# Tubular K-Joint under in-Plane Bending

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**Abstract:** ***Structural member that contains a discontinuity, will experience localized stresses near the discontinuity. Such discontinuities are called stress raisers and the regions in which they occur are called areas of stress concentration. In this study the effect of loading on variation of Stress Concentration Factor (SCF) for different brace-to-chord diameter ratio, $\beta$, and brace-to-chord thickness ratio, $\tau$, for a simple tubular gap K-joint are investigated. Case study IPB1 is for in-plane bending load acting on brace B of the model while Case study IPB2 is for in-plane bending loads acting simultaneously on braces B and A. Results shows that the highest value of SCF occurred when the brace-to-chord thickness ratio $\tau = 0.7$ and brace-to-chord diameter ratio, $\beta = 0.9$ with a magnitude of 1.8526. This is an increment of 24.36% for the same loading parameter on K-joint with $\tau = 0.7$.***

**Keywords:** tubular K-joint, structural modelling, in-plane bending, SCF analysis

## 1. Introduction

Tubular K-joint is one of the commonly used joints in industry where the size as well as its parameter ratio depend on its applications. It consists of a chord and two braces on the same side of the chord. Offshore industry is one of the main sectors where this type of joint usually employed. K-joints may be found in particular on jacket structure supporting a topside in typical offshore installations.

Analysis has been done on the effects of external loads on brace-to-chord diameter ratio as well as brace-to-chord thickness ratio. The stress concentration factor (SCF) for particular loading conditions were determined based on the resulting value of von-Mises stress.

## 2. Loading Consideration

Stresses are higher at certain part of the structure especially at intersection of structural members and in particular at welded joint holding together these members. These stresses are due to external loading as well as transferred load from adjoining members and it must be carefully addressed at the design stage of the structure [1]. Enough allowance must be considered to ensure that the structure is safe and capable to perform its intended function within its design life. Stresses at any location may increase in respond to loads and it increases linearly until the reaching the maximum magnitude of yield stress. Beyond this linear relationship the concerned structural elements are considered as fall into failed category. SCF for the K-joint were calculated based on the maximum stress over nominal stress at the location. Then the results were plotted and discussed as refer to Section IV. The multiplier applied to the nominal stress to reach the peak or maximum stress at the hot spot is called the stress concentration factor (SCF). The SCF is different from a joint geometry to another and is a measure of the joint strength, particularly its fatigue strength [2]. Recent review on SCF on tubular joints used in industry may be referred as in [3].
In this study, the in-plane bending loads are illustrated in Figure 1 shows the source of bending load applied at the end of braces that would induce stresses at the joint.

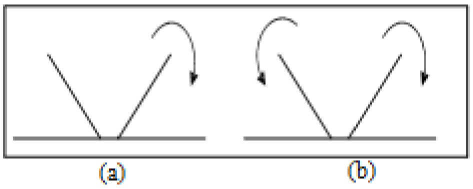



**Figure 1**: In-plane bending loading: (a) case study IPB1, (b) case study IPB2

In this study, two type of loading conditions were adopted. Firstly, in-plane bending load acting on brace B (case study IPB1) and secondly, in-plane bending loads acting on brace A and brace B (case study IPB2). The magnitude of in-plane load is 1.0 kN.

SCF is related to actual maximum stress at the discontinuity to the nominal stress. The factor is defined by the equation below:

$$SCF = \frac{\sigma_{max}}{\sigma_0} \quad (1)$$

where $\sigma_{max}$ is maximum stress and $\sigma_o$ is nominal stress.

