

On Matching Girth Domination Number of Graphs

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Abstract: A set M of edges of a graph G is a matching if no two edges in M are incident to the same vertex. A set S of vertices in G is a girth dominating set of G if every vertex in $V-S$ is adjacent to at least one vertex of girth graph is called the girth dominating set. the matching number is the maximum cardinality of a matching of G , while the girth domination number of G is the minimum cardinality taken over all girth domination number and is denoted by $\gamma_g(G)$ also the addition of any edge decreases the girth domination number denoted by $\gamma_{mg}(G)$.

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1. Introduction

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge [3] in 1958 and 1962 and O.Ore [8] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1200 papers related to domination in graphs in over 75 variation.

Throughout this paper, $G(V, E)$ a finite, simple, connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by $d(v)$, the maximum degree of a graph G is denoted by $\Delta(G)$. Let C_n a cycle on n vertices, P_n a path on n vertices by and a complete graph on n vertices by K_n . A graph is connected if any two vertices are connected by a path. A maximal connected subgraph of a graph G is called a component of G . The number of components of G is denoted by $\omega(G)$. The complement \bar{G} of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G . A tree is a connected acyclic graph. A bipartite graph is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2 . A complete bipartite graph is a bipartite graph with partitions of order $|V_1|=m$ and $|V_2|=n$, is denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a tree with one root vertex and $n-1$ pendant vertices. A bistar, denoted by $B(m, n)$ is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A wheel graph denoted by W_n is a graph with n vertices formed by

connecting a single vertex to all vertices of C_{n-1} . A Helm graph denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n .

The chromatic number of a graph G denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours. For any real number x , $\lceil x \rceil$ denotes the largest integer greater than or equal to x and $\lfloor x \rfloor$ the smallest integer less than or equal to x . A Nordhaus-Gaddum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs with no loops. The basic definitions and concepts used in this study are adopted from [11].

Given a graph $G = (V(G), E(G))$, the cardinality $|V(G)|$ of the vertex set $V(G)$ is the order of G is n . The distance $d_G(u, v)$ between two vertices u and v of G is the length of the shortest path joining u and v . If $d_G(u, v) = 1$, u and v are said to be adjacent.

For a given vertex v of a graph G , The open neighbourhood of v in G is the set $N_G(v)$ of all vertices of G that are adjacent to v .

The degree $\deg_G(v)$ of v refers to $|N_G(v)|$, and $\Delta(G) = \max\{\deg_G(v) : v \in V(G)\}$. The closed neighbourhood of v is the set $N_G[v] = N_G(v) \cup v$ for $S \subseteq V(G)$, $N_G(S) = \bigcup_{v \in S} N_G(v)$ and $N_G[v] = N_G(S) \cup S$. If $N_G[v] = V(G)$, then S is a dominating set in G . The minimum cardinality among dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$.

Definition [18]: In a connected graph G , a chord of a spanning tree T is a line of G which is not in T . Clearly the

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subgraph of G consists of T and any chord of T has exactly one cycle.

Definition [18]: If T is a regular of degree 2, every component is a cycle and regular graphs of degree 3 are called cubic.

Definition [18]: If all the edges of the girth are the edges of any other cycles in a graph G .

Theorem [18]: Let x be a line of a connected graph G , The following statements are equivalent (1) x is a bridge of G . (2) x is not on any cycle of G . (3) There exist points u and v of G s.t the line x is on every path joining u and v . (4) There exists a partition of v into subsets U and W s.t for any points $u \in U$ and $w \in W$ the line x is on every path joining u and w .

Theorem [18]: Let G be a connected graph with at least three points. The following statements are equivalent. (1) G is a block (2) Every two points of G lie on a common cycle (3) Every point and line of G lie on a common cycle (4) Every two lines of G lie on a common cycle (5) Given two points and one line of G , there is a path joining the points which contains the line (6) For every three distinct points of G , There is a path joining any two of them which contains the third.

2. Main Results

Definition 2.0: A set $S \subset V(G)$ is called a girth dominating set of G if every vertex in $V-S$ is adjacent to at least one vertex in the girth (cycle) graph of G . The minimum cardinality of a girth dominating set of G is called girth domination number of G denoted by $\gamma_g(G)$ also the addition of any edge decreases the girth domination number denoted by $\gamma_{mg}(G)$.

Example 2.1: For any graph $|G|=K_4=C_3+v=4$ and $UN(v_i)=C_3$ has girth dominating set of G with $\gamma_{mg}(G)=n-1=3$ for $n=4$ if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_3$. Hence $\gamma_{mg}(G)=n-1$ with $|M|=0$.

Example 2.2: For any graph $|G|=C_4. (k_2, k_2, k_2, k_2)$ and $UN(v_i)=C_3$ is a girth dominating set of G with $\gamma_{mg}(G)=n-1=3$ for $n=4$ if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_3$. Hence $\gamma_{mg}(G)=n-1$ with $|M|=2$.

