

Continued Education in Mathematics in a Collaborative Group: Working with Capacity and Volume

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Abstract: *We present the research carried out in the area of continuous training in a collaborative group, composed of Pedagogy students, Basic Education teachers, postgraduate students and university professors. In the group are addressed themes related to teaching and learning in Mathematics, primarily involving content worked in Early Childhood Education and Early Years of Elementary School. Actions are planned and coordinated horizontally, which indicates that all are heard and respected. The knowledge conveyed is differentiated, as we will outline in this extract of the investigation. In an activity on containers capacity and volume of solids, we bring different presentations, focusing on the action of the university professor, who organized knowledge of the content, which provided understanding and re-signification of the content by the other participants and the elaboration of differentiated strategies for work with students. Altogether, the participants theorize the concept of volume and elaborate strategies for learning relating it to the axis scales and measures, so little explored in the school for the childhood.*

Keywords: solids volume concept, capacity concept, childhood Mathematics, collaborative group, teacher training

1. Introduction

In order to teach mathematics, it is essential that teachers commit to their own learning, training and professional development during their careers (DAY, 2005). The concrete needs and ways of reaching this commitment vary according to the circumstances, the personal and professional histories and the dispositions in force in each moment, and participation in study groups with collaborative characteristics is one of these possibilities.

We present in this article the extract of an investigation developed within a collaborative group - the GEProMAI: Mathematics Teachers Study Group in the Early Years. This group meets every fortnight and seeks, together, to discuss - conceptually and in practice - contents that are worked on the Early Childhood Education and in the initial years of Elementary School. It is attended by teachers who work in public and private schools of Brazilian schools, Pedagogy students and university professors.

In a horizontal collaboration, the members of the group can present their suggestions, their professional elaborations, their knowledge, in actions where the deprivatization of the practice prevails, awakening the collaboration of other people of the group who are also involved in the process of improving teaching and learning of Mathematics (COCHRAN-SMITH, 2012).

The group has been meeting for five years. The common practice of meetings is to elect one or more subjects to be dealt with for a time, from a prior planning. The chosen theme is studied considering the theoretical contributions chosen by the participants that approach it. From these studies and from the experiences of the participants, discussions are held regarding the possible actions at the various levels of education that involve the content and preparation of activities to be developed with the students.

2. What we study about

Among the different subjects and contents conveyed in the meetings, the study of the measures was a fruitful perspective for the group, because it is articulated to the daily experiences of children and adults, and because it is a relevant topic, as indicated in the official documents that school curricula (BRAZIL, 1997, 2018).

Considering the presence and importance of measures in everyday life, Moura (1995) shows that the measure is present in situations of control of variations in the dimensions of objects, in displacements, in cooking, in artistic productions, among other situations, impregnated with cultural meanings arising from the human relations it represents and communicates.

For Caraça (1963: 29) "measuring consists of comparing two quantities of the same species - two lengths, two weights, two volumes, etc." It should be noted that comparing is essential in the measurement process, however, knowing, for example if one length is greater or less than another is not sufficient, in most cases it is important to know how many times one length fits in the other, which places emphasis in the importance of choosing the unit of measurement, that is, the establishment of the single comparison term for all quantities of the same species. Caraça (1963) points out that in the problem of measuring, it is possible to consider three phases and three distinct aspects: the choice of unit, the comparison with the unit and the numerical expression of the result of that comparison.

Among the studies related to measures at school, volume and capacity are themes understood as challenging (LIMA; BELLEMAIN, 2010). It is important to note that volume and capacity are understood as "the same greatness in different contexts" (LIMA; BELLEMAIN, 2010: 192) The

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volume is understood as space that a three-dimensional object occupies in the environment, in this perspective it is necessary to consider the objects and their volume as closely related but distinct elements, since it is possible to have different three-dimensional objects that have the same volume, which can occupy the same amount of space. When the object in question is a container that has an available internal space, we associate that space with the concept of capacity, that is, the capacity is the volume of the inner part of that object. (LIMA, BELLEMAIN, 2010)

In the episode we bring a study on volume and capacity and the teaching-learning strategies that were constructed in the interactions of the participants, where it is possible to highlight the importance of interlocution between the teachers of the different levels of education, re-signifying each one, in its specificity and with different vision as an essential element for teacher education. Two concepts deserve our understanding.

