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Comparative Study of Transformations for Time Dependent Signal and Analysis of the Earthquake Motion Dynamics Using HHT

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Abstract: ***This study examines the comparative study of the different transformations for the time dependent signals. The most famous transformations for the data analysis are the Fourier transform, the Fast Fourier transform and the Hilbert Huang transform. A brief introduction of the Hilbert Huang transform is presented and it is used for the analysis of the earthquake motion dynamics.***

Keywords: Fast Fourier Transform, Empirical mode Decomposition, Hilber Spectral Analysis, Hilbert Huang Transform, Earthquake

1. Introduction

Data are the only link we have with the unexplained reality; therefore, data analysis is the only way through which we can find out the underlying processes of any given phenomenon. As the most important goal of scientific research is to understand nature, data analysis is a critical ink in the scientific research cycle of observation, analysis, synthesizing, and theorizing. Because of the limitations of available methodologies for analyzing data, the crucial phase of data analysis has in the past been relegated to "data processing," where data are routinely put through certain well-established algorithms to extract some standard parameters. Most traditional data processing methodologies are developed under rigorous mathematic rules; and we pay a price for this strict adherence to mathematical rigor, as described by Einstein. In order not to deviate from mathematical rigor, we are forced to live in a pseudoreality world, in which every process is either linear or stationary and for most cases both linear and stationary.

For example, spectral analysis is synonymous with Fourier-based analysis. Methods based on the Fourier transform are almost synonymous with frequency domain processing of signals. The Fourier transform is essentially an integral over time. Thus, we lose all information that varies with time. All we can tell from the spectrum is that the signal has two distinct frequency components. In other words, we can comment on what happens a signal, not when it happens.

The real world is neither linear nor stationary; thus the inadequacy of the linear and stationary data analysis methods that strictly adhere to mathematical rigor is becoming glaringly obvious.

A popular choice to represent both time and frequency characteristics is the short-time Fourier transform (STFT)[1], which, simply put, transforms contiguous chunks of the input and aggregates the result in a 2 dimensional form, where one axis represents frequency and the other represents time. This representation is quite satisfactory. However, there are a number of reasons why it might not always work. First of all, the short time Fourier transform is parameterized by two important things, other than the signal itself- the number of bins into which the frequency range of the signal is partitioned, and the window function used for smoothing the frequencies.

There are a number of heuristics one can apply to make this representation more reasonable - like tweaking the parameters of the TFT, increasing the sampling frequency of the signal, or to use another time-frequency representation altogether. Unfortunately none of these methods are fully data driven, in that they rely very strongly on a parametric model of the data, and the representation is only as good as the model. A major drawback of time frequency distributions that depend on Fourier or wavelet models is that they don't allow for an "unsupervised" or data driven approach to time series analysis.

2. Motivation for Hilbert Huang Transform

The real world is neither linear nor stationary. A more suitable approach to revealing nonlinearity and nonstationarity in data is to let the data speak for themselves and not to let the analyzer impose irrelevant mathematical rules; that is, the method of analysis should be adaptive to the nature of the data. The combination of the well-known Hilbert spectral analysis (HAS) [2] and the recently developed empirical mode decomposition (EMD) [3], designated as the Hilbert-Huang transform (HHT) by NASA, indeed, represents such a paradigm shift of data analysis methodology. The HHT is designed specifically for analyzing nonlinear and nonstationary data.

The key part of HHT is EMD with which any complicated data set can be decomposed into a finite and often small number of intrinsic mode functions (IMFs). The instantaneous frequency defined using the Hilbert transform denotes the physical meaning of local phase change better for IMFs than for any other non-IMF time series. This decomposition method is adaptive and therefore highly efficient. As the decomposition is based on the local characteristics of the data, it is applicable to nonlinear and nonstationary processes.

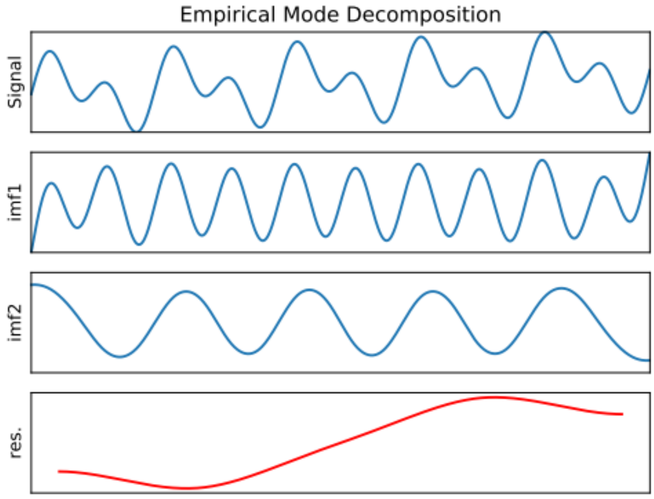

Figure 1: Empirical mode decomposition

The Empirical mode decomposition (EMD) has implicitly a simple assumption that, at any given time, the data may have many coexisting simple oscillatory modes of significantly different frequencies, one superimposed on the other. Each component is defined as an intrinsic mode function (IMF) satisfying the following conditions: (1) In the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one. (2) At any data point, the mean value of the envelope defined using the local maxima and the envelope defined using the local minima is zero. Let m1 be the mean of the lower and the upper envelope data x(t), then