Example 2.3: Every Corona graph of a girth graph G is $G \circ H = (C_4 - e) \circ k_1$ has a girth dominating set if $V-S = (G-S) \cup H_v$ and $|UN(H_v)| \neq C_4$ since if $|UN(H_v)| = C_4$ and $|UN(H_v)| = C_3$ where $v_i \in V-S$ and there exists $M_1, M_2 \in M$ such that $N(v_i) = u_i$ Suppose if $UN(v_i) = C_3$ or C_4 has a girth dominating set of G with $\gamma_g(G) = n-1$ or n with $|M| = 2$ or 1 hence we have $\gamma_g(G) = n-1$ or n for $n=4$ if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_3$ or C_4 also $|N(u_i) \cap (V-S)| = 1, i \neq 1$. Hence $\gamma_{mg}(G) = n-1$ or n with $|M| = 2$ or 1 respectively. Hence a graph G has a addition of any edge decreases the girth domination number.

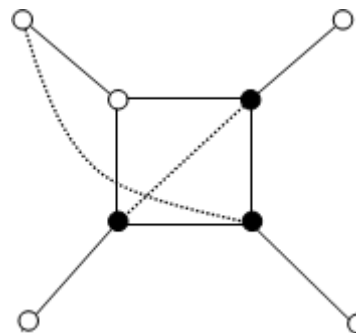


Figure 1: $G \circ H = (C_4 - e) \circ k_1, \gamma_{mg}(G) = n-1$ with $|M| = 2$.

Example 2.4: Every Corona graph of a girth graph G is $G \circ H = (K_4 - e) \circ k_2$ has a girth dominating set if $V-S = (G-S) \cup H_v$ and $|UN(H_v)| = C_4$.

Suppose if $UN(v_i) = C_3$ has a girth dominating set of G with $\gamma_g(G) = n-1=3$ for $n=4$ if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_3$. Hence $\gamma_{mg}(G) = n-1$ with $|M| = 2$.

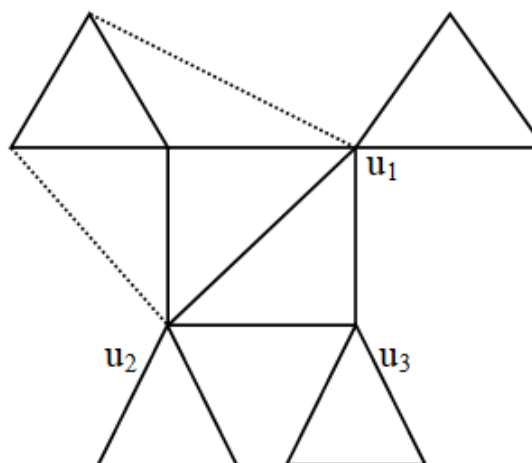


Figure 2: $G \circ H = (K_4 - e) \circ k_2, \gamma_{mg}(G) = n-1$ with $|M| = 2$.

Example 2.5: Every Corona graph of a girth graph G is $G \circ H = (K_n - e) \circ k_{n-1}$ has a girth dominating set if $V-S = (G-S) \cup H_v$ and $|UN(H_v)| = C_4$.

Suppose if $UN(v_i) = C_3$ is a girth dominating set of G with $\gamma_g(G) = n-1$ for all n if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_{n-1}$. Hence $\gamma_{mg}(G) = n-1$ with $|M| = n-1$.

Example 2.6: Every Corona graph of a girth graph G is $G \circ H = (C_4) \circ k_3$ has a girth dominating set if $V-S = (G-S) \cup H_v$ and $|UN(H_v)| = C_4$.

Suppose if $UN(v_i) = C_3$ has a girth dominating set of G with $\gamma_g(G) = n-1=3$ for $n=4$ if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_3$. Hence $\gamma_{mg}(G) = n-1$ with $|M| = 1$.

Example 2.7: Every Corona graph of a girth graph G is $G \circ H = (C_5 + e) \circ k_1$ has not a girth dominating set if $V-S = (G-S) \cup H_v$ since $|UN(H_v)| \neq C_4$ or C_3 where $v_i \in V-S$ and there exists $M_1, M_2 \in M$ such that $N(v_i) = u_i$

Suppose if $\cup N(v_i) = C_3$ or C_4 has a girth dominating set of G with $\gamma_{mg}(G) = n-2$ or $n-1$ for $n = 5$ if $\text{Max}\{d(u_i, u_j)\} \geq n-3, i \neq j$ where $u_i \in C_3$ or C_4 . Hence $\gamma_{mg}(G) = n-2$ or $n-1$ with $|M| = 2$ or 1 respectively.

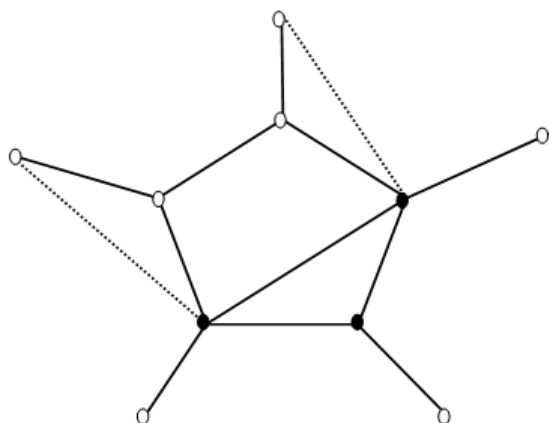


Figure 3: $G \circ H = (C_5 + e) \circ k_1, \gamma_{mg}(G) = n-2$ or $n-1$ with $|M| = 2$ or 1 respectively