The first one concerns the re-signification, both of concepts and of teaching practices. As found in Almeida (2017: 81),

Resignification is a term that has been used as a process of producing (new) meanings and (new) interpretations of what we know, do, say, know. In a context of formation that allows interaction with the other, exchanges about different points of view, the conversation about experiences and teacher knowledge, the teacher can produce a new meaning and a new interpretation about what he knows, does, says.

We can, thus, affirm that to re-signify is to learn from the experiences and the knowledge in action, from the interaction between the involved ones, in this case from the dialogues in different contexts.

The other concept is that of "surplus of insight." We take it from Bakhtin's (2011) statements, in the sense that it is possible for the other to perceive about us what we are sometimes unaware of. Hence, such surplus of insight is only perceptible in relation to the other, the reciprocal being true.

It should be noted that in this group, heterogeneous visions occur related to the level of teaching, but always focused on mathematical content and teaching practices. Such practices are always discussed, so that each one will configure them according to the age group with which they work.

For Fiorentini (2013: 68-69), this heterogeneity is seen as an element that adds quality to the group, since:

Basic school teachers, for example, bring as surplus of insight, in relation to trainers and future teachers, an experience knowledge related to the teaching of mathematics in schools and know the conditions and possibilities of certain tasks and practices. The knowledge they mobilize and produce are situated in the complexity of their practices, and this is the main reference in the processes of negotiating meanings and meanings during the elaboration of tasks, analysis of episodes or teaching-learning situations, the appropriation or validation of knowledge of academic

practice and also of those coming from academic research.

The university formators, on the other hand, have as surplus of insight, the theories and methodologies from which they produce analyzes, interpretations and understandings of the current school practices, with the purpose of problematizing and denaturalizing them.

It is in dialogue that we know each other better. From this perspective the surplus of insight of one participant / author can complement the other in which he/she, from the place he/she occupies, cannot identify.

The interactions derive from the experiences in the group and denote a teacher formation based on a constructive model, as evidenced by Fiorentini (2008). This model presupposes the existence of a continuous process of interactive and contextualized reflection on pedagogical and teaching practices, relating them to theory, articulating the training practices to professionals. This training model implies a partnership relationship between academics and teachers of Basic Education, who interact collaboratively to understand the challenges of the current educational context and seek joint solutions to the problems of the school.

The reflections presented in the analyzes were provoked by the dialogical and collaborative movement of the group, which resulted from the proposed activity, the presented report, the readings made, the debates, the exchanges and the different situations that derive from the discussions, since the collaboration is supported by the fact that different perspectives from different levels and places favor a profusion of experiences that, when they are reported and shared, provoke reflection on knowledge. This experience is related to the perspective of research as a posture (COCHRAN-SMITH; LYTLE, 1999).

For Cochran-Smith and Lytle (1999), the ways in which teachers collectively theorize, study and act on the problems and contexts of teaching practice in order to meet learning interests and provide better opportunities for students and their communities is understood as an investigative stance. The authors point out that the questioning of the teacher is a world view, a critical habit of the mind and a way of knowing about teaching, in that sense, the fundamental of research as posture is the idea that the educational practice is not simply instrumental, but also social and political and aims to deliberate on what should be done, on why to do it, who decides and what interests are met.

We agree with Almeida and Megid (2017: 178) when they affirm that the fundamental characteristics of a collaborative group are "voluntariness and spontaneous participation, indicated in the predisposition of the participants to work together with other teachers as well as the desire to be part of a particular group".

The motivation may be derived from a need for support for the teaching life, from the understanding of the complex everyday life, from its confrontation. But, also, the intention to develop differentiated activities, sharing projects,

researching their own practices and/or their peers, adding new knowledge to what is already known and realized.

In addition, one of the main characteristics of a collaborative group, as found in Fiorentini (2004), is the absence of an upright relationship. There are no passive spectators who obey the commands of an eventual trainer. On the contrary, each participant is responsible for the group, for study proposals, for the contributions they deem pertinent, both regarding new studies and the presentation of strategies developed in their places of performance.

