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Estimating and Forecasting Bitcoin Daily Returns Using ARIMA-GARCH Models

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Abstract: ***This paper provides an evaluation of the forecasting performance of hybrid ARIMA-GARCH model in forecasting Bitcoin daily price returns. We combined ARIMA and GARCH model with Normal, Student's t and Skewed student's t distributions. To make the series stationary, Bitcoin daily price data was transformed to Bitcoin daily returns. By using Box-Jenkins method, the appropriate ARIMA model was obtained and for capturing volatilities of the returns series GARCH (1,1) models with Normal, Student's t and Skewed student's t distributions was used. To evaluate the performance of the models, the study employs two measures, RMSE and MAE. The results reveal that ARIMA (2,0,1)-GARCH (1,1) with Normal distribution outperform the other three in terms of out-of-sample forecast with minimum RMSE and MAE.The findings can aid investors, market practitioners, financial institutions, policymakers, and scholars.***

Keywords: ARIMA, GARCH, Bitcoin returns, Hybrid ARIMA-GARCH

1. Background of the Study

Bitcoin is a cryptocurrency first introduced by an unknown or group of people using the name Satoshi Nakamoto [1]. A cryptocurrency is a digital asset designed to work as a medium of exchange that uses strong cryptography to secure financial transactions, control the creation of additional units, and verify the transfer of assets(Wikipedia.org accessed on Aug 20th, 2019). Bitcoinsare created as a reward for a process known as mining and they can be exchanged for other currencies, products, and services (Wikipedia.org accessed on Aug 20th, 2019). Bitcoin is the first widely used and traded cryptocurrency since 2009 when the Bitcoin software started to be available to the public and mining-the process of which new Bitcoins can be created and transactions can be recorded and verified on the blockchain begins [2].

As Bitcoin becomes increasingly popular, and the idea of decentralized and encrypted currencies catch on, more rival, alternative cryptocurrencies appear. But Bitcoin remains the most successful and widely accepted cryptocurrencies. Bitcoin prices have been negatively affected by several hacks or theft from cryptocurrency exchanges including thefts from Coincheck in January 2018, Coinrail and Bithumb in June, and Bancor in July 2018 and for the first six months of 2018, $761million worth of cryptocurrencies was reported stolen from exchanges (Wikipedia.org accessed on Aug 20th, 2019).

The purpose of this paper is to use time series methodology to predict the future returns and price of Bitcoin. At the same time, we want to compare the performance of ARIMA-GARCH models with Normal, Student's t and Skewed student's t distributions. As Bitcoin gradually has had a place in the financial markets and in portfolio management, time series analysis is a useful tool to examine the characteristics of Bitcoin prices and returns and extract important statisticsin order to forecast future values of the series.

Section 2 that follows this background of the study discusses review of related works, Section 3 discusses the methodology of this study and Section 4 discusses data analysis, while Section 5 gives the summary and conclusion of the paper.

2. Literature Review

Some studies have already been conducted on the financial and statistical characteristics of Bitcoin. Brandvold et al. [3] and Bouoiyour et al. [4], examined price discovery in the Bitcoin market, their findings reveal some significant relationship between Bitcoin prices, transaction use, and investors attractiveness. Kristoufek [5], and Gracia et al [6] are of the opinion that Bitcoin price is subject to unique factors which are substantially different from those affecting conventional, financial assets, such as internet search, information on google trends, and word of mouth information on social media.

Glaser et al. [7], in their finding, reveal that Bitcoin is mainly used and viewed as an asset rather than a currency. Dyhrberg [8] analyses the volatility of Bitcoin in comparison to the US Dollar and Gold using GARCH (1,1) and EGARCH (1,1). The study concluded that Bitcoin bears significant similarities to both assets, especially concerning hedgingcapabilities and volatility reaction to news, suggesting that Bitcoin can be a useful tool for portfolio management, risk analysis, and market sentiment analysis. The author replicates the study using T-GARCH (1,1) and finds similar conclusions [9].