Example 2.8: Every Corona graph of a girth graph G is $G \circ H = (C_5 - e) \circ k_1$ cannot have a girth dominating set if $V-S = (G-S) \cup H_v$ and $|\cup N(H_v)| = C_5$ since $|\cup N(H_v)| \neq C_5$ or C_4 or C_3 where $v_i \in V-S$ and there exists $M_1, M_2 \in M$ such that $N(v_i) = u_i$. Suppose if $\cup N(v_i) = C_3$ or C_4 or C_5 has a girth dominating set of G with $\gamma_g(G) \neq n-2$ with $|M| = 2$ hence we have $\gamma_g(G) = n-1$ or n for $n = 5$ if $\text{Max}\{d(u_i, u_j)\} \geq n-2, i \neq j$ where $u_i \in C_4$ or C_5 also $|N(u_i) \cap (V-S)| = 1, i \neq 1$. Hence $\gamma_{mg}(G) = n-1$ or n with $M=2$ or 1 respectively.

Theorem 2.9: Every Connected graph is of girth dominating set C_3 with $|M| \leq 3$ and its $\gamma_{mg}(G) = n - (n-3) \dots \dots \dots$ or $n-2$ or $n-1$ or n with $|M| = n - (n-3)$ or $\dots \dots \dots 3$ or 2 or 1 respectively.

Proof: Let C_n be the girth subgraph of G and $S = C_n$ with $V-S = G-S$. If $|\cup N(V-S)| = 3$ then C_n is the girth dominating set of G with $n \geq 3$. If $d(u_i, u_j) = 1$ where $u_i \in C_n$ since if $|\cup N(v_i)| = C_n$ is the girth subgraph of G where $v_i \in V-S$ and there exists $M_1, M_2 \in M$ such that $N(v_i) = u_i$ if $\text{Max}\{d(u_i, u_j)\} \geq n-1, i \neq j$ where $u_i \in C_{n-1}$ or C_n also $Nu_i \cap (V-S) = 1, i \neq 1$. Hence $\gamma_{mg}(G) = n-1$ or n with $|M| = 2$ or 1 respectively.

Suppose if $|\cup N(v_i)| = C_n$ and $|\cup N(v_i)| = C_n$ is the girth subgraph of G where $v_i \in V-S$ and there exists $M_1, M_2, M_3 \in M$ such that $N(v_i) = u_i$ if $\text{Max}\{d(u_i, u_j)\} \geq n-1, i \neq j$ where $u_i \in C_{n-2}$ or C_{n-1} or C_n also $|N(u_i) \cap (V-S)| = 1, i \neq 1$. Hence $\gamma_{mg}(G) = n-2$ or $n-1$ or n with $M=3$ or 2 or 1 respectively.

Suppose if $|\cup N(v_i)| = C_n$ and $|\cup N(v_i)| = C_n$ is the girth subgraph of G where $v_i \in V-S$ and there exists $M_1, M_2, \dots, M_n \in M$ such that $N(v_i) = u_i$ if $\text{Max}\{d(u_i, u_j)\} \geq n-1, i \neq j$ where $u_i \in C_{n-(n-3)}, \dots, C_{n-2}$ or C_{n-1} or C_n also $|N(u_i) \cap (V-S)| = 1, i \neq 1$. Hence $\gamma_{mg}(G) = n - (n-3)$ or $\dots \dots \dots$ or $n-2$ or $n-1$ or n with $|M| = n -$

$(n-3)$ or $\dots \dots \dots 3$ or 2 or 1 respectively. Hence a graph G has a girth domination number and addition of any edge decreases the girth domination number.

Theorem 2.10: Suppose for any graph $|G| = (k_4 - e) \cdot (k_{1,2}, k_{1,2}, k_{1,2}, k_{1,2})$ and $\cup N(v_i) = C_3$ has a matching girth dominating set of G with some conditions. Here if $|\cup N(v_i)| = 3$ or 4 its $\gamma_{mg}(G) = 3$ or 4 with $|M| = 2$ or 0 respectively.

Proof: Suppose for any graph $|G| = (k_4 - e) \cdot (k_{1,2}, k_{1,2}, k_{1,2}, k_{1,2})$ and if $|\cup N(v_i)| = 3$ then we have $\text{Max}\{d(u_i, u_j)\} = 2$ and its $|N(u_i) \cap (V-S)| = 2, i \neq 1$ hence we have $|N(u_1) \cap V-S| = Nu_4, u_2, v_{11}, v_{12} \cap V-S = 2, Nu_2 \cap V-S = Nu_1, u_3, v_{21}, v_{22} \cap V-S = 2, Nu_3 \cap V-S = Nu_4, u_2, v_{31}, v_{32} \cap V-S = 2, Nu_4 \cap V-S = Nu_3, u_2, v_{31}, v_{32} \cap V-S = 2$, but $N(u_4) \notin S$ and it is belongs to $V-S$. Hence $N(k_4) \neq u_i$. Hence $N(u_1) = |N(u_1) \cap (V-S)| \geq 2$ and its $\text{Max}\{d(u_i, u_j)\} = 2$ and now we have $|N(u_i) \cap V-S| \geq 2$ also its $\gamma_{mg}G = 3$ with $M=2$.