Two sets of boundary conditions had been used in the analytical study where the chord was simply supported at the end for in-plane moment loads. The thickness-to-diameter ratio of the chord ($t/D$) will influence the radial flexibility of the chord. The brace-to-chord diameter ratio ($\beta$) was a governing factor in the stress distribution due to the manner in which the load transfer is accomplished. The brace-to-chord thickness ratio ($\tau$) is an indication of the relative bending stiffness of the brace and chord and therefore, primarily governs the bending stress in the brace at the intersection. The inclusion of the angle of inclination of the brace to chord ($\theta$) is necessitated by the mechanism of the load transfer. These four parameters discussed above are applicable to each of the three joint types in SCF determination [4]

Parametric equations suggested for the SCFs on the chord and the brace side derived from joint with $\alpha = 12$ under balanced axial loading are given as follow [5];

$$SCF_{Chord} = \gamma^{0.91}(0.22 - \beta^2 + 1.92\beta)sin^{2.51}\theta$$
$$\times \left[\beta_{max}\left(\frac{-0.81}{\beta_{min}^3} + \frac{5.67}{\beta_{min}^2} - \frac{8.09}{\beta_{min}} + \frac{4.08}{\beta_{max}}\right)\right]$$
$$\times \tau^{1.26}[0.13 + 0.12\beta^{-0.41}arctan(\zeta)]$$
$$+\{\beta^{0.27}\gamma^{0.32}(-12.42\tau - 0.02)$$
$$\times \left[-0.1(\theta_{max} + \theta_{min})^{0.59\beta}\right]f(\alpha)\} \qquad (2)$$

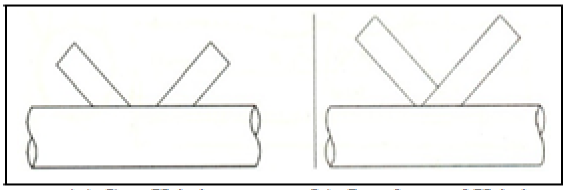
$$SCF_{Brace} = 0.26\tau(-0.18 - \beta^2 + 2.23\beta)sin^{3.12}\theta$$
$$\times \left[\beta_{max}\left(\frac{-0.17}{\beta_{min}^3} + \frac{1.76}{\beta_{min}^2} - \frac{3.19}{\beta_{min}} + \frac{2.27}{\beta_{max}}\right)\right]$$
$$\times [0.33 + 0.26\beta^{-0.90}arctan(\zeta)]$$
$$+\{(2 + 0.58^{\theta\tau})\tau^{0.01}\gamma^{0.88}$$
$$+2.31\ (\theta_{max} + \theta_{min})^{-0.03\beta}\ \} \qquad (3)$$

With;

$$f(\alpha) = 1 + [0.06\beta\alpha\tau^{50.62}exp(-14.55\gamma^{0.83}\alpha^{0.35})] \quad \text{for } \alpha > 12$$
$$f(\alpha) = 1.0 \quad \text{for } \alpha \le 12$$

## 3. Modelling of K-Joint

Two types of tubular K-joints usually used for structural assembly namely tubular gap K-joint and overlapping K-joint as illustrated in Figure 2. It consists of a chord and two braces on the same side of the chord.



(a) Gap K-joint (b) Overlapped K-joint

**Figure 2:** Types of tubular K-joint:

Basic parameters for the model of a K-joint used in this study is given in Table 1.

**Table 1:** Basic parameters for the tubular K-joint model [1]

| Parameters | Value |
|---|---|
| Chord Diameter, *D* | 0.100 m |
| Chord Thickness, *T* | 0.002 m |
| Chord Length, *L* | 0.600 m |
| Brace Length, *l* | 0.180 m |
| Gap distance, *g* | 0.020 m |
| Inclination Angle of brace to chord, $\theta A = \theta B$ | 45° |
| Chord diameter-to-2 times thickness ratio, $\gamma = D/2T$ | 25 |
| Chord 2 times length-to-diameter ratio, $\alpha = 2L/D$ | 12 |
| Gap-to-chord diameter ratio, $\xi = g/D$ | 0.2 |
| Young's Modulus, *E* | 210 GPa |
| Poisson's ratio, ν | 0.3 |

Analysis was performed on a model of gap K-joint. The in-plane loads then applied on braces A and B in analysis as illustrated in Figure 1 in order to obtain the von-Mises stress at certain related hot-spot area

## 4. Results and discussion

In-plane loads acting on braces A and B will result with in-plane bending moments which caused the deflection and deformation of braces in its own plane. These applied loads also cause the increase of stresses at joints between braces and chord, i.e. at crown and saddles as shown in Figure 3. The magnitude of SCF at these locations may be estimated.

