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Stresses of Transient Thermoelastic Problem in an Elliptical Cylinder

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Abstract: ***This paper is concerned with thermal stresses in an Elliptical Cylinder to determine the temperature gradient and stresses at any point of the Elliptical Cylinder. The integral transform techniques are used to find the solution of the problem. The results are expressed in terms of Bessel's function in the form of infinite series.***

Keywords: Elliptical Cylinder, Stresses, Marchi-Zgrablich Transform, Marchi-Fasulo Transform

1. Introduction

Grysa and Kozlowski (1982) have discussed one dimensional thermoelastic problems derived the heating temperature and the heat flux on the surface of an isotropic infinite slab and **Sirakowski and Sun** (1968) have studied the direct problems of finite length hollow cylinder and determined an exact solution. **Grysa** and **Cialkowski** (1980) and Further **Deshmukh** and **Wankhede** (1997) have studied an axisymmetric inverse steady state problem of thermoelastic deformation to determine the temperature, displacement and stress functions on the outer curved surface of finite length hollow cylinder. In this paper, an attempt has been made to determine the temperature gradient and stresses at any point of the cylinder.

2. Statement of the Problem

Consider a Elliptical Cylinder of length $2h$ occupying space D defined by $a \le r \le b$, $-h \le z \le h$. The thermoelastic displacement function is governed by the Poisson's equation [5]

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \frac{(1+\nu)}{(1-\nu)} a_t \varsigma \tag{1}$$

with $\phi = 0$ at $r = a$ and $r = b$ (2)

ν and a_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder. Consider the ς is the temperature of the cylinder and satisfying the differential equation,

$$\frac{\partial^2 \varsigma}{\partial r^2} + \frac{1}{r}\frac{\partial \varsigma}{\partial r} + \frac{\partial^2 \varsigma}{\partial z^2} = \frac{1}{k}\frac{\partial \varsigma}{\partial t} \tag{3}$$

Subject to the initial condition

$$\varsigma(r,z,t)\ \big|_{t=0} = F(r,z) \tag{4}$$

the boundary conditions

$$\left[\varsigma + k_1 \frac{\partial \varsigma}{\partial r}\right]_{r=a} = 0 \tag{5}$$

$$\left[\varsigma + k_2 \frac{\partial \varsigma}{\partial r}\right]_{r=b} = 0 \tag{6}$$

$$\left[\varsigma + \frac{\partial \varsigma}{\partial z}\right]_{z=-h} = g_1(r,t) \tag{7}$$

$$\left[\varsigma + \frac{\partial \varsigma}{\partial z}\right]_{z=h} = g_2(r,t) \tag{8}$$

Where, k is the thermal diffusivity of the material of the cylinder.

The radial and axial displacements U and W satisfying the uncoupled thermoelastic equations are

$$\nabla^2 U - \frac{U}{r^2} + (1+2\nu)^{-1}\frac{\partial l}{\partial r} = 2\frac{(1+\nu)}{(1-2\nu)} a_t \frac{\partial \varsigma}{\partial r} \tag{9}$$

$$\nabla^2 W + (1+2\nu)^{-1}\frac{\partial l}{\partial z} = 2\frac{(1+\nu)}{(1-2\nu)} a_t \frac{\partial \varsigma}{\partial z} \tag{10}$$

Where $l = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilatation and

$$U = \frac{\partial \phi}{\partial r} \tag{11}$$

$$W = \frac{\partial \phi}{\partial z} \tag{12}$$

The stress functions are given by

$$\tau_{rz}(a,z,t) = 0,\ \tau_{rz}(b,z,t) = 0,\quad \tau_{rz}(r,z,0) = 0, \tag{13}$$

And

$$\sigma_r(a,z,t) = p_1,\ \sigma_r(b,z,t) = -p_0,\ \sigma_z(r,0,t) = 0, \tag{14}$$

Where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The boundary conditions for the stress functions (13) and (14) are expressed in terms of the displacement components by the following relations:

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left[\frac{U}{r} + \frac{\partial W}{\partial z}\right] \tag{15}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{\partial U}{\partial r} + \frac{U}{r}\right] \tag{16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \tag{17}$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \tag{18}$$

Where $\lambda = \dfrac{2G\nu}{1-2\nu}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components.

