Generalised Model of Optimum Stratification Allocation Size and Variance for Linear and Non-Linear Cost Function Considering Other Allocation Procedures

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Abstract: ***In this paper, consideration is given to a generalised model for optimum allocation of multivariate sampling with linear and nonlinear cost function, given that there is only one varying cost.. The problem of determining the optimum allocation are formulated as a generalized programming problem and solved using Lagrange multiplier technique to minimize the variance subject to a given cost. A generalized sample size and variance is derived and other factors like when the cost is constant, when the sample size for a given cost is not known, when the sample size for a specified variance is not known and when a proportional allocation or equal allocation is required for a given cost and a proportionality constant. Data from the sample frame of the final result of 1991 census of Owo local government area of Ondo State, Nigeria which encompasses of 117 localities was extracted from the National Population Commission (NPC), its inhabitants is used as character of study for the purpose of numerical illustration.***

Keywords: Optimum Allocation, Compromise Allocation, Cost Functions, Sample Size, Proportionality Constant

1. Introduction

The development of statistical methodology used in survey sampling is aimed at estimating the mean or a particular character under study with high precision or least cost, the concept of stratification is used in statistical surveys when the subpopulations within an overall population is heterogeneous and it will be of advantage to sample each stratum independently. The problem of optimal allocation was initially proposed by Sukhatme et al. in 1984 in other to minimize the cost for a given precision or minimize the variance of the sample mean subject to cost.

When considering the method of stratification in sampling, certain issues are taking into consideration

1) How many strata are required
2) How boundaries will be set for each stratum and
3) Optimum allocation of sample size to each stratum

Optimum allocation in stratified random sampling is well known for a univariate population (Cochran 1977). The problem of determining strata boundaries in multivariate surveys was considered by Ashan et al. in 1983. When more than one character is defined on each unit of the population, it will not be feasible to use the individual optimum allocation to the strata unless there is a strong positive correlations between the characteristics under study, one have to use an allocation that is optimum in some sense for all the characters, which will result into a compromise allocation.

The cost of enumerating a character from stratum to stratum may also differ since characters differ, what is optimum for one character may not be optimum for other character Shazia Ghufran et al. 2011.

When the cost of transportation is considered with the cost of sampling, The cost function is of the form

$$C = c_0 + \sum_{n=1}^{L} (c_h n_h) + \sum_{n=1}^{L} \left(t_h \sqrt{n_h}\right)$$

Where c_0 is the overhead cost, c_h is the cost of enumerating a character per unit, n_h is the sample size in various strata and t_h is the cost of transportation or travel cost per unit. This is a nonlinear cost function.

Mohd Vaseem Ismail et al. in 2015 considered optimum allocation for multivariate sampling with nonlinear cost function, determination of optimum allocation was formulated as a nonlinear programming problem and solved for case of substantial travel cost inclusive using Lagrange multiplier technique. Manoj Kumar Sharma et al in 2015 also developed a MATLAB computer program to solve the same problem. If the travel cost between units are substantial the cost function is approximately

$$C = c_0 + \sum_{n=1}^{L} \left(t_h \sqrt{n_h}\right)$$

Shazia Ghufran et al. in 2011 used a different approach, the method of chanced constrained programming in considering optimum compromise allocation in multivariate stratified sampling with nonlinear function, NA Sofi et al. 2016 considered an integer programming technique known as branch and bound approach to obtain an optimum integer solution for a linear cost function in case of small samples to avoid rounding off allocations so as to get an optimal and feasible allocation, Khan et al 2003 in order to determine an optimum compromise allocation treated nonlinear programming problem as multistage decision problem and solved using dynamic programming technique, S.Tugba Sahin et al. 2011 to solve the problem of selecting sample size from strata under nonlinear cost constraint, applied Kunn-Tucker methods and goal programming constraint in case of multi objective decision making problems, swain et al. 2013 also solved the problem of a compromise optimum allocation using goal programming technique.

In this paper we will be considering various allocation procedures using a generalised model of optimum allocation size and variance for linear cost function and specified nonlinear by using langrage multiplier technique to solve.