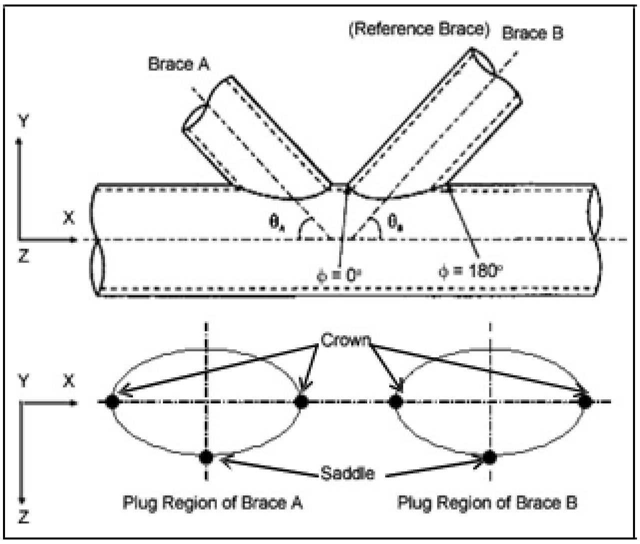
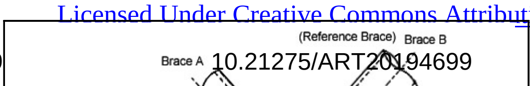



**Figure 3:** Locations of saddle and crown on Brace A and B

Figure 4 and Figure 5 shows a von Mises stress distribution of a K-joint under in plane bending loads. The contour of stresses in Figure 4 shows the response for IPB1 case in that the higher stress appears at the joint between brace B and the Chord. The stress response for the second load case, IPB2 is shown in Figure5. Stress response contour for each model shows a correct magnitude where higher stresses may occur at the joint between brace and chord. In load case IPB1, the stress is only increased in the vicinity of the joint due to single in-plane load acting on brace B. The hot- spot stress is still maintained at the same location as long as the same type of loading is acting on the FE model. However, the stress value on that critical area is totally different for various brace diameter and thickness in used. In Figure 4 and Figure 5, critical stress occurred at crown position due to the direction of loads. The critical area of load case IPB1 is located at the crown point ($\phi = 0^o$) of brace B. Whereas, in load case IPB2 the critical area occurred at the crown point ($\phi = 0^o$) both braces A and B.