If $|\cup N(v_i)| = 4$ then we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V-S)| = 2$ and we have $N(u_i) = (S, V-S)$ hence we have $\gamma_{mg}(G) = 4$ with $|M| = 0$.

Hence a graph G has a girth domination number and addition of any edge decreases the girth domination number.

Theorem 2.11: Suppose for any graph $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,3}, k_{1,3}, k_{1,3}, k_{1,3})$ has a matching girth dominating set with conditions if $|\cup N(v_i)| = 3$ or 4 and its $\gamma_{mg}(G) = 3$ or 4 with $|M| = 3$ or 0 respectively.

Proof: Suppose for any graph $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,3}, k_{1,3}, k_{1,3}, k_{1,3})$ has a matching girth dominating set with conditions if $|\cup N(v_i)| = 3$ or 4 and its $\gamma_{mg}(G) = 3$ or 4 with $|M| = 3$ or 0 respectively.

Suppose if $|\cup N(v_i)| = 3$, then we have the $\text{Max}\{d(u_i, u_j)\} = 2$ and its $|N(u_i) \cap (V-S)| = 3$

But at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap V-S| \geq 3$ and its $\gamma_{mg}G = 3$ with $M=3$.

If $|M| = 2$ that is $|\cup N(v_i)| \neq 4$ since we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V-S)| \neq 3$ hence at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$.

If $|M| = 2$ that is $|\cup N(v_i)| = 4$ since we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V-S)| = 3$ hence at least three $v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap V-S| = 4$ also $\gamma_{mg}G = 4$ with $M=0$ and its G is isomorphic to $(C_4 \text{ or } (k_4 - e)) \cdot (k_{1,3}, k_{1,3}, k_{1,3}, k_{1,3})$.

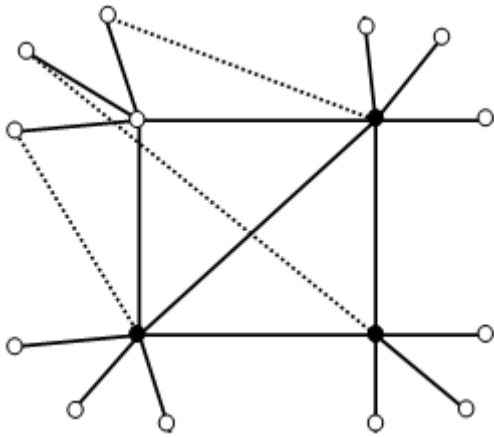


Figure 4: $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,3}, k_{1,3}, k_{1,3}, k_{1,3})$, $\gamma_{mg}(G) = 3 \text{ or } 4$ with $|M| = 3 \text{ or } 0$ respectively.

Theorem 2.12: Suppose for any graph $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,4}, k_{1,4}, k_{1,4}, k_{1,4})$ has a matching girth dominating set with conditions if $|\cup N(v_i)| = 3 \text{ or } 4$ and its $\gamma_{mg}(G) = 4$ with $|M| = 0$ respectively.

Proof: Suppose for any graph $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,4}, k_{1,4}, k_{1,4}, k_{1,4})$ if $|\cup N(v_i)| = 3$ then we have the $\text{Max}\{d(u_i, u_j)\} = 2$ and its $|N(u_i) \cap (V - S)| = 4$ but at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap V - S| \geq 3$ and its $\gamma_{mg}G = 3$ but M cannot be equal to 4 since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

Hence we must have $|\cup N(v_i)| = 4$. Hence if $|M| = 3$ then we can have at least one $v_i \in V - S$

and $\cup N(v_i) \neq C_3$ and if $|M| = 2$ then we have at least 2 $v_i \in V - S$ and $\cup N(v_i) \neq C_3$ and if $|M| = 1$ then we have at least 3 $v_i \in V - S$ and $\cup N(v_i) \neq C_3$. Hence we have $\cup N(v_i) = C_4$ and its $|N(u_i) \cap (V - S)| = 4$ also its $\text{Max}\{d(u_i, u_j)\} = 3$. Hence its $\gamma_{mg}(G) = 4$ with $|M| = 0$.

Theorem 2.13: Suppose for any graph $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,n}, k_{1,n}, k_{1,n}, k_{1,n})$ has a matching girth dominating set with the following conditions if $|\cup N(v_i)| = 3 \text{ or } 4$ and its $\gamma_{mg}(G) = 4$ with $|M| = 0$ respectively.