2. Problem Formulation

The main idea is to derive a generalized model for linear and nonlinear cost function to get the optimum stratum sample size provided we have only one varying cost and other cost incurred are fixed or not significant. Suppose c_0 represents the overhead cost and c_h is the cost of enumerating a character per unit.

For a linear cost, the total cost incurred
C = c_0 + $\sum_{n=1}^{L}(c_h n_h)$

If travel cost per stratum t_h is the only varying cost and highly significant, the cost function becomes

$$C = c_0 + \sum_{n=1}^{L}\left(t_h\sqrt{n_h}\right)$$

which is nonlinear.

Hence we can say,

$$C = c_0 + \sum_{n=1}^{L}\left(c_h n_h{}^{\beta}\right),$$

where β is the proportionality constant defined on a set of real numbers, such that β = ½ for a case in which it is only travel cost $t_h = c_h$ that is incurred, and β = 1 for a case of simple linear cost function.

We either minimize the cost function for a given precision or minimize the variance of the sample mean subject to cost the function

Hence, let $V(\bar{y}_{str})$ be the optimization function, given a proportionality constant β, let the total cost incurred C be the constraint function

Then we minimize

$$V(\bar{y}_{str}) = \sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{n_h}\right) - \sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{N_h}\right),$$

Subject to $C = c_0 + \sum_{n=1}^{L}\left(c_h n_h{}^{\beta}\right)$,

$n_h \geq 1$.

Using Lagrange multiplier technique, let $G(n_h, \lambda) = V(\bar{y}_{str}) + \lambda\left[c_0 + \sum_{n=1}^{L}\left(c_h n_h{}^{\beta}\right) - C\right]$

Differentiating G with respect to n_h,

$$\frac{\delta G}{\delta n_h} = -\sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{n_h{}^2}\right) + \lambda\beta\sum_{h=1}^{L}\left(c_h n_h{}^{\beta-1}\right)$$

Setting the above equation to zero

$$\sum_{h=1}^{L}\left[\lambda\beta\left(c_h n_h{}^{\beta-1}\right) - \left(\frac{W_h{}^2 S_{hj}{}^2}{n_h{}^2}\right)\right] = 0$$

This implies $\lambda\beta\left(c_h n_h{}^{\beta-1}\right) - \left(\frac{W_h{}^2 S_{hj}{}^2}{n_h{}^2}\right) = 0$

$$\lambda\beta\left(c_h n_h{}^{\beta-1}\right) = \left(\frac{W_h{}^2 S_{hj}{}^2}{n_h{}^2}\right);$$

$$n_h{}^{\beta+1} = \frac{1}{\lambda}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right);$$

Making n_h the subject of formula

$$n_h = \left(\frac{1}{\lambda}\right)^{1/(\beta+1)}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)} \quad (1)$$

Recall that $\sum_{h=1}^{L} n_h$ = n; summing up (1)

$$n = \left(\frac{1}{\lambda}\right)^{1/(\beta+1)}\sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)} \quad (2)$$

from (1) & (2) we can deduce that

$$\frac{n_h}{n} = \frac{\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)}}{\sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)}};$$

$$n_h = \frac{n\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)}}{\sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)}};$$

$$= \frac{n\,\frac{W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{c_h{}^{1/(1+\beta)}}}{\sum_{h=1}^{L}\frac{W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{c_h{}^{1/(1+\beta)}}} \quad (3)$$

Where n_h in equation (3) is the sample size of the h[th] stratum given the proportionality constant β recall that

$$V(\bar{y}_{str}) = \sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{n_h}\right) - \sum_{h=1}^{L}\left(\frac{W_h{}^2 S_{hj}{}^2}{N_h}\right);$$

Substituting n_h *in* equation (3) to $V(\bar{y}_{str})$

$$V(\bar{y}_{str}) = \frac{1}{n}\sum_{h=1}^{L}\left(\frac{W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{c_h{}^{1/(1+\beta)}}\right)\sum_{h=1}^{L}\left(W_h{}^{2\beta/(1+\beta)} S_{hj}{}^{2\beta/(1+\beta)} c_h{}^{1/(1+\beta)}\right) - \frac{1}{N}\sum_{h=1}^{L}\left(W_h S_{hj}{}^2\right) \quad *$$

Which is the generalised variance provided we have only one varying cost and other cost incurred are fixed or not significant given a proportionality constant β.