Our conception of collaboration is based on Saraiva and Ponte (2003:9). For the authors:

Collaboration between teachers and researchers can help to bridge the gap between teacher professional practice and educational research, as well as the separation of schools and universities and, ultimately, separation of theory and practice.

In the group in question, respect for the interventions of each participant as well as the involvement with the activities in it is fundamental for participation. There is no hierarchy and all manifestations occur for the development of each - in their teaching practices - and the group - which is always guided by the professional formation and development of the teacher, at all levels of education.

3. Context and Method

For this work, we selected an episode that occurred in a meeting of the GEProMAI in which the units of measure of volume and capacity were discussed. Data were collected through audio and video recordings and personal records of university teachers about the training. We emphasize that the videos and audios were revisited by the authors and the transcription was made.

In order to develop this qualitative research, we sought to value the detailed description of the context, the interactions among the participants highlighting aspects of the trainer's performance and evidencing the trainees' learning indicators from the formative actions. The analysis was developed in a narrative perspective "as a way of understanding the experience" (CLANDININ E CONELLY, 2011: 51). In this sense, we seek to understand the experience lived together by the participants of this collaborative group.

The action was involved in one of the meetings, which lasted approximately two hours, in which ten participants were present. One of the authors prepared the environment with packages and containers of different volumes and invited the other partners to observe all the material, and could even handle it.

The following are the dialogues and actions of the collaborative group that sought to understand and interpret the dynamics of the educational proposal in order to reflect collectively on the implications in pedagogical practice.

4. Working with measures

We will present and analyze an episode centered on capacity and volume measurements. The university professor (UP) started the conversation by introducing to the group some bottles and packages with capacity for different quantities. He/she pointed out that in the school it would be possible to ask the children to bring some containers of different shapes and sizes to enable comparison. Subsequently, UP makes an exploration of the materials, together with the group, establishing and combining that a bottle of alcohol of one liter will be understood as the measuring instrument with the desired unit. One of the participants (R) questioned whether the teacher establishes this or it is better to negotiate with the children which instrument will be set as the standard. UP responds that it is more interesting to combine this with the students.

He/She, then, presented some questions that could be asked in the development of work with children, especially with regard to estimation and also the conservation of continuous quantities. By showing two different containers you could ask the students: Where do you think more water fits? And less water? Among these bottles, are there two that can hold the same amount of water? And among those three, where does it fit most?

UP has clarified that it is important to present to children situations that challenge pre-established concepts, because depending on the age, children usually believe that the "taller" container has the greatest capacity. She had the water transferred to the higher vessel with the aid of a funnel, the group noted that it was not possible to pour all the water that was in the bottle used as a measuring instrument. He commented that this transfer procedure would have to be carried out by the children themselves. This action corroborates Nacarato's (2005, p.3), in order to enable a "true action on the part of the child and not a simple reproduction of what was said by the teacher". He/She also stressed the importance of repeating the same action, with different objects, so that they can estimate and compare the results of their hypotheses. It has also been shown that it is important for children to carry out the estimates and measurements on a number of occasions, with different materials, and to provide them with opportunities to develop measurement actions (MOURA, 1995).

In the sequence UP suggested as a measuring instrument a graduated cup, a utensil commonly used in the kitchen. He/She emphasized that it is important for the teacher to talk with the children about the different graduated units of measurement present in that utensil. In one of the marks printed on the container, there is a gradation in cups or cups - unconventional units. The ratio of the cups and cups to the milliliters and/or grams is also recorded. It is worth mentioning that talking about measures considered "unconventional", also used in daily contexts, allows to increase the knowledge and understanding of mathematical concepts, which makes learning more meaningful.

Following this description, UP pursued the activity by exploring other issues. He/She pointed to a certain place on

the container's marker and asked, "This is the half liter mark, do you believe that in that bottle (going to one of the containers) it's about half a liter?" A lot more or a lot less? And if I pour the water from this bottle into the graduated container, how many milliliters do you think it will measure? Each question was answered and discussed by the group. They made considerations related to the participants' estimates and to the actions that could occur if the activity was carried out with students.