Proof: Suppose for any graph $|G| = (C_4 \text{ or } (k_4 - e)) \cdot (k_{1,n}, k_{1,n}, k_{1,n}, k_{1,n})$ has a matching girth dominating set with the following conditions if $|\cup N(v_i)| = 3$ then we have the $\text{Max}\{d(u_i, u_j)\} = 2$ and its $|N(u_i) \cap V - S| = n$ but at least one $u_i \notin C_n = S$ hence $N(u_i) \cap V - S \geq n - 1$ and its $\gamma_{mg}(G) = 3$ but $|M|$ cannot be equal to $n \geq 3$ since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

Hence we must have $|\cup N(v_i)| = 4$. Hence if $|M| = 3$ then we can have at least $(n - 3)v_i \in V - S$ and $\cup N(v_i) \neq C_3$ and if $|M| = 2$ then we have at least $(n - 2)v_i \in V - S$ and $\cup N(v_i) \neq C_3$ and if $|M| = 1$ then we have at least $(n - 1)v_i \in V - S$ and $\cup N(v_i) \neq C_3$. Hence we have $\cup N(v_i) = C_4$ and its $|N(u_i) \cap (V - S)| = n$ also its $\text{Max}\{d(u_i, u_j)\} = 3$. Hence its $\gamma_{mg}(G) = 4$ with $|M| = 0$.

Theorem 2.14: Suppose for every Corona graph of a girth graph G is $G \circ H = |G| = (C_5 \text{ or } (C_5 + e)) \circ (k_1)$ has a girth dominating set if $V - S = (G - S) \cup H_v$ and $|\cup N(H_v)| \neq C_5$ since if $|\cup N(H_v)| = C_4$ and $|\cup N(H_v)| = C_3$ where $v_i \in V - S$ and there exists $M_1, M_2, M_3 \in M$ such that $N(v_i) = u_i$ and $\gamma_{mg}(G) = n - 2$ or $n - 1$ or n with $|M| = 2$ or 1 or 0 respectively.

Proof: Suppose for every Corona graph of a girth graph G is $G \circ H = |G| = (C_5 \text{ or } (C_5 + e)) \circ (k_1)$ has a girth dominating set if $\cup N(v_i) = C_3 \text{ or } C_4 \text{ or } C_5$ has a girth dominating set of G with $\gamma_g(G) = n - 2$ or $n - 1$ or n with $|M| = 2$ or 1 or 0 hence we have $\gamma_{mg}(G) = n - 2$ or $n - 1$ or n for $n = 5$ if $\text{Max}\{d(u_i, u_j)\} \geq n - 3, i \neq j$ where $u_i \in C_3 \text{ or } C_4 \text{ or } C_5$ also $N(u_i) \cap (V - S) = 1, i \neq 1$. Hence $\gamma_{mg}(G) = n - 2$ or $n - 1$ or n with $|M| = 2$ or 1 or 0 respectively.

Suppose if $|\cup N(v_i)| = n - 2$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| = 1$ but at least one $v_i \in V - S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 1$ also $\gamma_{mg}(G) = 3$ with $|M| = 2$.

If $|M| = 1$ that is $|\cup N(v_i)| \neq 3$ since we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V - S)| \neq 1$ hence at least one $v_i \in V - S$ is non adjacent with $u_i \in C_n$.

If $|M| = 1$ that is $|\cup N(v_i)| = 4$ since we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V - S)| = 1$ hence at least three $v_i \in V - S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap V - S| = 1$ also $\gamma_{mg}G = 4$ with $M = 1$.

If $|M| = 0$ that is $|\cup N(v_i)| \neq 4$ since we have $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap (V - S)| \neq 1$ hence every $v_i \in V - S$ is adjacent with $u_i \in C_n$.

If $|M| = 0$ that is $|\cup N(v_i)| = 5$ since we have $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap (V - S)| = 1$ hence every $v_i \in V - S$ is adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 1$ also $\gamma_{mg}(G) = 5$ with $|M| = 0$. Hence a graph G has a addition of any edge decreases the girth domination number.

Theorem 2.15: Suppose for every Corona graph of a girth graph G is $G \circ H = |G| = (C_5 \text{ or } (C_5 + e)) \circ (k_2)$ has a girth dominating set if $V - S = (G - S) \cup H_v$ and $|\cup N(H_v)| \neq C_5$ since if $|\cup N(H_v)| = C_4$ and $|\cup N(H_v)| = C_3$ where $v_i \in V - S$ and there exists $M_1, M_2 \in M$ such that $N(v_i) = u_i$ and $\gamma_{mg}(G) = n - 2$ or $n - 1$ or n with $|M| = 2$ or 1 or 0 respectively

Proof: Suppose for every Corona graph of a girth graph G is $G \circ H = |G| = (C_5 \text{ or } (C_5 + e)) \circ (k_2)$ if $\cup N(v_i) = C_3 \text{ or } C_4 \text{ or } C_5$ has a girth dominating set of G with $\gamma_g(G) = n - 2$ or $n - 1$ or n with $|M| = 4 \text{ or } 3 \text{ or } 2 \text{ or } 1 \text{ or } 0$ hence we have $\gamma_{mg}(G) = n - 2$ or $n - 1$ or n for $n = 5$ if $\text{Max}\{d(u_i, u_j)\} \geq n - 3, i \neq j$ where $u_i \in C_3 \text{ or } C_4 \text{ or } C_5$ also $N(u_i) \cap (V - S) \geq 2, i \neq 1$. Hence $\gamma_{mg}(G) = n - 2$ or $n - 1$ or n with $|M| = 2$ or 1 or 0 respectively.