3. Case of Fixed Cost

If the cost c_h of collecting information from h[th] stratum is constant given a proportionality constant β; $c_h = c$

$$C = c_0 + c\sum_{n=1}^{L} n_h{}^{\beta}$$

Hence;

$$n_h = \frac{n\frac{W_h^{\frac{2}{1+\beta}} S_{hj}^{\frac{2}{1+\beta}}}{c_h^{\frac{1}{1+\beta}}}}{\sum_{h=1}^{L}\frac{W_h^{\frac{2}{1+\beta}} S_{hj}^{\frac{2}{1+\beta}}}{c_h^{\frac{1}{1+\beta}}}};$$

$$= \frac{\frac{n}{c^{1/(1+\beta)}} W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{\frac{1}{c^{1/(1+\beta)}}\sum_{h=1}^{L} W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}};$$

$$n_h = \frac{n \quad W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{\sum_{h=1}^{L} W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}};$$

and

$$V(\bar{y}_{str})$$

$$= \frac{1}{n}\sum_{h=1}^{L}\left(W_h{}^{2/(\beta+1)} S_{hj}{}^{2/(\beta+1)}\right)\sum_{h=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)}\right) - \frac{1}{N}\sum_{h=1}^{L}\left(W_h S_{hj}{}^2\right)$$

Where n_h is the sample size of the h[th] stratum, c is the fixed cost and $V(\bar{y}_{str})$ is the variance given the proportionality constant β

4. Case of Unknown Sample Size for a Given Cost

Given a proportionality constant β,

$$C = c_0 + \sum_{n=1}^{L}\left(c_h n_h{}^{\beta}\right),$$

$$C - c_0 = \sum_{n=1}^{L}\left(c_h n_h{}^{\beta}\right)$$

From equation (1)

$$n_h = \left(\frac{1}{\lambda}\right)^{1/(\beta+1)}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{1/(\beta+1)};$$

$$n_h{}^{\beta} = \left(\frac{1}{\lambda}\right)^{\beta/(\beta+1)}\left(\frac{W_h{}^2 S_{hj}{}^2}{\beta c_h}\right)^{\beta/(\beta+1)} \quad (4)$$

Substituting equation (4) into C - c_0

C - c_0

$$= \left(\frac{1}{\lambda}\right)^{\beta/(\beta+1)}\left(\frac{1}{\beta}\right)^{\beta/(\beta+1)}\sum_{n=1}^{L}\left(c_h{}^{1/(\beta+1)}\left(W_h{}^2 S_{hj}{}^2\right)^{\beta/(\beta+1)}\right)$$

(5)

From equation (4)

$$\left(\frac{1}{\lambda}\right)^{\beta/(\beta+1)} = n_h{}^{\beta}(\beta)^{\beta/(\beta+1)}\left(\frac{c_h}{W_h{}^2 S_{hj}{}^2}\right)^{\beta/(\beta+1)}; \quad (6)$$

Substituting (6) into (5)

$$C - c_0 = n_h{}^{\beta}(\beta)^{\beta/(\beta+1)}\left(\frac{c_h}{W_h{}^2 S_{hj}{}^2}\right)^{\beta/(\beta+1)}\left(\frac{1}{\beta}\right)^{\beta/(\beta+1)}\sum_{n=1}^{L}\left(c_h{}^{1/(\beta+1)}\left(W_h{}^2 S_{hj}{}^2\right)^{\beta/(\beta+1)}\right);$$

$$C - c_0$$

$$= n_h{}^{\beta}\left(\frac{c_h}{W_h{}^2 S_{hj}{}^2}\right)^{\beta/(\beta+1)}\sum_{n=1}^{L}\left(c_h{}^{1/(\beta+1)}\left(W_h{}^2 S_{hj}{}^2\right)^{\beta/(\beta+1)}\right)$$