More than that, the need for children to manipulate the utensils and to transfer the liquid individually from one container to another was emphasized. These experiences are much more consistent than mere observation.

The group considered that the activities, which mobilize the comparison and the numerical expression of the measurement results, also needed to be explored with the children.

Continuing the possibility of using materials to work with measurements, UP presented two equal "open cubes", made in transparent acrylic, with the dimensions of 10 cm x 10 cm x 10 cm (1000 cm³ or 1 dm³). He commented that children usually do not believe its capacity is a liter. He/She also showed a bottle of 500ml capacity soda, which children usually thought would contain 600ml because they had more contact with bottles of that capacity.

UP pointed out that this issue of talking to children about measures, comparing where it was more or less important was also important to problematize the advertisements shown on television, in the brochures of supermarkets and other trades. The UP considerations regarding the measures and the relationship with a daily mathematics seem to meet the considerations presented by Moura (1995) in evidencing cultural and social aspects of the measure:

It is in day-to-day relationships that the measure appears impregnated with cultural meanings of the human relations it represents and communicates, as well as: beauty in art and architecture, balance in engineering, communication of social phenomena in statistics. From the educational point of view, it is interesting to observe that, as a source of communication, it can serve as a means of manipulating the truth of social phenomena. (MOURA, 1995: 43)

As for the importance of knowledge related to the measures and relationships that they establish with the Mathematics we use in everyday life, the teachers commented that sometimes, when comparing the price and quantity of a given product, a larger package, when compared to the proportions, costs cheaper than the same product marketed in a smaller package. However, its shelf life is shorter. In this case, the need and/or the use made of this product should be a factor to consider which one is more interesting to be acquired. When it comes to a little used product, or purchased exclusively for a particular purpose, without consuming the entire amount, which initially seems to be advantageous, it can generate waste and more expense.

UP has shown that thinking about the cost-benefit issue is not only linked to the price-quantity relationship, but also to the use of the product, the type of packaging and the storage conditions that the consumer has to keep the product adequate for consumption during the expiration date.

In the continuity of the meeting, he/she showed the cube of 1000 cm³ and questioned if all the water of the bottle of alcohol (1 liter) would fit inside him. As everyone already knew about the dimensions of the cube, the answer was yes. However, if such an experiment were carried out with children in schools, they would probably respond differently.

UP returned to the dimensions of the cube of 1000 cm³ and explained that this would support the idea of the cubic centimeter, cubic meter. He/She highlighted that it was a cube with 10 cm of edge, and that 10 cm is the same as 1 decimeter. Therefore, that cube was 1dm³ in volume, which corresponds to 1 liter. Consequently, 1 liter equals one thousand milliliters and equals 1 dm³.

After the conceptual conversations about comparisons and transformations, UP showed a misshapen stone and said he wanted to know its volume, and questioned how to get that information. How do you measure it? After some comments indicating the difficulties due to the configuration of the stone, UP placed the stone in one of the 1000 cm³ cubes and poured the water from the other cube that was full. The teachers observed that by filling the cube with water with water, some of the liquid was left in the other cube. UP again questioned the group. Then he put the remaining water in the graduated cup and measured the amount of water that corresponded to the volume of the stone. R verbalized the result: 90 ml, or 90 cm³.

After this experience, the participants commented on how the work with Mathematics can be interesting and creative and that these activities would arouse children's interest, allowing them to think about measures and Mathematics.

In his narrative, K emphasized the importance of the diversity of activities for teaching the concept of measure of volume and capacity, highlighting the methodological aspect used in the meeting. He reported on his personal learning with the activities and discussions and presented a contribution to the group, introducing in the narrative other concepts related to the measurement of volume and capacity, highlighting the interlocution with the texts in their learning process.

It was possible to observe that the activities with measures of capacity and volume were configured as important experiences in the face-to-face meeting. Participants commented on satisfaction in re-meaning content on measures in a pleasurable and playful way, and stressed that group learning could also be developed with children in school.

In this episode on the volume and capacity measures, there was a greater role of UP, which presented to the group the materials and some ways of exploring them with the children in teaching and learning the measures of capacity

and volume. It was one of the few meetings in which the researcher of the university coordinated more directly an activity in the GEProMAI, acting explicitly as a trainer in the group. This happened because UP has experience as a teacher in both Basic Education and Higher Education, and because it was who had worked with this theme in different situations, having knowledge to share.

