

# Measurement of Susceptibility of a Liquid or a Solution by Quincke's Method

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**Abstract:** Here we measure susceptibility of given solution ( $MnSO_4$ ,  $H_2O$ ) by quinckes method.

**Keywords:** Quincke's Method

## 1. Introduction

The Quincke's method is used to determine magnetic susceptibility of diamagnetic or paramagnetic substances in the form of a liquid or an aqueous solution. When an object is placed in a magnetic field, a magnetic moment is induced in it. Magnetic susceptibility  $X$  is the ratio of the magnetization  $I$  (magnetic moment per unit volume) to the applied magnetizing field intensity  $H$ . The magnetic moment can be measured either by force methods, which involve the measurement of the force exerted on the sample by an inhomogeneous magnetic field or induction methods where the voltage induced in an electrical circuit is measured by varying magnetic moment. The Quincke's method like the Gouy's method belongs to the former class. The force  $f$  on the sample is negative of the gradient of the change in energy density when the sample is placed,

$$f = \frac{d}{dx} \left[ \frac{1}{2} \mu_0 (\mu_r - \mu_{ra}) H^2 \right] = \frac{1}{2} \mu_0 (X - X_{ra}) \frac{dH^2}{dx} \quad (1)$$

Here  $\mu_0$  is permeability of the free space and  $\mu_r$ ,  $\chi$  and  $\mu_{ra}$ ,  $X_a$  are respectively relative permeability and susceptibility of the sample and the air which the sample displaces. The force acting on an element of area  $A$  and length  $dx$  of the liquid column is  $fAdx$ , so the total force  $F$  on the liquid is

$$F = A \int f dx = \frac{A \mu_0}{2} (X - X_a) (H^2 - H_0^2) \quad (2)$$

where the integral is taken over the whole liquid. This means that  $H$  is equal to the field at the liquid surface between the poles of the magnet and  $H_0$  is the field at the other surface away from the magnet. The liquid (density  $\rho$ ) moves under the action of this force until it is balanced by the pressure exerted over the area  $A$  due to a height difference  $h$  between the liquid surfaces in the two arms of the U-tube. It follows that

$$F = Ah(\rho - \rho_a)g$$

$$\text{Or } \chi = \chi_a + \frac{2}{\mu_0} g(\rho - \rho_a) \frac{h}{(H^2 - H_0^2)} \quad (3)$$

In actual practice  $X_a$ , density of air  $\rho_a$  and  $H_0$  are negligible and can be ignored and the above expression simplifies to

$$X = \frac{2\rho gh}{\mu_0 H^2} \quad (4)$$

This equation shows that by plotting  $h$  as a function of  $H^2$ , the susceptibility  $X$  (called the volume susceptibility) can be determined directly from the slope of the straight line graph. It is a dimensionless quantity. This expression is in S.I. units in which  $\rho$ ,  $g$ ,  $h$  and  $H$  are measured in  $kg/m$ ,  $m/s$ ,  $m$  and  $amp.turn/m$  respectively.

$$X = X_a + 2g(\rho - \rho_a) \frac{h}{(H^2 - H_0^2)} \quad (5)$$

$$X = \frac{2\rho gh}{H^2} \quad (6)$$



where  $\rho$ ,  $g$ ,  $h$  and  $H$  are measured in  $\text{g/cm}^3$ ,  $\text{cm/s}^2$ ,  $\text{cm}$  and gauss respectively. The volume susceptibilities in the two systems of units are related as  $X$  (SI units) =  $4\pi X$  (CGS units).

Brief description of the apparatus

- 1) Quinck's tube with stand.
- 2) Sample :  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$
- 3) R.D. Bottle.
- 4) Travelling Microscope, TVM-02  
Horizontal scale: 180mm with screw gauge type motion.  
Vertical scale: 150mm with screw gauge type motion.  
Least count: 0.01mm on both scales.
- 5) Electromagnet, EMU-50T  
Pole Pieces:  $\phi$  50mm tapered to  $\phi$  20mm.  
Field: 9.5 KG at 10mm air-gap.
- 6) Constant Current Power Supply, DPS-50  
Current: -0-4 A (Smoothly adjustable)  
Line load Regulation :  $\pm 0.1\%$ .
- 7) Digital Gaussmeter, DGM-102  
Range: 0-2 KG & 0-20 KG  
Accuracy:  $\pm 0.5\%$

Detailed specifications are as per the datasheets.

The apparatus is complete in all respect, except a Chemical Balance required for measuring weights.