Making $n_h{}^{\beta}$ subject of formula

$$n_h{}^{\beta} = \left(\frac{W_h{}^2 S_{hj}{}^2}{c_h}\right)^{\beta/(\beta+1)}\frac{C - c_0}{\sum_{n=1}^{L}\left(c_h{}^{1/(\beta+1)}\left(W_h{}^2 S_{hj}{}^2\right)^{\beta/(\beta+1)}\right)};$$

This implies

$$n_h = \left(\frac{W_h{}^2 S_{hj}{}^2}{c_h}\right)^{1/(\beta+1)}\left(\frac{C - c_0}{\sum_{n=1}^{L}\left(c_h{}^{1/(\beta+1)}\left(W_h{}^2 S_{hj}{}^2\right)^{\beta/(\beta+1)}\right)}\right)^{1/\beta};$$

Recall that

n = $\sum_{n=1}^{L} n_h$

Hence;

$$n = \sum_{n=1}^{L} n_h = (C - c_0)^{1/\beta}\frac{\sum_{n=1}^{L}\left(W_h{}^{2/(\beta+1)} S_{hj}{}^{2/(\beta+1)} c_h{}^{-1/(\beta+1)}\right)}{\left[\sum_{n=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)} c_h{}^{1/(\beta+1)}\right)\right]^{1/\beta}}$$

5. Case of Unknown Sample Size for a Specified Variance V_0

Given a proportionality constant β,

$$V_0 = \frac{1}{n}\sum_{n=1}^{L}\left(W_h{}^{2/(\beta+1)} S_{hj}{}^{2/(\beta+1)} c_h{}^{-1/(\beta+1)}\right)\sum_{n=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)} c_h{}^{1/(\beta+1)}\right) - \frac{1}{N}\sum_{h=1}^{L} W_h S_{hj}{}^2$$

This implies

$$V_0 + \frac{1}{N}\sum_{h=1}^{L} W_h S_{hj}{}^2 = \frac{1}{n}\sum_{n=1}^{L}\left(W_h^{\frac{2}{\beta+1}} S_{hj}^{\frac{2}{\beta+1}} c_h^{-\frac{1}{\beta+1}}\right)\sum_{n=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)} c_h{}^{1/(\beta+1)}\right);$$

Hence;

$$n = \frac{\sum_{n=1}^{L}\left(W_h{}^{2/(\beta+1)} S_{hj}{}^{2/(\beta+1)} c_h{}^{-1/(\beta+1)}\right)\sum_{n=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)} c_h{}^{1/(\beta+1)}\right)}{V_0 + \frac{1}{N}\sum_{h=1}^{L} W_h S_{hj}{}^2}$$

if cost is constant for a specified variance V_0;

$$n = \frac{\sum_{n=1}^{L}\left(W_h{}^{2/(\beta+1)} S_{hj}{}^{2/(\beta+1)}\right)\sum_{n=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)}\right)}{V_0 + \frac{1}{N}\sum_{h=1}^{L} W_h S_{hj}{}^2};$$

6. Case of Proportional Allocation for A Given Cost C

If the sampler decides to allocate the sample sizes based on set proportion for each stratum given a proportionality constant β,

Then

$$n_h = nw_h$$

Recall that

$C = c_0 + \sum_{n=1}^{L}(c_h n_h{}^{\beta});$

$C - c_0 = \sum_{n=1}^{L} c_h n_h{}^{\beta};$

Substituting n_h as nw_h into $C - c_0$;