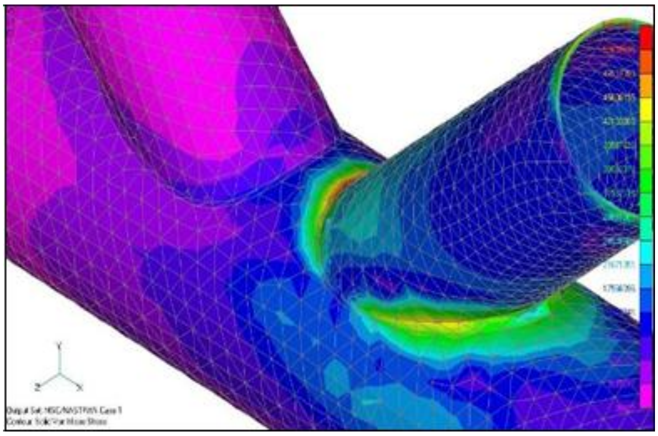


**Figure 4:** Gap K-joint (β = 0.7; τ = 0.7) under in-plane bending on brace B, case study IPB1

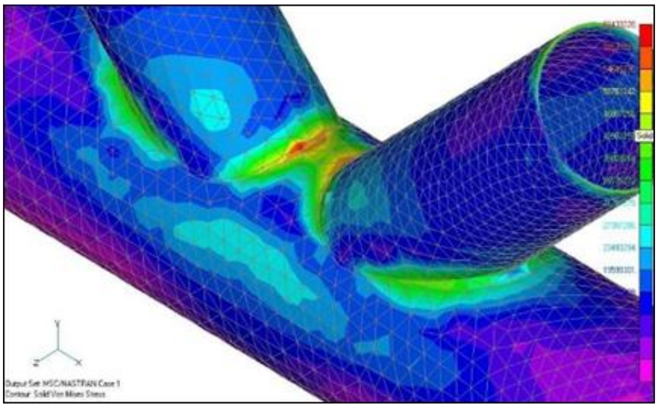


**Figure 5:** Gap K-joint (β = 0.7; τ = 0.7) under in-plane bending on braces A and B, case study IPB2.

The response of tubular K-joint models in term of SCF for a certain loading cases applied on a joint model with geometric parameters given in Table 1. Effect of brace-to-chord diameter ratio (β) on SCF and effect of brace-to-chord thickness ratio (τ) on SCF are analyzed.

Result for SCF analysis on the K-joint where the τ value of 0.7 and β value of 0.7 is shown in Table 2. The same values of nominal stress were considered for both load cases. In Table 2, the highest stress concentration factor occurred when the model is acting with in-plane loading on brace B (load case IPB1) with SCF value is 1.581788. This is due to in-plane loading producing a bending stress onto the chord surface. Tables 3 and 4 shows the results of SCF value for β = 0.5, 0.6 and 0.7 under load cases IPB1 and IPB2 respectively.

Results shows that SCF is more sensitive to the variation in τ value for both load cases. In IPB1 load case, the variation of 15.29% occurred when τ varies between 0.7 to 0.9 for β=0.7. For load case IPB2, where in-plane bending load acts on both braces, the SCF is more sensitive to the variation in β value. The variation of 24.36% occurred when β varies between 0.7 to 0.9 for τ=0.7. Maximum von-Mises stress, $\sigma_{vM}$ is 56.1239 MPa and located at the crown position in load case IPB1. Maximum SCF is 1.852617 and located at the crown position.

**Table 2:** Von-Mises stress, nominal stress and SCF for IPB1 and IPB2 load cases

| Types of Loading | Von Mises Stress, $\sigma_{VM}$ (MPa) | Nominal Stress, $\sigma_0$ (MPa) | SCF | Critical Location |
|---|---|---|---|---|
| IPB1 | 56.1239 | 35.4813 | 1.581788 | Crown |
| IPB2 | 54.6684 | 35.4813 | 1.540767 | Crown |

**Table 3:** SCF value for β = 0.5, 0.6 and 0.7 under load case IPB1

| Brace-to-Chord Dia. Ratio, β=*d/D* | 0.5 | 0.6 | 0.7 | Increment (%) |
|---|---|---|---|---|
| SCF (τ = 0.7) | 1.420674 | 1.483563 | 1.581788 | 11.34 |
| SCF (τ = 0.8) | 1.525456 | 1.576540 | 1.646236 | 7.92 |
| SCF (τ = 0.9) | 1.553774 | 1.620547 | 1.791356 | 15.29 |
| Increment (%) | 9.37 | 11.26 | 13.25 | |

**Table 4:** SCF value for β = 0.5, 0.6 and 0.7 under load case IPB2.

| Brace-to-Chord Dia. Ratio, β=*d/D* | 0.5 | 0.6 | 0.7 | Increment (%) |
|---|---|---|---|---|
| SCF (τ = 0.7) | 1.410321 | 1.369610 | 1.540767 | 12.50 |
| SCF (τ = 0.8) | 1.563780 | 1.447001 | 1.704470 | 17.79 |
| SCF (τ = 0.9) | 1.713740 | 1.489685 | 1.852617 | 24.36 |
| Increment (%) | 21.51 | 8.77 | 20.24 | |

Graphs showing the relationship between SCF and β under the in-plane loading are illustrated in Figure 6 and Figure 7. Figure 6 shows the response of SCF versus brace-to-chord diameter ratio, β for IPB1 load case. The trend of the response is that SCF is increase with the increment in β values from β = 0.5 through β = 0.7. This indicates that bigger brace diameter will induce higher stress at the joint with the chord. This relationship shows that SCF values does not influenced by eccentricity problem for τ = 0.7, τ = 0.8 and τ = 0.9 when the model is under in-plane loading on brace B.