3. Methodology

In this section, the paper discusses the techniques that feature prominently in this study. These are ARIMA, ARIMA- GARCH models with (Normal, Student-t, and Skewed Student-t distribution)

3.1 The Box-Jenkins for ARIMA Model

Auto-Regressive Integrated Moving Average (ARIMA) model is one of the time series forecasting methods which says that the current value of a variable can be explained in terms of two factors; a combination of lagged values of the same variable and a combination of a constant term plus a moving average of past error terms. To build an ARIMA model one essentially use Box-Jenkins methodology [10], which is an iterative process and involves four stages; Identification, Estimation, Diagnostic Checking, and forecasting. As the Box Jenkins (AR, MA, ARMA or ARIMA) models are based on the time series stationary, if underlying series is non-stationary, then first it is converted into a stationary series either by using differencing approach or taking logarithms or regressing theoriginal series against time and by taking the error terms of this regression [11]. The series stationary was tested by applying the ADF-Augmented Dickey-Fuller [12] and PP-Phillips-Perron unit root tests [13]. ADF was performed for the scenario with a constant, without a constant and with a trend [14]. If it is needed for the time series to have one differential operation to achieve stationarity, it is a I(1) series. Time series is I(n) in case it is to be differentiated for n times to achieve stationarity. Therefore, ARIMA (p, d, q) models are used for the non-stationary time series, specifically the autoregressive integrated average models, where d is the order of differentiation for the series to become stationary.

Box-Jenkins ARIMA is known as ARIMA (p, d, q) model where p is the number of autoregressive (AR) terms, d is the number of difference taken and q is the number of moving average (MA) terms. ARIMA models always assume the variance of data to be constant. The ARIMA (p, d, q) model can be represented by the following equation:

$$y_t = \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \quad (1)$$

Where $\varepsilon_t \sim N(0, \sigma_t^2)$,p and q are the number of autoregressive terms and the number of lagged forecast errors, respectively.

The identification of modeling the conditional mean value is based on the analysis of estimated autocorrelation and partial autocorrelation function (ACF, PACF). These estimations may be strongly inter-correlated, it is therefore recommended not to insist on unambiguous determination of the model order, but to try more models. We must not forget to carry out the verification, which is based on a retrospective review of the assumptions imposed on the random errors.

Validation of ARMA (p, q) models is based on minimizing the AIC (Akaike's information criterion) and BIC (Schwarz's information criterion) criteria. Given that financial data are very often characterized by high volatility, it is necessary to test the model for ARCH effect, i.e. presence of conditional heteroscedasticity [11]. Regarding heteroscedasticity, it is, therefore, a situation where the condition of finite and constant variance of random components is violated. If ARCH test indicates that the variance of residuals is nonconstant, we can use ARCH family models for capturing volatilities of model

3.2. GARCH Models

GARCH models are used mainly for modeling financial time series that present time-varying volatility clustering. The general GARCH (q, p) model for the conditional heteroscedasticity according to Bollerslev [15] has the following form:

$$y_t = \mu_t + z_t \quad (2)$$

Where
μ_t is conditional mean of y_t
z is the shock at time t

$$z_t = \sigma_t \varepsilon_t \quad (3)$$

Where
$\varepsilon_t \rightarrow$ iid N(0,1).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i z_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (4)$$

Where
σ_t^2 is the conditional variance of y_t
α_0 is a constant term
q is the order of the ARCH terms
p is the order of the GARCH terms
α_i and β_j are the coefficient of the ARCH and GARCH parameters respectively

With constrains

$$\alpha_0 > 0$$
$$\alpha_i \geq 0, \text{for } i = 1,2,\ldots,q$$
$$\beta_j \geq 0, \text{for } j = 1,2,\ldots,p$$
$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$$

3.3 The Densities

The GARCH models are estimated using a maximum likelihood (ML) approach. The logic ofML is to interpret the density as a function of the parameters set, conditional on a set of sampleoutcomes. This function is called the likelihoodfunction. financial time-series often exhibits non-normality patterns,i.e. excess kurtosis and skewness. Bollerslev and Wooldridge [16] propose a Quasi MaximumLikelihood method (hereafter QML) that is robust to departure from normality. Indeed, Weiss [17] and Bollerslev and Wooldridge [16] show that under the normality assumption, the QML estimator is consistent if the conditional mean and the conditional variance are correctlyspecified. This estimator is, however, inefficient with the degree of inefficiency increasing with thedegree of departure from normality [18].

Since it may be expected that excess kurtosis and skewness displayed by the residuals ofconditional heteroscedasticity models will be reduced when a more appropriate distribution isused, we consider three distributions in this study: TheNormal, the Student-t (including a "tail"parameter) and the Skewed Student-t (including a "tail" parameter and an asymmetric parameter).