If $|\cup N(v_i)| = n - 2$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| = 2$ but at least $4v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| \neq 2$ also $\gamma_{mg}(G) = 3$ but $|M|$ cannot be equal to 4 since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

If $|M| = 3$ and also if $|\cup N(v_i)| = n - 2$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| = 2$ but at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| \neq 2$ also $\gamma_{mg}(G) \neq 3$ but $|M|$ cannot be equal to 4 since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

If $|M| = 2$ that is $|\cup N(v_i)| \neq 3$ since we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least $2v_i \in V-S$ is non adjacent with $u_i \in C_n$.

If $|M| = 2$ that is $|\cup N(v_i)| = 4$ since we have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least $2v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 1$ also $\gamma_{mg}(G) = 4$ with $M = 2$.

If $|M| = 0$ that is $|\cup N(v_i)| \neq 4$ since we have $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap (V - S)| \neq 1$ hence every $v_i \in V-S$ is adjacent with $u_i \in C_n$.

If $|M| = 0$ that is $|\cup N(v_i)| = 5$ since we have $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap (V - S)| = 2$ hence every $v_i \in V-S$ is adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 2$ also $\gamma_{mg}(G) = 5$ with $|M| = 0$.

Theorem 2.16: Suppose for any Corona graph of a girth graph G is $G \circ H = |G| = (C_6 \text{ or } (C_6 + e)) \circ (k_2)$ has a girth dominating set if $V-S = (G-S) \cup H_v$ and $|\cup N(H_v)| \neq C_5$ since if $|\cup N(H_v)| = C_4$ and $|\cup N(H_v)| = C_3$ where $v_i \in V-S$ and there exists $M_1, M_2 \in M$ such that $N(v_i) = u_i$ and $\gamma_{mg}(G) = n-3$ or $n-2$ or $n-1$ or n with $|M| = 4$ or 3 or 2 or 1 or 0 .

Proof: Suppose for any Corona graph of a girth graph G is $G \circ H = |G| = (C_6 \text{ or } (C_6 + e)) \circ (k_2)$ if $|\cup N(v_i)| = C_3$ or C_4 or C_5 or C_6 has a girth dominating set of G with $\gamma_{mg}(G) = n-3$ or $n-2$ or $n-1$ or n with $|M| = 4$ or 3 or 2 or 1 or 0 hence we have $\gamma_{mg}(G) = n-3$ or $n-2$ or $n-1$ or n for $n = 6$ if $\text{Max}\{d(u_i, u_j)\} \geq n-4$, $i \neq j$ where $u_i \in C_3$ or C_4 or C_5 or C_6 also $|N(u_i) \cap (V - S)| \geq 2$, $i \neq 1$. Hence $\gamma_{mg}(G) = n-2$ or $n-1$ or n with $|M| = 2$ or 1 or 0 respectively.

If $|\cup N(v_i)| = n - 3$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 4$ and its $|N(u_i) \cap (V - S)| = 2$ but at least $6v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| \neq 2$ also $\gamma_{mg}(G) = 3$ but $|M|$ cannot be equal to 6 since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

If $|M| = 3$, If $|\cup N(v_i)| = n - 3$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 4$ and its $|N(u_i) \cap (V - S)| = 2$ but at least $3v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap$

$(V - S)| \neq 2$ also $\gamma_{mg}(G) \neq 3$ but $|M|$ cannot be equal to 3 since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

If $|M| = 4$ that is $|\cup N(v_i)| \neq 3$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least $4v_i \in V-S$ is non adjacent with $u_i \in C_n$.

If $|M| = 4$ that is $|\cup N(v_i)| = 4$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least $4v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 2$ also $\gamma_{mg}(G) = n-2=4$ with $M=4$.

If $|M| = 3$ that is $|\cup N(v_i)| \neq 3$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$.

If $|M| = 3$ that is $|\cup N(v_i)| = 4$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| \neq 2$ also $\gamma_{mg}(G) \neq n-2=4$ with $|M| = 3$.

If $|M| = 2$ that is $|\cup N(v_i)| \neq 4$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 2 = 4$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence every $v_i \in V-S$ is adjacent with $u_i \in C_n$.

If $|M| = 2$ that is $|\cup N(v_i)| = 5$ since we have $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap (V - S)| = 2$ hence at least $2v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 2$ also $\gamma_{mg}(G) = 5$ with $M = 2$.

If $|M| = 1$ that is $|\cup N(v_i)| = 5$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 2$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$.

If $|M| = 1$ that is $|\cup N(v_i)| = 5$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 2$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| \neq 2$ also $\gamma_{mg}(G) \neq n-1=5$ with $|M| = 1$.

If $|M| = 0$ that is $|\cup N(v_i)| \neq 5$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 2$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence at least one $v_i \in V-S$ is non adjacent with $u_i \in C_n$.

If $|M| = 0$ that is $|\cup N(v_i)| = 6$ since we have $\text{Max}\{d(u_i, u_j)\} = n - 1$ and its $|N(u_i) \cap (V - S)| \neq 2$ hence every $v_i \in V-S$ adjacent with $u_i \in C_n$ but $|N(u_i) \cap (V - S)| = 2$ also $\gamma_{mg}(G) = n=6$ with $M=0$.

