Reliability Computation of System Reliability for the New Rayleigh Pareto Distribution

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Abstract: ***In this paper presents the reliability computation of system reliability when the applied stress and strength follows the New Rayleigh Pareto distribution. The New Rayleigh Pareto Distribution (NRPD) is considered as a simple model to make component reliability and may place a good fit for failure data and also provide more appropriate information about hazard rate. The results may be applied to semiconductor devices. Maximum likelihood estimator are used here.***

Keywords: The New Rayleigh Pareto distribution, Reliability Computation: Stress – Strength Model, Maximum Likelihood Estimator

1. Introduction

Many lifetime data used for statistical analysis follows a particular statistical distribution. Recent days technological world nearly everyone depends on the continued functioning a broad collection of complex machinery and equipment for our every day to day life security, safety, ability to move easily and economic welfare. We expect our electronic devices, lights, hospital monitoring control, next generation aircraft, nuclear power plants ect., to function whenever we need them. It will be fail, the results can be disastrous illness or even loss of life.

Statistical distributions have long been employed in the assessment of semiconductor device and product reliability. The use of the exponential distribution which more preferred over mathematically very difficult distribution, such as the Weibull and the lognormal among others, suggest that most of the engineers interest the application of simple model to find out failure rates and reliability results quickly. It is therefore proposed that the new Rayleigh Pareto distribution be considered as a simpler alternative which in some situations , may place a good fit for failure data and provide more appropriate information about reliability and hazard rates. The New Rayleigh Pareto distribution is also used to appropriate representation of the lower tail of the distribution of random variable having fixed lower bound.

In the concept of 'strength-reliability', the stress-strength model describes the life of a component, which has a random strength Y and is subjected to a random stress X . The component fails at the instant that the stress applied to it exceeds the strength , and the component will function satisfies whenever $Y > X$. Thus, $Y > X$ is a measure of component reliability. It has number of applications in engineering concepts, deterioration of rocket motors ect., the reliability estimation of a single component stress-strength version has been considered by several authors assuming various lifetime distributions for the stress-strength random variates. Enis and Geisser (1971), Downtown (1973),Awad and Gharraf (1986), McCool (1991), Nandi and Aich (1994),Kundu and Gupta (2005, 2006), Arulmozhi (2003).

In this paper we compute the reliability of a system's reliability when the applied stress and strength follow the probability distribution of New Rayleigh Pareto distribution.

2. The Model

Let us consider the New Rayleigh Pareto distribution with probability density function (pdf)

$$g(x) = \frac{\lambda}{\alpha^{\lambda}} x^{\lambda-1} e^{-\left(\frac{x}{\alpha}\right)^{\lambda}} \qquad (1)$$

Where $0 < x < \infty, \lambda > 0, \alpha > 0.$

The cumulative distribution function (cdf) of the NRPD is given by

$$G(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\lambda}} \qquad (2)$$

Where $0 < x < \infty, \lambda > 0, \alpha > 0.$

Where $\alpha \succ 0$ is a scale parameter and $\lambda \succ 0$ is the shape parameter

The theoretical mean and variance are derived as ;

$$Mean = E(x) = \alpha\Gamma\left(\frac{\lambda+1}{\lambda}\right)$$

$$Variance = \alpha^2\Gamma\left(\frac{\lambda+2}{\lambda}\right) - \left[\alpha\Gamma\left(\frac{\lambda+1}{\lambda}\right)\right]^2$$

The corresponding survival function is

$$S(x) = e^{-\left(\frac{x}{\alpha}\right)^{\lambda}}$$

The hazard function is

$$h(x) = \frac{\lambda}{\alpha^{\lambda}} x^{\lambda-1}$$

Time to failure is given by

$$F(t) = 1 - \exp\left[-\int_0^t h(x)dx\right]$$

$$= 1 - \exp\left[-\int_0^t \frac{\lambda}{\alpha^{\lambda}} x^{\lambda-1} dx\right]$$

By integrating we get

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\lambda}}$$

3. Maximum Likelihood Estimator (MLE)

Let $x_1, x_2, x_3, \ldots\ldots, x_n$ be a random sample of size n. We suppose that the lifetime of these components follows the New Rayleigh Pareto distribution and then the likelihood function is given by

$$L(\alpha,\lambda)=\prod_{i=1}^{n}[g(x_i)]$$

$$=\prod_{i=1}^{n}\left[\frac{\lambda}{\alpha^{\lambda}}x_i^{\lambda-1}e^{-\left(\frac{x_i}{\alpha}\right)^{\lambda}}\right] \quad (3)$$

Then log-likelihood function is given by

$$\ln L(\lambda,\alpha)=n\log\lambda-n\lambda\ln\alpha+(\lambda-1)\sum_{i=1}^{n}\ln x_i-\frac{\lambda\sum_{i=1}^{n}x_i}{\alpha} \quad (4)$$