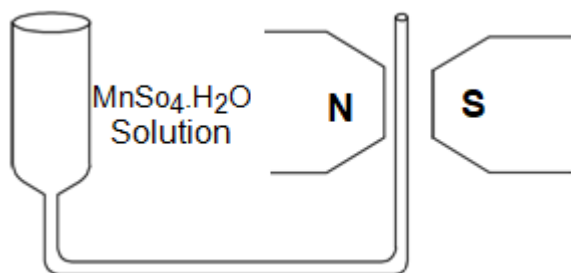


Figure 1: Schematic of Quinck's Setup

## 2. Experimental Set Up

A schematic diagram of Quincke's set up is shown in Fig.1. One limb of the glass U-tube is very narrow (about 2 - 3 mm in diameter) and the other one quite wide. The result is that a change in the level of the liquid in the narrow limb does not affect the level in the wider limb. The narrow limb is placed between the pole pieces of an electromagnet shown as N-S such that the meniscus of the liquid lies symmetrically between N-S. The length of the limb should be sufficient enough to keep the lower extreme end of this limb well outside the field of the magnet. The rise or fall  $h$  on applying the field is measured by means of a traveling microscope fitted with a micrometer scale of least count of order  $10^{-3}$  cm.

## 3. Experimental Procedure

- 1) Test and ensure that each unit (Electromagnet and Power Supply) is functioning properly
- 2) Measure the density  $\rho$  of the specimen (liquid or solution) by specific gravity bottle. If the mass of empty

bottle is  $w_1$ , filled with specimen  $w_2$  and filled with water  $w_3$ , then

$$\rho = \rho_{\text{water}} \frac{w_2 - w_1}{w_3 - w_1} \quad (7)$$

- 3) *Scrupulous* cleaning of the tube is essential. *Thoroughly* clean the Quincke's tube, rinse it well with distilled water and dry it (preferably with dry compressed air). Do not use the tube for longer than one laboratory period without recleaning it.
- 4) Keep the Quincke's tube between the pole pieces of the magnet as shown in Fig.1. The length of the horizontal connecting limb should be sufficient to keep the wide limb out of the magnetic field.
- 5) Fill the liquid in the tube and set the meniscus centrally within the pole pieces as shown. Focus the microscope on the meniscus and take reading.
- 6) Apply the magnetic field  $H$  and note its value from the calibration, which is done earlier as an auxiliary experiment. Note whether the meniscus rises up or descends down. It rises up for paramagnetic liquids and solutions while descends down for diamagnetics. Readjust the microscope on the meniscus and take reading. The difference of these two readings gives  $h$  for the field  $H$ . The magnetic field between the poles of the magnet does not drop to zero even when the current is switched off. There is a residual magnetic field  $R$  which requires a correction.
- 7) Measure the displacement  $h$  as a function of applied field  $H$  by changing the magnet current in small steps. Plot a graph of  $h$  as a function of  $H^2$ .

## 4. Results

The mass susceptibility  $X'$  is given by  $X/\rho$  and the molar susceptibility  $X''$  by  $M X'$  where  $M$  is the molecular weight of the specimen. In the case of solutions, correction must be made for the diamagnetic contribution of water. If the number of water molecules per unit volume is not very different in the solution from that in pure water,

$$X_{\text{solution}} = X_{\text{salt}} + X_{\text{water}} \quad (8)$$

In general if  $m_s$  is the mass of the salt dissolved in  $m_w$  of water,

$$X'_{\text{solution}} = \frac{m_s}{m_s + m_w} X'_{\text{salt}} + \frac{m_w}{m_s + m_w} X'_{\text{water}} \quad (9)$$

Mass susceptibility of the salt,  $X'_{\text{salt}}$  can be obtained from this relation.

In a paramagnetic substance there are non-interacting permanent magnetic dipoles. The magnetizing field tends to align these parallel to the field. Thermal effects on the other hand tend to destroy this alignment. The result is that, for  $kT \gg \mu_0 \mu_B H$ , the volume susceptibility  $X_{\text{salt}}$  at temperature  $T$  is given by

$$X_{\text{salt}} = \frac{I}{H} = \frac{N \mu_0 (\mu_B)^2}{3kT} = \frac{C}{T} \text{ (S.I. units)} \quad (10)$$

$$X_{\text{salt}} = \frac{N (\mu_B)^2}{3kT} = \frac{C}{T} \text{ (C.G.S. units)}$$

where  $k$  is Boltzmann constant,  $N$  ( $= N_A \rho_{\text{salt}} / M$ ) is the number of ions per unit volume and  $\mu_B$  is Bohr magneton.  $p$

is the magneton number which may be calculated from the measured value of  $\chi_{\text{salt}}$  ( or  $X'_{\text{salt}} = X_{\text{salt}} / \rho_{\text{salt}}$  ) and compared with the theoretical value depending on  $L, S$  and  $J$  of the magnetic ion as given in Appendix I. Eq.10 expresses Curie law and the constant  $C$  is called the Curie constant. The above expression assumes that the magnetic field acting on each ion is just the applied field and contributions due to neighbouring magnetic ions are neglected. For dilute paramagnetic materials this approximation is valid.

