2 > 0, \alpha > 0 \quad (10)$$

Then from, [6] we evaluate cascade reliability.

a) 1-cascade reliability,

$$R_1 = P(X_1 > Y) = \int_0^\infty \overline{G_1}(y) f(y) dy$$

$$= \int_0^\infty e^{-\left(\frac{y}{\alpha}\right)^{\lambda_1}} \frac{\lambda_2}{\alpha^{\lambda_2}} y^{\lambda_2 - 1} e^{-\left(\frac{y}{\alpha}\right)^{\lambda_2}} dy$$

$$= \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

b) 2-cascade reliability,

$$R_2 = P(X_1 > Y, X_2 > Y) = \int_0^\infty G_1(y) \overline{G_2}(y) f(y) dy$$

$$= \int_0^\infty \left(1 - e^{-\left(\frac{y}{\alpha}\right)^{\lambda_1}}\right) e^{-\left(\frac{y}{\alpha}\right)^{\lambda_1}} \frac{\lambda_2}{\alpha^{\lambda_2}} y^{\lambda_2 - 1} e^{-\left(\frac{y}{\alpha}\right)^{\lambda_2}} dy$$

$$= \frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)(2\lambda_1 + \lambda_2)}$$

Similarly,

a) 3- cascade reliability,

$$R_3 = \frac{2\lambda_1^2 \lambda_2}{(\lambda_1 + \lambda_2)(2\lambda_1 + \lambda_2)(3\lambda_1 + \lambda_2)}$$

b) 4- cascade reliability,

$$R_4 = \frac{6\lambda_1^3 \lambda_2}{(\lambda_1 + \lambda_2)(2\lambda_1 + \lambda_2)(3\lambda_1 + \lambda_2)(4\lambda_1 + \lambda_2)}$$

In general n- cascade reliability,

$$R_n = \frac{(n-1)! \lambda_1^{n-1} \lambda_2}{\prod_{i=1}^{n} (i\lambda_1 + \lambda_2)} \quad (11)$$

4. Simulation and Graphical Study

In this section, the simulation work and graphical presentation by determining cascade and system reliability is carried out. In the table 1 strength parameter is constant i.e., 4 and stress parameter is steadily increasing it from 1 to 5.5. In the table 2 stress parameter is constant i.e., 4 and the strength parameter steadily increasing it from 1 to 5.5. In the table 3 the strength parameter is increasing it from 1 to 5.5 and stress parameter decreasing it from 5.5 to 1. In table 4 stress parameter is increasing it from 1 to 5.5 and strength parameter is decreasing it from 5.5 to 1. Now we determine the cascade reliability by using (11) and then next compute the system reliability using (4) . Next we plot the graphs by

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taking the progressive values of strength and stress against cascade and system reliability respectively.

Table 1: Cascade and system reliability

λ_1	λ_2	R_1	R_2	R_3	R_4	R_s
4	1	0.2	0.097561	0.059259	0.038612	0.395433
4	1.25	0.238	0.115607	0.06864	0.043245	0.465587
4	1.5	0.273	0.131868	0.076555	0.046662	0.527813
4	1.75	0.304	0.146597	0.08324	0.049113	0.583298
4	2	0.333	0.16	0.088889	0.050794	0.633016
4	2.25	0.36	0.172249	0.093659	0.05186	0.677767
4	2.5	0.385	0.183486	0.09768	0.052436	0.718218
4	3	0.429	0.20339	0.103896	0.052495	0.788352
4	3.5	0.467	0.220472	0.108213	0.051555	0.846907
4	4	0.5	0.235294	0.111111	0.05	0.896405
4	4.5	0.529	0.248276	0.112941	0.048081	0.93871
4	5	0.556	0.25974	0.11396	0.045967	0.975223
4	5.5	0.579	0.269939	0.11436	0.043768	1.007014

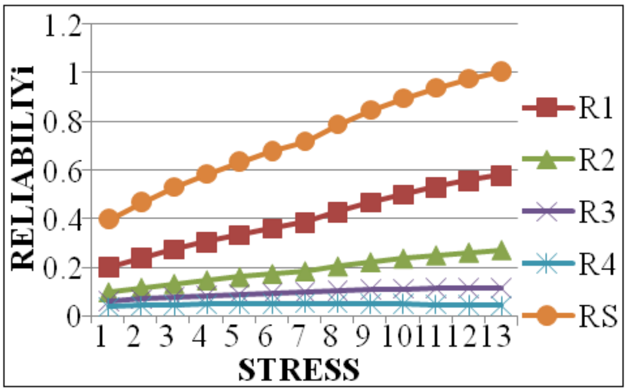

Figure 1: Cascade and system reliability (for constant strength parameter 'λ_1')