3.3.1 Gaussian

The Normal distribution is by far the most widely used distribution when estimating and forecastingGARCH models. If we express the mean equation as:

$$y_t = E[y_t|\varphi_{t-1}] + \varepsilon_t \quad (5)$$

And $\varepsilon_t = z_t\sigma_t$

thelog-likelihood function of the standard normal distribution is given by

$$L_T = -\frac{1}{2}\sum_{t=1}^{T}[\ln(2\pi) + \ln(\sigma_t^2) + z_t^2], \quad (6)$$

Where T is the number of observations.

3.2.2 Student-t

For a Student-*t* distribution, the log-likelihood is

$$L_T = \ln[\Gamma(v + 1/2)] - \ln[\Gamma(v/2)] - 0.5\ln[\pi(v-2)]$$
$$-0.5\sum_{t=1}^{T}[\ln\sigma_t^2 + (1+v)\ln\left(1 + \frac{z_t^2}{v-2}\right) \quad (7)$$

Where v is the degree of freedom,

$2 < v \le \infty$ and $\Gamma(.)$ is the gamma function. When $v \to \infty$, we have the normal distribution, so that the lower v the fatter the tails.

3.3.3 Skewed Student-t

Skewness and kurtosis are important in financial applications in many respects (in asset pricingmodels, portfolio selection, option pricing theory or Value-at-Risk among others). Therefore, adistribution that can model these two moments looks appropriate. Quite recently, Lambert andLaurent [19]-[20], extend the Skewed Student density proposed by Fernandez and Steel [21]to the GARCH framework. For a Standardized (zero mean and unit variance) Skewed Student,the log-likelihood is:

$$L_T = \ln[\Gamma(v + 2/2)] - \ln[\Gamma(v/2)] - 0.5\ln[\pi(v-2)] + \ln\left(\frac{2}{\xi + 1/2}\right)$$
$$+\ln(s) - 0.5\sum_{t=1}^{T}[\ln\sigma_t^2 + (1+v)\ln(1 + \frac{sz_{t+m}}{v-2}\xi - I_t)] \quad (8)$$

Where ξ is the asymmetry parameter, v the degree of freedom of the distribution, $\Gamma(.)$ is the gamma function.

$$I_t = \begin{cases} 1 \text{ if } z_t \ge -\frac{m}{s} \\ -1 \text{ if } z_t < -\frac{m}{s} \end{cases}$$

$m = \frac{\Gamma(v+1/2)\sqrt{v-2}}{\sqrt{\pi}\Gamma(v/2)}(\xi - \frac{1}{2})$ and

$s = \sqrt{\xi^2 + \frac{1}{\xi^2} - 1 - m^2}$ see Lambert and Laurent [20] for more details.

3.4. Hybrid ARIMA-GARCH Model

In order to recommend a hybrid ARIMA-GARCH model, two stages should be applied. In the first stage, we use the best ARIMA model that fits on stationary and linear time series data while the residuals of the linear model will contain the non-linear part of the data. In the second stage, we use the GARCH model in order to contain non-linear residuals patterns. This hybrid model, which combines ARIMA and GARCH model containing nonlinear residuals patterns, is applied to analyze and forecast the returns of Bitcoin.

3.5. Estimation of Hybrid ARIMA-GARCH Model

The hybrid ARIMA-GARCH model is a nonlinear time series model which combines a linear ARIMA model with the conditional variance of a GARCH model. The estimation procedure of ARIMA and GARCH models are based on maximum likelihood method. Parameters' estimation in logarithmic likelihood function is done through nonlinear Marquardt's algorithm [22]. The logarithmic likelihood function has the following equation:

$$\ln L[(y_t), \theta] = \sum_{t=1}^{T}\{\ln\left[D(z_t(\theta), \mu] - \frac{1}{2}\ln[\sigma_t^2(\theta)]\right\} \quad (9)$$

Where $\theta$is the vector of the parameters that have to be estimated for the conditional mean, conditional variance, and density function, $z_t$denoting their density function, $D(z_t(\theta), \mu]$is the log-likelihood function of, for $[(y_t), \theta]$a sample of T observation. The maximum likelihood estimator $\hat{\theta}$for the true parameter, vector is found by maximizing (10) [23].