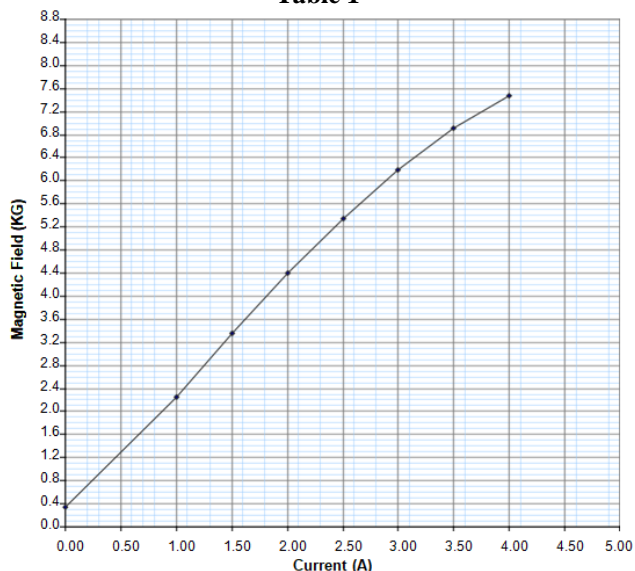
Record of Observations And Calculation

(The following is a record of observations corresponding to an experiment performed with  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$  in our laboratory).

1) Preparation of solution

- (a) Mass of beaker ( $m_b$ ) = 29.857 g
- Mass of beaker + water ( $m_{bw}$ ) = 62.585 g
- Mass of beaker + water + salt ( $m_{bws}$ ) = 82.565 g
- Mass of water ( $m_w = m_{bw} - m_b$ ) = 32.728 g
- Mass of dissolved salt ( $m_s = m_{bws} - m_{bw}$ ) = 19.980 g

Table 1



(b) Determination of density  $\rho$ :

- Mass of specific gravity bottle ( $w_1$ ) = 19.698 g
- Mass of specific gravity bottle + water ( $w_3$ ) = 44.973 g
- Mass of specific gravity bottle + solution ( $w_2$ ) = 56.173 g

$$\rho = \frac{w_2 - w_1}{w_3 - w_1} = \frac{36.475}{25.275} = 1.443 \text{ g/cm}^3$$

- 2) Ambient Temperature 293 K
- 3) Calibration of magnetic field as a function magnetizing current:-

Procedure:

- 1) Fix the air gap between the pole pieces of the electromagnet to the minimum distance required to insert Quinck's tube without touching the pole pieces.
- 2) Measure the air gap. Each time the air gap changes, the graph will change.
- 3) Mount the Hall probe of the Digital Gaussmeter, DGM-102 in the wooden stand provided and place it at the centre of the air gap such that the surface of the probe is parallel to the pole pieces. The small black crystal in the probe is its transducer, so this part should be at the centre of the air gap.
- 4) Connect the leads of the Electromagnet to the Power Supply, bring the current potentiometer of the Power Supply to the minimum. Switch on the Power Supply and the Gaussmeter.
- 5) Slowly raise the current in the Power Supply and note the magnetic field reading in the Gaussmeter.
- 6) Plot the graph between the current and the magnetic field. This graph will be linear for small values of the current and then the slope will decrease as magnetic saturation occurs in the material of the pole pieces.

Table 1

S.No.	Current (A)	Magnetic Field (KG)
1	0	0.34
2	1	2.26
3	1.5	3.35
4	2	4.4
5	2.5	5.34
6	3	6.19
7	3.5	6.91
8	4	7.47

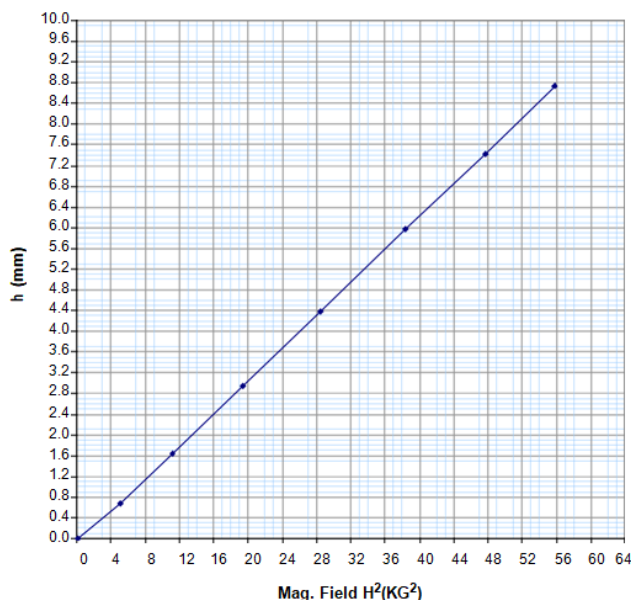


Figure: Rise of solution level (h) as a function of magnetic field (H)