$C - c_0 = \sum_{n=1}^{L} c_h (nw_h{}^{\beta});$

$C - c_0 = n^{\beta} \sum_{n=1}^{L} c_h w_h{}^{\beta};$

Making n^{β} subject of formula

$$n^{\beta} = \frac{C - c_0}{\sum_{n=1}^{L} c_h w_h{}^{\beta}};$$

Hence;

$$n = \left(\frac{C - c_0}{\sum_{n=1}^{L} c_h w_h{}^{\beta}}\right)^{1/\beta}$$

7. Case of Equal Allocation for a Given Cost C

If the sampler decides to partition each stratum to L disjoint stratum given a proportionality constant β,

Then;

$$n_h = \frac{n}{L};$$

Recall that;

$$C = c_0 + \sum_{n=1}^{L}(c_h n_h{}^{\beta});$$

$$C - c_0 = \sum_{n=1}^{L} c_h n_h{}^{\beta};$$

Substituting n_h as $\frac{n}{L}$

$$C - c_0 = \sum_{n=1}^{L} c_h \left(\frac{n}{L}\right)^{\beta};$$

$= \left(\frac{n}{L}\right)^{\beta} \sum_{n=1}^{L} c_h;$

$= \frac{n^{\beta}}{L^{\beta}} \sum_{n=1}^{L} c_h;$

Making n^{β} subject of formula

$$n^{\beta} = \frac{L^{\beta}(C - c_0)}{\sum_{n=1}^{L} c_h};$$

Hence;

$$n = L\left[\frac{(C - c_0)}{\sum_{n=1}^{L} c_h}\right]^{1/\beta}$$

This is applicable when the sample size for a given cost is not known.

8. Numerical Illustration

Stratum h	W_h	c_h	S_{hj}	$S_h{}^2$
1	0.726	₦2	104.12	10840.83
2	0.274	₦4	143.54	20605.2
Total	1.000	₦6		

N = 117, N_1 = 85, N_2 = 32, assuming n = 48, C = ₦130 and c_0= ₦10

Recall that the generalised stratum size given the proportionality constant β

$$n_h = \frac{n \frac{W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{c_h{}^{1/(1+\beta)}}}{\sum_{h=1}^{L} \frac{W_h{}^{2/(1+\beta)} S_{hj}{}^{2/(1+\beta)}}{c_h{}^{1/(1+\beta)}}}$$

For $\beta = 1$;

$$n_h = \frac{n \frac{W_h S_h}{c_h{}^{1/2}}}{\sum_{h=1}^{L} \frac{W_h S_h}{c_h{}^{1/2}}};$$

$$n_1 = \frac{48(53.45)}{53.54+19.66} = \frac{2565.6}{73.12} = 35;$$

$$n_2 = \frac{48(19.66)}{53.54+19.66} = \frac{934.68}{73.12} = 13$$

The generalised variance given the proportionality constant β

$$V(\bar{y}_{str}) = \frac{1}{n} \sum_{h=1}^{L} \left(\frac{W_h{}^{2/(\beta+1)} S_{hj}{}^{2/(\beta+1)}}{c_h{}^{1/(\beta+1)}}\right) \sum_{h=1}^{L}\left(W_h{}^{2\beta/(\beta+1)} S_{hj}{}^{2\beta/(\beta+1)} c_h{}^{1/(\beta+1)}\right) - \frac{1}{N}\sum_{h=1}^{L}(W_h S_{hj}{}^2)$$

If $\beta = 1$

$$V(\bar{y}_{str}, \beta{=}1) = \frac{1}{n} \sum_{h=1}^{L} \left(\frac{W_h S_{hj}}{c_h{}^{1/2}}\right) \sum_{h=1}^{L} W_h S_{hj} c_h{}^{1/2} - \frac{1}{N}\sum_{h=1}^{L}(W_h S_{hj}{}^2)$$

$= \frac{1}{48}(73.12)(185.56) - \frac{1}{117}(13516.27)$

=282.67- (115.52)

=167.15

This implies that there is a very high variance between the number of inhabitants of the localities of Owo local government.