3.6. Diagnostic Checking of Hybrid ARIMA-GARCH Model

The diagnostic tests of hybrid ARIMA-GARCH models are based on residuals. Residuals' normality test is employed with Jarque and Bera test [24]. Ljung and Box [25] (Q-statistics) statistic for all time lags of autocorrelation is used for the serial correlation test. Also, for the conditional heteroscedasticity test, we use thesquared residuals of autocorrelation function.

3.7. Forecast Evaluation

This study adopted two very popular measures for evaluating the forecast accuracy of the series and these are: Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). These measures are evaluated by assessing their returns. The one with the lowest error measure is judged the best. These measures are defined as follows.

Mean Absolute Error (MAE) is given by:

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|(X_t - \bar{X})^2 - \hat{p}_t| \quad (10)$$

and Root Mean Squared Error (RMSE) is given by:

$$RMSE = \left[\frac{1}{N}\sum_{t=1}^{N}|(X_t - \bar{X})^2 - \hat{p}_t|\right]^{1/2} \quad (11)$$

Where:

X_t: The return of the horizon before the current time t

$\bar{X}$: The average return

$\hat{p}_t$: Is the forecast value of the conditional variance over n steps ahead horizon of the current time t

4. Data Analysis

The data used in this study are the daily closing prices of Bitcoin from Jan 1st, 2012 to July 31st, 2019, which corresponds to a total of 2769 observations. The estimation process is run using 7 years of data (2012-2018) while the remaining are used for forecasting. The data is compiled from Bitstamp, the largest Bitcoin exchange, and covers a daily database denominated in US dollar, which is the main currency against which Bitcoin is the most traded.The

Bitcoin prices are transformed into their returns so that we obtain stationary series. The transformation is

$$r_t = 100 * [Iny_t - Iny_{t-1}] \qquad (12)$$

Table 1 show some key statistics of the data before transformation which shows high standard deviation and none normality.

Table 2 presents some key statistics of the data after transformation. Skewness and excess kurtosis are clearly observed, leading to a high valued Jarque and Bera (1987) test which indicates non-normality of the distribution.
In the following **Figure 1,**we present the closing price of Bitcoin before transformation which seems to be nonstationary.

In figure 2, we present the closing returns of Bitcoin price after transformation. From **figure 2**, the daily closing returns of Bitcoin seems to be stationary. The conformation in stationarity of the returns of Bitcoin price is done with Dickey-Fuller [12] and Phillips-Perron [13] unit root tests. The results of **Table 3** confirm that the returns of Bitcoin price are stationary in their level.

Table 1: Descriptive Statistics of Daily Bitcoin Price

Mean	2227.456
Median	511.87
Maximum	19187.78
Minimum	4.23
Standard deviation	3387.488
Skewness	1.864031
Kurtosis	3.090124
Jarque-Bera	2717.929
Probability	0.000000
Observations	2769

Table 2: Descriptive Statistics of Daily Bitcoin Returns

Mean	0.280326
Median	0.222228
Maximum	33.748619
Minimum	-66.394803
Standard deviation	4.746170
Skewness	-1.243758
Kurtosis	21.575621
Jarque-Bera	54648.6373
Probability	0.000000
Observations	2769

Figure 1: Time Plot of daily Bitcoin closing price from January 1, 2012, to July 31st, 2019.

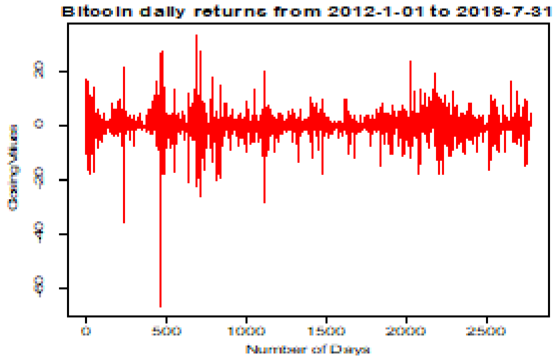

Figure 2. Time Plot of daily Bitcoin closing returns from January 1, 2012, to July 31st, 2019.