While performing the Quinck's tube experiment, if the air gap is kept same as in the above experiment, you can determine the magnetic field at any specific current, just by looking at the graph. Note there may also be some magnetic hysteresis present and for a given current, the field may be slightly different, depending on whether the current is increasing or decreasing.

Rise of solution level ( $h$ ) as a function of magnetic field ( $H$ ):-

## References

- [1] S. Grant and W.R. Phillips, "Electromagnetism", (Wiley)  
 [2] <http://www.agoenvironmental.com/sites/default/files/pdf/diamagnetic%20element%20list.pdf>

Table 2

S.No.	Current (A)	H (KG)	$H^2$ (KG <sup>2</sup> )	Height of liquid level (mm)	Rise of solution h (mm)
1	0	0.34	0.12	10.14	0
2	1	2.26	5.11	10.81	0.67
3	1.5	3.35	11.22	11.78	1.64
4	2	4.4	19.36	13.09	2.95
5	2.5	5.34	28.52	14.53	4.39
6	3	6.19	38.32	16.12	5.98
7	3.5	6.91	47.75	17.57	7.43
8	4	7.47	55.8	18.38	8.74

From the graph of  $h$  vs  $H^2$

$$\frac{h}{H^2} = \frac{7.5 \times 10^{-1}}{47 \times 10^6} = 15.96 \times 10^{-9}$$

Calculation:

1) Putting  $\rho = 1.443$ ,  $g = 980$ ,  $\frac{h}{H^2} = 15.96 \times 10^{-9}$  in

$$X = \frac{2\rho gh}{H^2}, \text{ we get} \quad (6)$$

$$X = 45.14 \times 10^{-6}$$

2)  $X'_{\text{sol}} = \frac{X}{\rho} = \frac{45.14 \times 10^{-14}}{1.443} = 31.28 \times 10^{-6}$

3)  $X'_{\text{sol}} = \frac{m_s}{X'_{\text{salt}}} + \frac{m_w}{m_s + m_w} X'_{\text{water}}$

$$X'_{\text{salt}} = (X'_{\text{sol}} - \frac{m_w}{m_s + m_w} X'_{\text{water}}) \frac{m_s + m_w}{m_s}$$

Putting,  $X'_{\text{sol}} = 31.28 \times 10^{-6}$ ,  $m_w = 32.728$  g,  $m_s = 19.980$  g,

$$X'_{\text{water}} = -0.72 \times 10^{-6} \quad X'_{\text{salt}} = 83.7 \times 10^{-6}$$

4)  $X'_{\text{salt}} = X'_{\text{salt}} \times \text{molecular weight of MnSO}_4 \cdot \text{H}_2\text{O}$

$$= 83.7 \times 10^{-6} \times 169$$

$$= 14145 \times 10^{-6} \text{ C.G.S. unit}$$

Internationally accepted value of  $X''$  for  $\text{MnSO}_4 \cdot \text{H}_2\text{O}$  at 293 °K is  $14200 \times 10^{-6}$

## Appendix I

Some constants and conversion factors:

Boltzmann constant $k$	$1.381 \times 10^{-23} \text{ Joule/}^\circ\text{K}$
Avogadro's number $N_A$	$6.0225 \times 10^{26} \text{ per kg mole}$
Bohr magneton $\alpha_B$	$9.272 \times 10^{-24} \text{ A}\cdot\text{m}^2$
	$9.272 \times 10^{-21} \text{ erg/gauss}$

Permeability of free space  $\alpha_0$   $4\pi \times 10^{-7}$

1 amp.tum/m	$4\pi \times 10^{-3} \text{ gauss}$
Mass susceptibility of water	$-9.0 \times 10^{-9} \text{ m}^3/\text{kg}$
Molecular weight of $\text{MnSO}_4 \cdot \text{H}_2\text{O}$	169
Density of $\text{MnSO}_4 \cdot \text{H}_2\text{O}$	$2.95 \times 10^3 \text{ kg/m}^3$