Theorem 2.17: Suppose for any graph $|G| = (C_5 \text{ or } (C_5 + e)) \cdot (k_{1,n}, k_{1,n}, k_{1,n}, k_{1,n}, k_{1,n})$ has a matching girth dominating set with the following conditions if $|\cup N(v_i)| = 3$ or 4 or 5 . And $\gamma_g(G) = n-2$ or $n-1$ or n with $|M| = 3$ or 0 .

Proof: Suppose for any graph $|G| = (C_5 \text{ or } (C_5 + e)) \cdot (k_{1,n}, k_{1,n}, k_{1,n}, k_{1,n}, k_{1,n})$ if $|\cup N(v_i)| = 3$ then we have the $\text{Max}\{d(u_i, u_j)\} = 2$ and its $|N(u_i) \cap (V - S)| =$

3 but at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq n + 2$ and its $\gamma_{mg}(G) = n + 1$ but $|M|$ cannot be equal to $n + 1$ since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

Hence we must have $|\cup N(v_i)| = 3$. Hence if $|M| = 3$ then we can have at least $(n-3)v_i \in V-S$ and $\cup N(v_i) \neq C_3$ and if $|M| = 2$ then we have at least $(n-2)v_i \in V-S$ and $\cup N(v_i) \neq C_3$ and if $|M| = 1$ then we have at least $(n-1)v_i \in V-S$ and $\cup N(v_i) \neq C_3$. similarly if $|\cup N(v_i)| = 4$ then we can have $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap V-S| = n$ but at least one $u_i \notin C_n = S$ hence $N(u_i) \cap V-S \geq n + 1$ and we must have $\gamma_{mg}(G) = n + 1$ since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$. Hence we must have $|\cup N(v_i)| = 4$.

If $|M| = 4$ then we can have at most $(n-4)v_i \in V-S$ and $\cup N(v_i) \neq C_3$. If $|M| = 2$ then we can have at least $(n-2)v_i \in V-S$ and $\cup N(v_i) \neq C_3$ that is $3n+5=V$ and if $|M| = 1$ then we can have at least $(n-2)v_i \in V-S$ and $\cup N(v_i) \neq C_3$. By similar way $|\cup N(v_i)| = 5$

Then we can have $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap V-S| = n$. Hence we have $N(u_i) \cap V-S = n$ and its $N(u_i) \cap V-S = n$ also its $\text{Max}\{d(u_i, u_j)\} = 4$. Hence its $\gamma_{mg}(G) = 5$ with $|M| = 0$.

Theorem 2.18: Suppose for any graph $|G| = (C_6 \text{ or } (C_6 + e))$. $(k_{1,3}, k_{1,3}, k_{1,3}, k_{1,3})$ has a matching girth dominating set with condition if $|\cup N(v_i)| = 3 \text{ or } 4 \text{ or } 5 \text{ or } 6$ and $\gamma_{mg}(G) = n-3 \text{ or } n-2 \text{ or } n-1 \text{ or } n$ with $|M| = 3 \text{ or } 2 \text{ or } 1 \text{ or } 0$

Proof: Suppose for any graph $|G| = (C_6 \text{ or } (C_6 + e))$. $(k_{1,3}, k_{1,3}, k_{1,3}, k_{1,3})$ if $|\cup N(v_i)| = 3$ then we have the $\text{Max}\{d(u_i, u_j)\} = 2$ and its $|N(u_i) \cap (V - S)| \neq n$ but it has $n=6$, then $|G| = K_n = C_{n-3} + 3v = n$, $C_{n-3} = n - 3v$, that is $3v = 6 - (n-3)$ and $V-S = 6m + 3v = 6m + 3$ but $3m + 1$ vertices are non adjacent with C_3 and $(n-3)m + (n-3) - (n-5) = 3m + 2$ are the vertices adjacent with C_3 at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = 3$ but $|M|$ cannot be equal to $3m + 1$ since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$. If $m=0$, we have $\gamma_{mg}(G) = 3$ with $|M| = 3$.

Suppose if $|\cup N(v_i)| = 4$ then we have the $\text{Max}\{d(u_i, u_j)\} = 3$ and its $|N(u_i) \cap (V - S)| \neq n$ but it has $n=6$, then $|G| = K_n = C_{n-2} + 2v = n$, $C_{n-2} = n - 2v$, that is $2v = 6 - (n-2)$ and $V-S = 6m + n - (n-2) = 6m + 6 - (6-2) = 6m + 6 - 4 = 6m + 2$ but $2m + 0$ vertices are non adjacent with C_4 and $(n-2)m + (n-4) - (n-6) = 4m + 2$ are the vertices adjacent with C_4 at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = 4$ but $|M|$ cannot be equal to $2m$ since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$. If $m=0$, we have $\gamma_{mg}(G) = 4$ with $|M| = 2$.

Suppose if $|\cup N(v_i)| = 5$ then we have the $\text{Max}\{d(u_i, u_j)\} = 4$ and its $|N(u_i) \cap (V - S)| \neq n$ but it has $n=6$, then $|G| = K_n = C_{n-1} + v = n$, $C_{n-1} = n - v$, that is

$v = 6 - (n-1)$ and $V-S = 6m + n - (n-1) = 6m + 6 - (6-1) = 6m + 5 = 6m + 1$ but we have $[n - (n-1)]m + ov = [6 - (6-1)]m + 0 = (6-5)m = m$ vertices are non adjacent with C_5 and $(n-1)m + (n-1) - (n-2) = 5m + 1$ are the vertices adjacent with C_5 at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = 5$ but $|M|$ cannot be equal to m since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$. If $m=0$, we have $\gamma_{mg}(G) = 5$ with $|M| = 1$ and hence if $m=1$ it does not have $\gamma_{mg}(G) \neq 5$.