To estimate the MLE's of λ, α, partially differentiate with respect to λ and equating to zero and partially differentiate with respect to α and equating to zero respectively;

$$\frac{\partial \ln L(\lambda,\alpha)}{\partial\lambda}=\frac{n}{\lambda}-n\ln\alpha+\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i}{\alpha}=0 \quad (5)$$

$$\frac{\partial \ln L(\lambda,\alpha)}{\partial\alpha}=-\frac{n\lambda}{\alpha}+\frac{\sum_{i=1}^{n}x_i}{\alpha_i^2}=0 \quad (6)$$

From (5) we obtain MLE of λ as a function of α is given by

$$\hat{\lambda}=\frac{n}{n\ln\alpha-\sum_{i=1}^{n}\ln x_i+\frac{\sum_{i=1}^{n}x_i}{\alpha}}$$

From (6), we estimate MLE of α as follows

$$\frac{-n}{\alpha}+\frac{\sum_{i=1}^{n}x_i}{\alpha^2}=0$$

$$\hat{\alpha}=\frac{\sum_{i=1}^{n}x_i}{n}=\bar{x}$$

4. Reliability Computation

Let Y represents the strength of an item with density function

$$g(y)=\frac{\lambda_1}{\alpha^{\lambda_1}}y^{\lambda_1-1}e^{-\left(\frac{y}{\alpha}\right)^{\lambda_1}},\ 0<y<\alpha,\lambda_1>0,\alpha>0$$

Where α is the scale parameter and λ_1 is the shape parameter and the probability density function of stress X is given by

$$f(x)=\frac{\lambda_2}{\alpha^{\lambda_2}}x^{\lambda_2-1}e^{-\left(\frac{x}{\alpha}\right)^{\lambda_2}},0<x<\alpha,\lambda_2>0,\alpha>0$$

That the item may suffer λ_2 and α are the shape parameter and scale parameter respectively.

Let X and Y be two independent random variables having the density functions $g(x), f(y)$ respectively.

M.A. Beg and N. Singh gave estimation of $P(X>Y)$ for Pareto distribution.

If X and Y are independent the probability that Y (strength), X (stress) then the reliability of the component is given by

$$R=P(y>x)=\int_0^{\infty}\int_0^{y}f(x)g(y)dydx$$

$$=\int_0^{\infty}\int_0^{y}\frac{\lambda_1\lambda_2}{\alpha^{\lambda_1}\alpha^{\lambda_2}}x^{\lambda_2-1}y^{\lambda_1-1}e^{-\left(\frac{y}{\alpha}\right)^{\lambda_1}}e^{-\left(\frac{x}{\alpha}\right)^{\lambda_2}}dydx \quad (7)$$

Using (7) , we have

$$R=\int_0^{\infty}\frac{\lambda_1}{\alpha^{\lambda_1}}y^{\lambda_1-1}e^{-\left(\frac{y}{\alpha}\right)^{\lambda_1}}\left(1-e^{-\left(\frac{y}{\alpha}\right)}\right)^{\lambda_2}dy$$

On simplification , the reliability of the system is

$$R=\frac{\lambda_1}{\lambda_1+\lambda_2}$$

If stress and strength have same scale parameter α, then it shows that in defining the life of component initial value of α doesn't matter.

5. Example

A r-out-of-n-System

This system is operational if and only if at least r components out of the system's n components are operational. An example might be a radar network having n radar control stations which can be cover a certain area under control and only if at least r of its service stations are operational. The structure of the system function is given by

$$\psi(x)=1 \qquad if\sum_{i=1}^{n}x_i\geq r$$

$$=0 \qquad \text{elsewhere}$$

A particular case of this system when r=n is called a series system. It is operational if and only if all of its elements are above. One can verify easily that the structure function for a series system can be represented as

$$\psi(x)=\prod_{i-1}^{n}x_i=\min_{1\leq j\leq n}x_i$$

Here we can observe that of a series system as an information transmission system having n " stations " in which the information is transmitted properly only if all the stations are operational. Other extreme case of r-out-of-n-system is r=1. Such a system is known as parallel system. We can observe that a parallel system as a system with one main operating unit and (n-1) extra units in stand by. The system is down if and only iff all n units are not operational. The structure of the system function for a parallel system is given by

$$\psi(x) = 1 - \prod_{i=1}^{n}(1 - x_i) = \max_{1 \le j \le n} x_i$$

6. Conclusions

The procedure for considering reliability system when applied the stress- strength follows the New Rayleigh Pareto distribution may be used as a single model to assess component reliability. The advantage or merit of this distribution is that the hazard rate, as opposed to the exponential distribution's is not be a constant and may be more useful in forecasting the original stages of component's life. It is suggested that this distribution be given consideration when analyzing the electronic devices failure data.

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