$$h1 = x(t) - m1 \tag{1}$$

By construction, h1 is expected to satisfy the definition of the IMF. However if changing a local zero from a rectangular to a curvilinear coordinate system may introduce new extrema and further adjustments are needed then repeat of the above procedure is applied. The shifting process has to be repeated as many times as is required to make the extracted signal satisfy the definition of an IMF. After k times of iterations.

$$h1k = h1(k-1) - m1k \tag{2}$$

The approximate local envelope symmetry condition is satisfied, and h1k becomes the IMF c1, i.e. c1 = h1k

Hilbert spectral analysis (HSA) s a signal analysis method applying the Hilbert Transform to compute the instantaneous frequency of signals. For any function x(t) its Hilbert transform

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \tag{3}$$

Empirical Mode Decomposition

where P is the Cauchy principal value of the singular integral. With the Hilbert transform y(t) of the function x(t), we obtain the analytic function,

$$z(t) = x(t) + \iota y(t) \tag{4}$$

where $\iota = \sqrt{-1}$

$$a(t) = (x^2 + y^2)^{\frac{1}{2}}, \quad \theta = tan^{-1}\frac{y}{x} \tag{5}$$

Here a is the instantaneous amplitude, and θ is the instantaneous phase function. The instantaneous frequency is simply

$$\omega = \frac{d\theta}{dt} \tag{6}$$

With both amplitude and frequency being a function of time, we can express the amplitude (or energy, the square of amplitude) in terms of a function of time and frequency, H(ω, t). An example of obtaining an IMF from an arbitrarily given time series is displayed in Figure 1. The details regarding the working of the HHT are discussed in the reference [4].

3. Analysis of the Earthquake Motion Dynamics

Hilbert-Huang transform (HHT) can be widely used for analyzing dynamic and earthquake motion recordings [5] in studies of seismology and engineering. Earthquake data are inherently nonstationary because the recordings are the result of propagation of various type waves with different amplitude, frequency, and wave speed in soil media that are likely nonlinear. It should be noted that earthquake recordings are often viewed as nonstationary, nonlinear data. Because earthquake motion data are nonstationary, Fourier spectral analysis of the data fails to capture the energy distribution of events over both time and frequency.

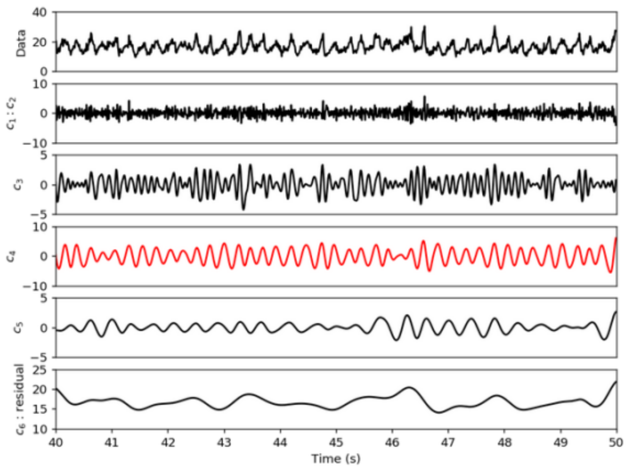

Figure 2: Empirical mode decomposition of the time dependent signal

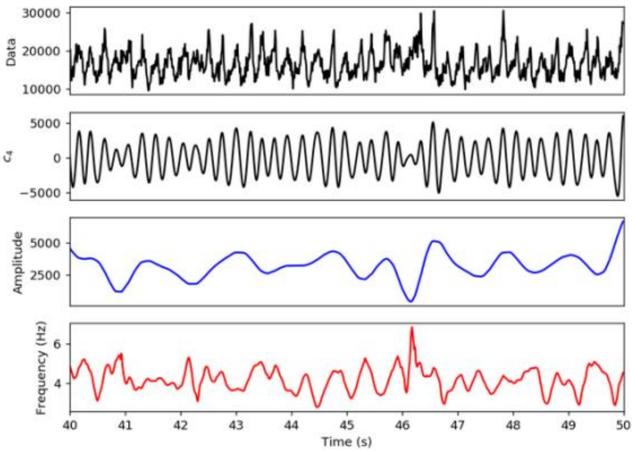

Figure 3: Hilbert Spectral Analysis of the Time dependent signal

In this section, we use the wave recording to illustrate features of HHT analysis in nonstationary data processing, which are also compared with those of traditional approaches for data processing. We now demonstrate the rationale for use of HHT in analyzing the earthquake motion. The data for the earthquake motion has been from the reference [6]. On the data we apply the EMD and the Hiplbert Spectrum as shown the figures 2 and figure 3.