If we test for β = 1 in few cases,

For example;

1. If the sampler decides that the strata should be divided into two equal parts for a given cost

Then L=2

$$n = L\left[\frac{(C - c_0)}{\sum_{n=1}^{L} c_h}\right]^{1/\beta}$$

for β =1,

$$n = L \frac{(C - c_0)}{\sum_{n=1}^{L} c_h}$$

$$= 2\frac{120}{6} = 40$$

2. If the sampler decides that the ratio of the 1st stratum to the second stratum is 7:3 for a given cost

Then $w_1 = 0.7$, $w_2 = 0.3$

$$n = \left(\frac{C - c_0}{\sum_{n=1}^{L} c_h w_h{}^{\beta}}\right)^{1/\beta}$$

for $\beta = 1$,

$$n = \frac{C - c_0}{\sum_{n=1}^{L} c_h w_h}$$

$$= \frac{120}{2.6} = 46.15 \approx 46$$

9. Conclusion

Stratification reduces variance or inequalities in a bid to reduce sampling error or increase precision, the objective of a sampler is to have the optimum precision, hence we have been able to derive a general model for a linear or nonlinear cost function given a proportionality constant β for stratum sample size and variance provided that we have only one varying cost and other cost incurred are fixed or not significant, considering factors like when cost is constant in all strata and when sample size for a given cost of the survey or for a specified variance is not known.

References

[1] Adebola, F.B., and Ajayi. (2014). "Comparison of allocation procedure in stratified random sampling of skewed populations under different distributions and sample sizes". *International Journal of innovative science, engineering and technology*. Volume 1 issue 8.

[2] Apantaku, F.S. (2014). "Comparing the Optimal allocation in Stratified and Post-stratified sampling using multi items". *Journal of natural science, engineering and technology*.

[3] Ashan, M.J. and Khan, S.U. (1982). "Optimum allocation in multivariate stratified random sampling with over head cost". Metrika 29, 71-78.

[4] Ashan, M.J. and Khan, S.U.(1983). "Minimizing a nonlinear function arising in stratification through approximation by a quadratic function". *Journal of Ind. Society Statist. Operation Research*, Vol. 4, 1-4.

[5] Bakhshi, Z.H., Khan, M.F. and Ahmad, Q.S. (2010). "Optimal sample numbers in multivariate stratified sampling with a probabilistic cost constraint", *International journal of mathematics and applied statistics*. Volume 1, No. 2, pp. 111-120.

[6] Beardwood, J., Halton, J.H and Hammersley J.M. (1959). "The shortest path through many points". Mathematical proceedings of the Cambridge philoso No. 4pp. 299-327.

[7] Chernayak, O.L. and Chornous, G. (2000). "Optimal allocation in stratified sampling with a nonlinear cost function". *Theory of stochastic process*, Vol. 6 (22), 3-4, pp. 6-17.

[8] Dalenius, T. and Hodges, J.L. (1959), "Minimum variance stratification". *Journal American statistical association*. Vol. 54, 88-101.

[9] Dantzig, G.B, Fulkerson, D.R. and Johnson, S.M. (1954). "Solution of a largescale travelling salesman problem", *Operation research*, Vol. 2, pp. 393-410.

[10] Diaz-Garcia, J.A. and Cortez, L.U. (2006). "Optimum allocation in multivariate stratified sampling, multi objective programming". Comunicacion Tecnica No. I-06-07/28-03-206.

[11] Fabian, C.O. (2002). "Sample survey theory with application". Afro-orbis publication LTD, ISBN 978-35253-7-9.

[12] Faisal M., Ali I., Singh Y and Bari A., (2012). "Allocation in multivariate stratified surveys with nonlinear random cost function". *American journal of operations research*, Vol. 2, 100-105.

[13] Ghufran, S., Khowaja, S. and Ashan M.J. (2011). "Multi objective optimal allocation problem with probabilistic nonlinear constraint". *International journal of engineering, science and technology* Volume 3, No. 6, pp 135-145.

[14] Ghufran, S., Khowaja, S. and Ashan M.J. (2012). "Optimum multivariate stratified sampling design with travel cost, A multivariate Integer nonlinear programming approach". *Communication in statistics simulation and computation*. Vol. 41, 598-610.