Table 2: Cascade and system reliability

λ_1	λ_2	R_1	R_2	R_3	R_4	R_s
1	4	0.8	0.133333	0.038095	0.014286	0.985714
1.25	4	0.762	0.14652	0.047265	0.019694	0.975383
1.5	4	0.727	0.155844	0.055004	0.024752	0.962872
1.75	4	0.696	0.162319	0.061418	0.029313	0.948702
2	4	0.667	0.166667	0.066667	0.033333	0.933333
2.25	4	0.64	0.169412	0.070917	0.036822	0.91715
2.5	4	0.615	0.17094	0.074322	0.039815	0.900462
3	4	0.571	0.171429	0.079121	0.044505	0.866484
3.5	4	0.533	0.169697	0.081923	0.047788	0.832741
4	4	0.5	0.166667	0.083333	0.05	0.8
4.5	4	0.471	0.162896	0.083775	0.051407	0.768667
5	4	0.444	0.15873	0.083542	0.052214	0.738931
5.5	4	0.421	0.154386	0.082841	0.052572	0.710852

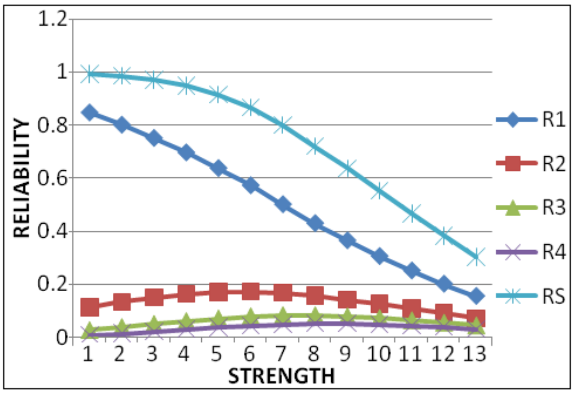

Figure 2: Cascade and system reliability (for constant stress parameter 'λ_2')

Table 3: Cascade and system reliability

λ_1	λ_2	R_1	R_2	R_3	R_4	R_s
1	5.5	0.846	0.112821	0.026546	0.008383	0.993903
1.25	5	0.8	0.133333	0.038095	0.014286	0.985714
1.5	4.5	0.75	0.15	0.05	0.021429	0.971429
1.75	4	0.696	0.162319	0.061418	0.029313	0.948702
2	3.5	0.636	0.169697	0.071451	0.037279	0.914791
2.25	3	0.571	0.171429	0.079121	0.044505	0.866484
2.5	2.5	0.5	0.166667	0.083333	0.05	0.8
3	2.25	0.429	0.155844	0.083117	0.052495	0.720027
3.5	2	0.364	0.141414	0.079192	0.05197	0.636212
4	1.75	0.304	0.124861	0.072646	0.049113	0.550968
4.5	1.5	0.25	0.107143	0.064286	0.044505	0.465934
5	1.25	0.2	0.088889	0.054701	0.038612	0.382202
5.5	1	0.154	0.070513	0.044322	0.031796	0.300478

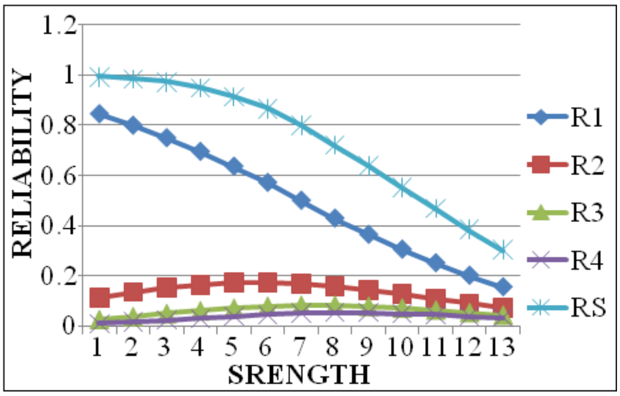

Figure 3: Cascade and system reliability (for increasing strength parameter 'λ_1')

Table 4: Cascade and system reliability

λ_1	λ_2	R_1	R_2	R_3	R_4	R_s
5.5	1	0.154	0.070513	0.044322	0.031796	0.300478
5	1.25	0.2	0.088889	0.054701	0.038612	0.382202
4.5	1.5	0.25	0.107143	0.064286	0.044505	0.465934
4	1.75	0.304	0.124861	0.072646	0.049113	0.550968
3.5	2	0.364	0.141414	0.079192	0.05197	0.636212
3	2.25	0.429	0.155844	0.083117	0.052495	0.720027
2.5	2.5	0.5	0.166667	0.083333	0.05	0.8
2.25	3	0.571	0.171429	0.079121	0.044505	0.866484
2	3.5	0.636	0.169697	0.071451	0.037279	0.914791
1.75	4	0.696	0.162319	0.061418	0.029313	0.948702
1.5	4.5	0.75	0.15	0.05	0.021429	0.971429
1.25	5	0.8	0.133333	0.038095	0.014286	0.985714
1	5.5	0.846	0.112821	0.026546	0.008383	0.993903