Table 3: Unit root tests of the returns of Bitcoin

Test	Test Stat	Mackinnon critical values
ADF	-37.9891	1% -3.96 5% -3.41 10% -3.12
PP	-54.3139	1% -3.4357 5% -2.8631 10% -25677

4.1 Choice of the best model

Once stationary have been addressed, the next step is to identify the order (the p, d, and q) of the autoregressive and moving average terms. The primary tools for doing this are Akaike information criterion, Schwartz information criterion, and Hannan Quinn information criterion. That is the model that gives minimum A

4.1.1 ARIMA Model Identification

Table 4: AIC of ARIMA (p, d, q)

Order	AIC
(0,0,1)	-5862.983
(0,0,2)	-5862.051
(1,0,0)	-5862.886
(1,0,1)	-5861.584
(1,0,2)	-5860.047
(2,0,0)	-5862.077
(2,0,1)	*-5864.612*
(2,0,2)	-5857.967

(*) minimum value to criterion
From table 4, we observed that the optimal model is ARIMA (2, 0, 1) that is based on the selection criterion AIC.

4.2 Model estimation

After an optimum model has been identified, the model estimation methods make it possible to estimate all the parameters of the ARIMA model.

Table 5: The estimated model of ARIMA (2,0,1)

Variable	coefficient	Std.error	p-value
Constant	0.0003927	0.000313	0.2096
AR(1)	0.8115002	0.086643	0.00000***
AR(2)	0.0425337	0.024744	0.0856
MA(1)	-0.8456408	0.084552	0.00000***

4.3 Justifications for ARIMA-GARCH model

We have already known from the excess kurtosis that an obvious fat tails displayed inour series, a typical evidence of heteroskedastic effects as clustering of volatility. We will also use Box-Pierce [25]and Engle LM test [26], to test for the presence of ARCH effect using ARIMA (2,0,1) residuals.

Table 6: ARIMA (2,0,1) residuals test

Type of test	Test stat	P-vale
Ljung-Pierce (R^2)	249.23	0.00000
ARCH LM-test (R)	282.51	0.00000

R denote residuals

The results of **Table 6** confirm the present of ARCH effect in the model. Given the ARCH effects on the returns of Bitcoin price, we proceed with the estimations of hybrid ARIMA-GARCH models to examine the volatilities that exist in the related returns of Bitcoin. To catch this cluster we should use ARIMA as well as GARCH models. Thus, in the levels this time-series on returns of Bitcoin prices we have to find out the appropriate hybrid ARIMA-GARCH model. Estimation parameters' is held with Maximum Likelihood method.

4.4 Estimation of ARIMA (2,0,1)-GARCH models

4.4.1 Why use GARCH models (1,1)

According to Javed and Mantalos [27], numerous studies that investigate model selectionfor the GARCH models find that the "performance of the GARCH (1,1) model issatisfactory". Javed and Mantalos claim that the first lag is sufficient to capture themovements of the volatility. To be able to compare the results, I will for the purpose of this study used GARCH (1,1) model with different distribution such as normal, student-t, and skewed-student-t.

Table 7: Estimate of ARIMA(2,0,1)-GARCH(1,1) model with Normal distribution

Mean equation	Coefficient	p-values
AR(1)	9.377e-01	0.00000***
AR(2)	9.284e-03	0.757
MA(1)	-9.077e-01	0.000000***
Variance equation	Coefficient	p-value
α_0	7.131e-05	0.000000***
α_1	2.192e-01	0.000000***
β_1	7.699e-01	0.000000***
Diagnostic test	Test stat	p-value
Ljung-Box R^2 Q(10)	2.188098	0.994685
Ljung-Box R^2 Q(15)	6.588718	0.968075
Ljung-Box R^2 Q(20)	8.87306	0.984331
LM Arch Test	2.883514	0.996312
Jarque-Bera Test	12804.39	0.0000000
Log-likelihood	3541.236	

AR and MA denote the autoregressive and moving average terms respectively. R^2 Q(.) are Ljung-Box Q-statistic of squared residuals at lag 10, 15, 20 respectively. *,**,*** indicate statistical significant at 1%, 5%, 10%.