[15] Javed, Bakhshi, Z.H. and Khalid, M.M. (2009). "Optimum allocation in stratified sampling with random cost". *International review of pure and applied mathematics*. Volume. 5, No. 2, pp. 363-370.

[16] Khan, M.G.M., Ahmad, N. and Khan, S. (2009). "Determining the optimum stratum boundaries using mathematical programming". *Journal of mathematical modelling and algorithm, springer* Netherland. DOI 10.1007/s10852-009-9115-3, 8(4), 409-423.

[17] Khan, M.G.M, Khan, E.A. and Ashan, M.J. (2008). "Optimum allocation in multivariate stratified sampling in presence of nonresponse". *Journal of Indian society of Agricultural statistics*, Vol. 59(2), 146-150.

[18] Khan, M.G.M., Khowaja, S., Ghufran, S., Varsheney, R. and Ashan, M.J. (2010). "A comparative study of various compromise criteria in multiple qualitative response stratified sampling". *Journal of statistics sciences*, Volume.2, No.1, pp. 1-2.

[19] Khowaja, S., Ghufran, S. and Ashan, M.J. (2011). "Estimation of population means in multivariate stratified random sampling". *Communication in Statistics simulation and computation*. Vol. 40:5, 710-718.

[20] Kokan, A.R. and Khan, S.U. 1(967). "Optimum allocation in multivariate surveys, an analytical solution". *Journal of the royal statistics society series*Vol. B.29, 115-125.

[21] Kozak, M. (2006)."Sample allocation in multivariate surveys". *Communication in statistics simulation and computation*. Vol. 35(4), 901-910.

[22] Kumar M., Vaseem M. and Kumar V., (2015). "Computer programming in MATLAB for optimum allocation for multivariate stratified sampling". *International Journal of computer application*. Volume 117, No. 10, 0975-8887.

[23] Naz S., Shad Y. and Shabbir J., (2014). "Compromise allocation for mean estimation in stratified random sampling using auxiliary attributes when some observations are missing". *International Journal of business and social science*. Volume 5, No. 13.

[24] Neyman, J. (1934). "The two different aspects of the representative's methods, the method stratified sampling and the method of purposive selection". *Journal of the royal statistics society*. 97, 558-606.

[25] Olayiwola, O.M. (2013). "Efficiency of Neyman allocation procedure over other allocation procedures in stratified random sampling". *American journal of theoretical and applied statistics*. Vol. 2(5): 122-127.

[26] Rizvi, S.E.H., Gupta, J.P. and Bhargava, M. (2002). "Optimum stratification based on auxiliary variable foe compromise allocation". Metron, Vol. 28(1), 201-215.

[27] Sahin, S.T. (2011). "Determination of sample size selecting from strata under nonlinear cost constraint by using goal programming and khun-tucker methods". *Gazi university journal of science*, Vol. 24(2), 249-262.

[28] Sofi, N.A., Ahmad A., Maqbool, S. and Bhat Bilal Ahmad. (2016). "A branch and bound approach to optimal allocation in stratified sampling". *Mathematical theory and modelling*. Volume 6, No 4.

[29] Sukhatme, P.V., Sukhatme, B.V., Sukhatme, and Asok, C. (1984). "Sampling with varying probabilities". Ann. Inst. Stat. Math., Vol. 24, 485-494.

[30] Swain. (2013). A note on optimal allocation in stratified random sampling. Revista investigacion operational. Volume 34. No. 2, 161-165.

[31] Ullah A., Sabbir J.,Hussain Z., and Al-zahrani, B. (2014). "Estimation of finite population mean in multivariate stratified sampling under cost function using goal programming". *Journal of applied mathematics*. Article ID686579.

[32] Wright, T. (2012). "The equivalence of Neyman allocation for sampling and equal proportions for apportioning the U.S. house of representatives". The American Statistician, Volume 66, No. 4, 212-224.

[33] Vaseem M., Kumar M., Khan E.A. and Naseer K.,(2015). "A note on optimum allocation with nonlinear cost function". *International journal of applied information system*. Volume 8. No. 2.