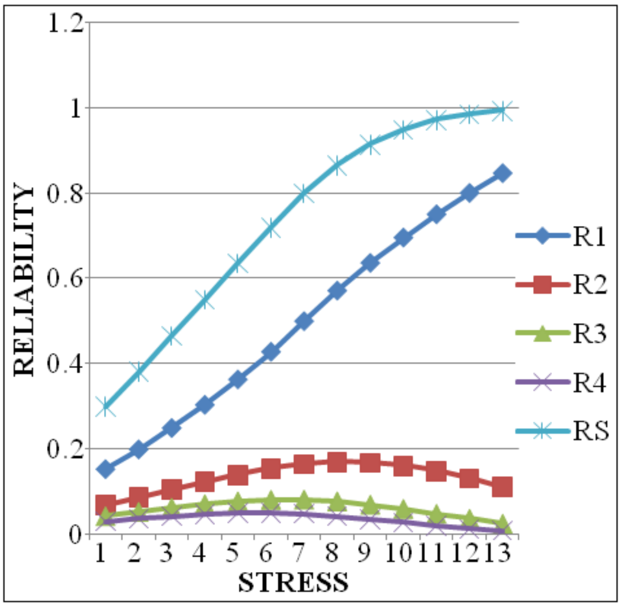

Figure 4: Cascade and system reliability (for increasing stress parameter ‘ λ_2 ‘)

5. Conclusion

In this paper the reliability formulae for an n-cascade system using new rayleigh-pareto strength-stress distribution is found out. We observe that the system reliability R_s reaches to unity (one) if we choose strength parameter constant and stress parameter steadily increasing. In addition, if we increase strength parameter and decrease stress parameter the cascade reliability little bit increases and then decreases but the system reliability steadily increases and reaches to unity (one). Hence we conclude that the cascade and system reliability is found to be constant if strength and stress parameters are steadily increasing.

References

[1] S.N.N. Pandit and G.L. Srivastav, studies in Cascade Reliability 1, IEEE Transactions Reliability, Vol. 24,No.1, 1975, pp. 53-56.

[2] A.C.N.Raghavachar, B. Kesava Rao, and S.N.N.Pandit, The Reliability of a Cascade System with Normal Sztress and Strength distribution, Advances in Space Research, Vol. 2, 1987, pp. 49-54.

[3] Rekha and T. shyam Sundar, Reliability of a Cascade System with Exponential Strength and Gamma Stress, Microelectronics Reliability, Vol. 37, No. 4, 1997, pp. 683-685.

[4] T. Shyam sundar, Case Study of Cascade Reliability with Weibull Distribution, International journal of Engineering and Innovative Technology, Vol. 1, No. 6.2012, pp 103-110.

[5] T. Shyam sundar, Case Study of Cascade Reliability with Rayleigh Distribution, IJCTEE, Vol. 2, No. 3, 2012, pp. 78-87.

[6] T.S. Uma Maheshwari , and N. Swathi, Cascade Reliability of Stress-strength System When Strength Follows Mixed Exponential Distribution ,ISOR Journal of mathematics, Vol.4,No. 5,2013, pp.27-31.

[7] T.S. Uma Maheshwari, A. Rekha, E. Anjan Rao and A.RChar, Reliability oof Cascade Sysstem with Normal Stress andd Exponential Strength, Microelectronics Reeliability, Vol.. 333, No. 7, 1993, pp. 929-936.

[8] Nalabolu Swathi, T.S. Uma Maheshwari, Reliability Analysis of a Redunddantt Ccascade System by Unising Markovian Approach, Journal of Applied Mathematics and Physics, Vol. 3, No. 7, 2015, pp.911-920.

[9] R.R.Mutkekar and S.B.Munoli, Estimation of Reliability for Stress-Strength Cascade Modell, Open Journall of Statistics, Vol. 6, No. 5, 20116, pp. 873-881.

[10] G. VVaasanthiand B. Venkata Rao, Fuzzy Moddelling of Selection of overall Best Performer, DJ Journal of Engineering and Applied Maathematics, Vol. 2, No. 1, 2016, pp. 1-6.

[11] Jamshaid-ul-Rahman, Muhammad Raheel Mohyuddin, Naveed Anjum and Rehan Butt, Modeling of two Inter connected Spring Carts and Minimization of Energy, DJ Journal of Engineering and Applied mathematics, Vol.2, No. 1, 2016, pp. 7-11.

[12] Dhananjeya Reddy, Cascade and System Reliability for Exponential Distributions, DJ Journal of Engineering and Applied Mathematics, Vol. 2(2), 2016. pp. 1-8.