The attention of the group, the questioning, the concern with the personal records and the novel look of the teachers when handling a bottle with a different format or the cube of 10 cm of edge occurred possibly because the participants had few (or none) experiences of that kind in their journey as students and/or as teachers. It was possible to provoke the participants of the group and to observe that the report and the shared reflections were constituted in new learning. This situation is in keeping with the idea of collaboration that we experience in the group, by making public the individual knowledge, and also in the sense of the close and active interaction between the university and the school in a group that is located at the border between these two spaces. From this perspective, the university approaches the pedagogical practice of schools and assists teachers in different issues related to teaching and learning in the classroom, and to their own formation, while at the same time attending the interest of teachers who seek support theoretical-methodological approach of the academy to deal with the challenges of everyday school life (TANAKA; PASSOS, 2015).

It is important to point out that, collaboratively, we understand and defend that "no didactic material - manipulative or otherwise - constitutes salvation for the improvement of mathematics teaching. Its effectiveness, or not, will depend on how it is used" (NACARATO, 2005, p.5). This is the central point of our questions: which materials to use? How? When to use them? What meanings can be produced from its use?

It is important to note that from the interaction between the participants the articulation between the school emerged mathematics, whose learning could be facilitated by the use of the materials, with everyday mathematics, highlighting social aspects of its use, as well as some more conceptual elaborations such as the transformations of units, which are common in teaching this content.

5. Conclusion

Group meetings generally produce re-significances for participants. Many terms used in everyday life, not always anchored in more precise mathematical concepts, are discussed and incorporated by the participants, both in their daily practices and in didactic situations.

In this text we bring the understandings related to volume and capacity. We noticed that, for some of those teachers, these terms were used synonymously. Measures of volume and capacity were little worked and, where they occurred, the focus was on capacity, sometimes confused with volume.

The possibility that, in a collaborative environment, experiencing situations where capacity and volume were in different ways studied, brought advances to those involved. In a natural and respectful way, each one was able to present what he / she understood about the situation experienced, their doubts, their insecurities, re-signifying this important content, so much for the use in the actions of the daily ones as in the configurations of teaching work.

At the end of the analysis of this episode, it is relevant to consider that, like the other meetings of the group, the interlocations about the meeting reverberated in the virtual interactions. On this day, Gi initiates a collaborative writing - text produced by one of those who participated in the meeting, published in *Google Docs* and complemented by other members (ALMEIDA and MEGID, 2017). Gi emphasizes the conceptual learning about units of measurement of capacity and volume. He remembered UP's words in saying that measuring is comparing. It is necessary to identify a unit of measurement and compare it with what one intends to measure to arrive at a numerical expression. He also recalled some equivalences: $1 \text{ dm}^3 = 1 \text{ L} = 1000 \text{ mL}$; $1 \text{ dm}^3 = 1000 \text{ cm}^3 = 1000 \text{ mL}$ (correspondence of capacity measurements with volume measurements).

When narrating about the conceptual aspect related to the act of measuring and the processes involved in it, Gi organized information and systematized it. In this sense, the production and sharing of narratives about the experience of teacher training can be configured in a fertile and appropriate way of producing knowledge and experiences that lead to reflection on pedagogic doing (CONNELLY; CLANDININ, 1995).

The reverberations of the face-to-face meeting are observed in the collaborative narratives, both by the Gi statements that evidenced, among other issues, an attention to the conceptual aspects of the measure, as well as the considerations of K, when he highlighted the problem of stone volume and added the concept of volume, anchored in Van De Walle (2009). Such reverberations help in understanding mathematical and pedagogical contents important to the teacher who "teaches-learns" Mathematics in childhood, and allows changes in the practices with the different classes, making them more exploratory, problematizing and playful.

We emphasize that the environment of respect and collaboration characteristic of GEProMAI, the collaborative production, through *Google Docs*, besides presenting themselves as an opportunity for interaction and production of meanings about the teaching and learning of Mathematics, are configured as important and unusual way of collaboration.

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