If $|\cup N(v_i)| = 6$ then we have the $\text{Max}\{d(u_i, u_j)\} = 5$ and its $|N(u_i) \cap (V - S)| \neq n$ but it has $n=6$, then $|G| = K_n = C_n + 0v = n$, $C_n = n$, that is $v = 6 - (n)$ and $V-S = 6m + n - (n) = 6m + 6 - (6) = 6m$ but we have $[n - (n)]m + ov = [6 - (6)]m + 0 = (0)m$ vertices are non adjacent with C_6 and $(n)m = 6m$ are the vertices adjacent with C_6 at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = 6$. If $m=0$, we have $\gamma_{mg}(G) = 6$ with $|M| = 0$.

Theorem 2.19: Suppose for any graph $|G| = (C_n \text{ or } (C_n + e))$. $(k_{1,m}, k_{1,m}, k_{1,m}, k_{1,m})$ has a matching girth dominating set with conditions if $|\cup N(v_i)| = 3 \text{ or } 4 \text{ or } 5 \text{ or } 6 \dots n$ with $|M| = n$ since by theorem every connected graph is of girth dominating set C_n with $|M| \leq n$ and its $\gamma_{mg}(G) = n-r$ with $|M| \leq n - r$ but it is $|M| = m$ since we have $m \leq n-r$.

Proof: Suppose for any graph $|G| = (C_n \text{ or } (C_n + e))$. $(k_{1,m}, k_{1,m}, k_{1,m}, k_{1,m})$ if $|\cup N(v_i)| = n$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 1$ and its $|N(u_i) \cap (V - S)| \neq n$ but it has n vertices, then $|G| = K_n = C_n + 0v = n$, $C_n = n$, that is $v = n - (n)$ and $V-S = nm + n - (n) = nm + n - (n) = nm$ but we can have $[n - (n)]m + ov = [n - (n)]m + 0 = (0)m$ vertices are non adjacent with C_n and $(n)m = nm$ are the vertices adjacent with C_n at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = n$ with $|M| = 0$.

If $|\cup N(v_i)| = n - 1$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 2$ and its $|N(u_i) \cap (V - S)| \neq n - 1$ but it has n vertices, then $|G| = K_n = C_{n-1} + 1v = n$, $C_{n-1} = n - 1$, that is $v = n - (n-1)$ and $V-S = nm + [n - (n-1)]m = nm + m = m(n+1)$ but we can have $[n - (n-1)]m + ov = [n - (n)]m + 0 = m$ vertices are non adjacent with C_{n-1} and $(n-1)m + [n - (n-1)]v = (n-1)m + 1$ are the vertices adjacent with C_{n-1} at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = n-1$ with $|M| \leq n$ but it is $|M| = m$ since we have $m \leq n-1$.

Suppose if $|\cup N(v_i)| = n - 2$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - 3$ and its $|N(u_i) \cap (V - S)| \neq n - 2$ but it has n vertices, then $|G| = K_n = C_{n-2} + 2v = n$, $C_{n-2} = n - 2$, that is $v = n - (n-2)$ and $V-S = nm + [n - (n-2)]m = nm + 2m = m(n+2)$ but we can have $[n - (n-2)]m + ov = 2m$ vertices are non adjacent with C_{n-2} and $(n-2)m + [n - (n-2)]v = (n-2)m + 2$ are the vertices adjacent with C_{n-2} at least one $u_i \notin C_n = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = n-2$ with $|M| \leq n$ but it is $|M| = m$ since we have $m \leq n-2$.

If $|\cup N(v_i)| = n - r = m$ then we have the $\text{Max}\{d(u_i, u_j)\} = n - r - 1$ and its $|N(u_i) \cap (V - S)| \neq n - r - 1$ but it has n vertices, then $|G| = K_n = C_{n-r} + rv = n$, $C_{n-r} = n - r$, that is $v = n - (n-r)$ and $V - S = n - (n-r) + [n - (n-r)]m = n - (n-r) + 2(n-r) = (n-r)(n+2)$ but we can have $[n - (n-r)]m + ov = rm$ vertices are non adjacent with C_{n-r} and also $(n-r)m + [n - (n-r)]v = (n-r)m + r = nm - rm + r$ are the vertices adjacent with C_{n-r} at least one $u_i \notin C_{n-r} = S$ hence $N(u_i) = |N(u_i) \cap (V - S)| \geq m$ and its $\gamma_{mg}(G) = n - r$ with $|M| \leq n - r$ but it is $|M| = m$ since we have $m \leq n - r$.

3. Conclusion

In this paper we found an upper bound for the girth domination number and Relationships between Matching girth domination number and characterized the corresponding extremal graphs. Similarly also the addition of any edge decreases the girth domination number denoted by $\gamma_{mg}(G)$. with other graph theoretical parameters can be considered.

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