Table 8: Estimate of ARIMA(2,0,1)-GARCH(1,1) model with Student's t distribution

Mean equation	Coefficient	p-values
AR(1)	0.9219392	0.00000***
AR(2)	0.0554118	0.0122*
MA(1)	-0.9562585	0.000000***
Variance equation	Coefficient	p-value
α_0	0.0000581	0.0466*
α_1	0.6063956	0.0183*
β_1	0.7710283	0.000000***
Shape	2.3286485	0.000000**
Diagnostic test	Test stat	p-value
Ljung-Box R^2 Q(10)	1.931536	0.9968384
Ljung-Box R^2 Q(15)	2.769039	0.9997563
Ljung-Box R^2 Q(20)	3.233868	0.9999922
LM Arch Test	2.552368	0.9979635
Jarque-Bera Test	43022.95	0.0000000
Log-likelihood	3862.174	

AR and MA denote the autoregressive and moving average terms respectively. R^2 Q(.) are Ljung-Box Q-test- statistic of squared residuals at lag 10, 15, 20 respectively. *,**,*** indicate statistical significant at 1%, 5%, 10%.

Table 9: Estimate of ARIMA(2,0,1)-GARCH(1,1) model with Skewed Student's t distribution

Mean equation	Coefficient	p-values
AR(1)	0.9072000	0.00000***
AR(2)	0.0591800	0.00706**
MA(1)	-0.947100	0.000000***
Variance equation	Coefficient	p-value
α_0	0.0000653	0.07108
α_1	0.6853000	0.04104*
β_1	0.7701000	0.000000***
Shape	2.3286485	0.000000***
Skew	0.9535000	0.000000***
Diagnostic test	Test stat	p-value
Ljung-Box R^2 Q(10)	1.962815	0.9968384
Ljung-Box R^2 Q(15)	2.836047	0.9997563
Ljung-Box R^2 Q(20)	3.304784	0.9999922
LM Arch Test	2.589192	0.9979635
Jarque-Bera Test	42417.99	0.0000000
Log-likelihood	3864.775	

AR and MA denote the autoregressive and moving average terms respectively. R^2 Q(.) are Ljung-Box Q-statistic of squared residuals at lag 10, 15, 20 respectively. *,**,*** indicate statistical significant at 1%, 5%, 10%.

The results on **Table 6**, **Table 7**and**Table 8** shows that ARIMA-GARCH(1,1) models has adequately captured the persistent in volatility and there is no ARCH effect left in the residuals from the selectedmodels except the presence of non-normality.

4.5 ARIMA –GARCH Model Forecast Evaluation

The full sample is from January 31st, 2012 to July 31st, 2019. To test which predicting model is better, we choose data from January 31st 2012 to January 31st 2018 to build up the prediction function. Then we use the data from February 1st 2018 to July 31st 2019 out-of-sample forecast.

Table 10: The evaluation of forecasting results of ARIMA(2,0,1)-GARCH(1,1) (normal, student-t, skewed student-t)

Model	RMSE	MAE
ARIMA	0.04102977	0.04102956
ARIMA-GARCH(Normal)	0.03722042	0.03721840
ARIMA-GARCH(Student-t)	0.05918818	0.05911808
ARIMA-GARCH(Skewed student-t)	0.06106578	0.06096372

4.6. Comparative Analysis

From **Table 10**it can be seen that ARIMA(2,01)-GARCH(1,1) with normal distribution outperforms other models. From the point of view of the RMSE it returned a forecasterror of 0.03722042 followed by ARIMA (2,0,1) with a forecast errorof 0.04102977, ARIMA (2,0,1)-GARCH (1,1) with student-t distribution is 0.05918818 and ARIMA (2,0,1)-GARG (1,1) with skewed student-t is 0.06106578. For the MAEARIMA (2,0,1)-GARCH (1,1) with normal distribution still post the best result with forecast accuracy of0.03721840; ARIMA (2,0,1) 0.04102956; ARIMA (2,0,1)-GARCH (1,1)with student-t distribution0.05911808 and ARIMA (2,0,1)-GARCH (1,1) with skewed student-t 0.06096372.

5. Summary and Conclusion

This paper aim to create a hybrid model combining ARIMA model with GARCH models of high volatility in order to analyze and forecast the return of Bitcoin price. To make the Bitcoin price stationary, the Bitcoin price are transformed to Bitcoin returns. In order to find the most optimal lags, different AR and MA lags were tested using the Box-Jenkins method. The most appropriate obtained model among different models using AIC is ARIMA (2,0,1). As financial time series like Bitcoin returns may possess volatility, an attempt is made to model this volatility using GARCH (1,1) model with Normal, Student's t and Skewed Student's t distributions.

The results of the paper showed that ARIMA (2,0,1)-GARCH (1,1) model provides the optimal results and improves forecasting in relation to other models.

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