3. Solution of the Problem

The equations (1) to (18) constitute the mathematical formulation of the problem under consideration. Applying finite Marchi-Zgrablich transform [4] to the equation (3), one obtains

$$-\mu_m^2 \bar{\varsigma} + \frac{d^2\bar{\varsigma}}{dz^2} = \frac{1}{k}\frac{d\bar{\varsigma}}{dt} \tag{19}$$

Further applying finite Marchi-Fasulo transform [6] to the equation (19), one obtains

$$\frac{d\bar{\varsigma}^*}{dt} + kp^2\bar{\varsigma}^* = k\omega(m,t) \tag{20}$$

where $p^2 = a_n^2 + \mu_m^2$

$$\omega(m,t) = \frac{P_n(h)}{\alpha_1}\bar{g}_2(m,t) - \frac{P_n(-h)}{\alpha_2}\bar{g}_1(m,t)$$

Equation (20) is the first order differential equation, whose solution is given by

$$\bar{\varsigma}^* = l^{-kp^2t}[\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt'] \tag{21}$$

Where, X is constant.

Applying inversion of finite Marchi-Fasulo transform and Marchi-Zgrablich transform to the equation (21)

We obtain

$$\varsigma = \sum_{m,n=1}^{\infty} \frac{S_0(k_1,k_2,\mu_m r)P_n(z)}{\mu_m \lambda_n} l^{-kp^2t} \times [\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt'] \tag{22}$$

4. Determination of Thermoelastic Displacement

Substituting the value of $\varsigma(r,z,t)$ from equation (22) in equation (1), one obtains the thermoelastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t) = \frac{a_t r^2}{4}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)P_n(z)}{\mu_m\lambda_n} \times l^{-kp^2t}(\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \tag{23}$$

Using equation (23) in equation (11) and (12), one obtains the radial and axial displacement U and W as

$$U = \frac{a_t}{4}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{P_n(z)}{\mu_m\lambda_n} l^{-kp^2t} (\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \times [r^2 S_0'(k_1,k_2,\mu_m r) + 2rS_0(k_1,k_2,\mu_m r)] \tag{24}$$

$$W = \frac{a_t r^2}{4}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m\lambda_n}P_n'(z)\, l^{-kp^2t} \times (\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \tag{25}$$

Equation (24) and (25) are the radial and axial displacement.

5. Determination of Stress Functions

Using equations (24) and (25) in equations (15) to (18), the stress functions are obtained as

$$\begin{aligned}\sigma_r = {} & \frac{a_t}{4}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{l^{-kp^2t}}{\mu_m\lambda_n}(\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \\ & \times\{(\lambda+2G)P_n(z)[r^2 S_0''(k_1,k_2,\mu_m r) + 4rS_0'(k_1,k_2,\mu_m r) \\ & + 2S_0(k_1,k_2,\mu_m r)] + \lambda[P_n(z)(rS_0'(k_1,k_2,\mu_m r) + 2S_0(k_1,k_2,\mu_m r)] + \\ & r^2 P_n''(z)S_0(k_1,k_2,\mu_m r)]\}\end{aligned} \tag{26}$$

$$\begin{aligned}\sigma_z = {} & \frac{a_t}{4}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{l^{-kp^2t}}{\mu_m\lambda_n}(\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \\ & \times\{(\lambda+2G)r^2 P_n''(z)S_0(k_1,k_2,\mu_m r) + \lambda P_n(z)[r^2 S_0''(k_1,k_2,\mu_m r) \\ & + 5rS_0'(k_1,k_2,\mu_m r) + 4S_0(k_1,k_2,\mu_m r)]\}\end{aligned} \tag{27}$$

$$\begin{aligned}\sigma_\theta = {} & \frac{a_t}{4}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{l^{-kp^2t}}{\mu_m\lambda_n}(\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \\ & \times\{(\lambda+2G)P_n(z)(rS_0'(k_1,k_2,\mu_m r) + 2S_0(k_1,k_2,\mu_m r) \\ & + \lambda[P_n(z)(r^2 S_0''(k_1,k_2,\mu_m r) + 4rS_0'(k_1,k_2,\mu_m r) \\ & + 2S_0(k_1,k_2,\mu_m r)) + P_n''(z)S_0(k_1,k_2,\mu_m r)]\}\end{aligned} \tag{28}$$

$$\begin{aligned}\tau_{rz} = {} & \frac{Ga_t}{2}\frac{(1+\nu)}{(1-\nu)}\sum_{m,n=1}^{\infty}\frac{P_n'(z)}{\mu_m\lambda_n}l^{-kp^2t}(\bar{F}^* + X + k\int_0^t \omega(m,t')l^{kp^2t'}dt') \\ & \times(r^2 S_0'(k_1,k_2,\mu_m r) + 2rS_0(k_1,k_2,\mu_m r)]\end{aligned} \tag{29}$$

Equation (27), (28) and (29) are the thermal stresses functions.

6. Conclusion

In this paper, we discussed completely the inverse unsteady-state problem of thermoelastic deformation of an elliptical cylinder for upper plane surface where the temperature is maintained at zero on the curved surface and the lower plane surface of the cylinder. The temperature, displacement and thermal stresses that are obtained by using the integral transform techniques.

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