Figure7 shows the non-linear behavior between the SCF and the brace-to-chord ratio, β under in-plane loading condition in load case IPB2 where both braces were loaded. The line τ = 0.7, τ = 0.8 and τ = 0.9 have negative slope from β= 0.5 to β = 0.6 and then follows with a positive trend to β= 0.7. This condition occurs due to eccentricity within the model. Maintaining the gap distance between two braces at 0.02 m and chord diameter at 0.1 m for the FE model, the eccentricity of joint will be zero when β= 0.5657. Therefore, the slope is negative when β is less than 0.5657. On the other hand, when β more than 0.5657, the graph shows a positive slope as shown in Figure 7. In both load cases, the SCF magnitudes were increases with the increment in τ values.

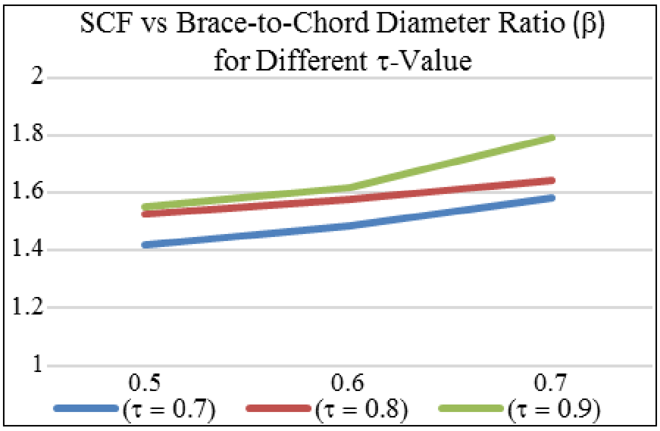



**Figure 6:** SCF versus β for different τ value, load case IPB1

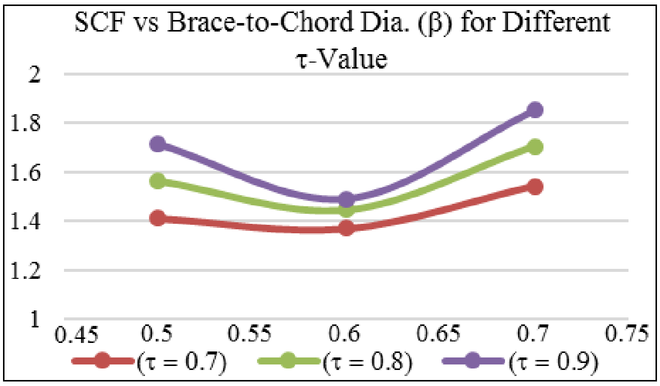



**Figure 7:** SCF versus β for different τ value, load case IPB2

## 5. Conclusions

Results of investigation on tubular gap K-joint under in-plane loading cases were presented. Main conclusions may be summarized as follows;

The SCF on the K-joint under in-plane loads is more sensitive to variation in in τ value for both load cases. Nominal stress is 35.48 MPa for both load cases. In IPB1 load case, the variation of 15.29% occurred when τ varies between 0.7 to 0.9 for β=0.7. For load case IPB2, where in-plane bending load acts on both braces, the SCF is more sensitive to the variation in β value where the variation of 24.36% occurred when β varies between 0.7 to 0.9 for τ=0.7. Maximum von-Mises stress, $\sigma_{VM}$ is 56.1239 MPa and located at the crown position in load case IPB1. Maximum SCF is 1.852617 and located at the crown position.

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