In contrast, the EMD can reveal important characteristics of the waves with just a few IMF components, while the Hilbert spectrum in HSA shows a clear picture of temporal-frequency energy distribution. The distribution of amplitude (energy) in time-frequency domain, H(w, t), in HHT can be regarded as a skeleton form of that in continuous wavelet analysis, which has been widely pursued in the wavelet community.

4. Results and Discussion

In section I, we discussed about the different transformations and their applicability for the time dependent signals. In section II, we presented the need of the Hilbert Huang transform, a brief introduction on it where we found the HHT is composed of the Empirical mode decomposition (EMD) and Hilbert Spectral analysis (HSA), and its dominance over the other transformations. In section III, we studied the earthquake motion dynamics using the Hilbert Huang transform. We found that we can be able to estimate the amplitude and frequency as shown in Figure 2.

5. Conclusion

The different transformations such as Fourier transform, the Fast Fourier transform and the Hilbert Hunag transform were studied in the scope of the time dependent signal. It was found that HHT offers a potentially viable method for nonlinear and nonstationary data analysis, especially for time-frequency -energy representations. It has been tested widely in various applications other than geophysical research. The earthquake motion dynamic was studied using the HHT as an application. In most cases studied HHT gives result much sharper than most of the traditional analysis methods. And in most cases, it reveals true physical meanings. One of the major drawbacks of the EMD is mode mixing like for a signal of a similar scale residing in the different IMF components. In order to make the methods more robust, rigorous and friendlier in application and analytic mathematical foundation is needed.

A confidence [7] limit is always desirable in any statistical analysis, for it provides a measure of reliability of the results. The EMD is an empirical algorithm and involves a prescribed stoppage criterion to carry out the sifting move. Therefore, a confidence limit of the EMD is a desirable quantity.

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Appendix

Code1

frompyhht:visualizationimportplotimfs
frompyhhtimportEMD
importnumpyasnp
$t = np:linspace(0; 1; 1000)$
$modes = np:sin(2 * np:pi * 5 * t) + np:sin(2 * np:pi * 10 * t)$
$x = modes + t$

```
decomposer = EMD(x)
imfs = decomposer:decompose()
plotimfs(x; imfs; t)
```

Code2

```
fromhhtpywrapper:eemdimportEEMD
importmatplotlib:pyplotasplt
importnumpyasnp

tstart = int(np.fix(40 / dt))
tend = int(np.fix(50 / dt))

hinoise = np:sum(eemdpostprocessing:imfs[:; :
2]; axis = 1)
c3 = eemd
postprocessing:imfs[:; 2]
c4 = eemd
postprocessing:imfs[:; 3]
c5 = eemd
postprocessing:imfs[:; 4]
lownoise = np:sum(eemdpostprocessing:imfs[:; 5 :
]; axis = 1)

plt.figure()
plt.subplot(611)            rate[tstart:tend]/1000,
plt.plot(time[tstart:tend],
'k')
plt.xticks([])
plt.yticks([0, 20, 40])
plt.xlim([40, 50])          hinoise[tstart        :
plt.ylabel('Data')
plt.subplot(612)
plt.plot(time[tstart:tend],
tend]=1000;                 0                     k   0   )

plt:xticks([])
plt:yticks([-10; 0; 10])
plt:xlim([40; 50])
plt:ylabel(r0c1 : c2')
plt.subplot(613)
plt.plot(time[tstart:tend], c3[tstart:tend]/1000, 'k')
plt.xticks([])
plt.yticks([-5, 0, 5])
plt.ylabel(r'c3')
plt.xlim([40, 50])
plt.subplot(614)
plt.plot(time[tstart:tend], c4[tstart:tend]/1000, 'r')
plt.xticks([])
plt.yticks([-10, 0, 10])
plt.xlim([40, 50])
plt.ylabel(r'c4')
plt.subplot(615)
plt.plot(time[tstart:tend], c5[tstart:tend]/1000, 'k')
plt.xticks([])
plt.yticks([-5, 0, 5])
plt.xlim([40, 50])
plt.ylabel(r'c5')
plt.subplot(616)
plt.plot(time[tstart:tend], lownoise[tstart :
tend]=1000;0 k0)
plt:yticks([10; 15; 20; 25])
plt:xticks(np:arange(40; 51))
plt:xlim([40; 50])
plt:xlabel(0Time(s)0)
plt:ylabel(r0c6 : residual')
plt.show()
```