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A Mathematical Model to Solve Cost Oriented Two Sided Assembly Line Balancing by Exact Solution Approach

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Abstract: ***Purpose**: The main aim of this paper is to develop a new mathematical model for the mixed model two-sided assembly line balancing problem (MTALBP) generally occur in plants producing large-sized high-volume products such as buses or trucks. **Methodology**: In this paper, the proposed mathematical model is applied to solve problem of two-sided mixed-model assembly line balancing problem to optimize the cost of machinery, worker and transportation in mated stations of an assembly line. Proposed mathematical model is solved using a branch and bound algorithm on lingo 17.0 solver. **Findings**: Based on the computational result, cost-oriented line efficiency that is obtained by reducing the cost of machinery, worker and transportation in mated stations of the assembly line is good as compare to station oriented line efficiency. **Practical implications**: Since the problem is well known as NP-hard problem a benchmark problem is solved and the result of the study can be beneficial for assembly of the mixed model products and to reduce total worker cost and overall cost. **Originality**: On the basis of literature review this paper is first to address cost-oriented mixed-model two-sided assembly line balancing problem using the exact solution approach.*

Keywords: Two-sided assembly line balancing, Mixed model, Mathematical model, Lingo solver

1. Introduction

An assembly line is a flow oriented production system, which consists of a number of workstations that are connected by material handling system like a conveyor or moving belt. Assembly line balancing problem is determining optimal assignment of tasks to workstations by considering some constraints to obtain anefficient assembly line to satisfy the customer demands on time.

Assembly lines can be divided into two different groups based on product characteristics and some technical requirements: (i) one-sided assembly lines, and (ii) two-sided assembly lines. While only one restricted side (either left (L) or right (R) side) is used in one sided assembly lines, both left and right sides are used in two-sided assembly lines. Two-sided assembly lines are usually constructed to produce large-sized high volume products such as buses, trucks, automobiles, and some domestic products.

In terms of the various numbers of product models assembled on the line, assembly lines can also be classified as single-model assembly lines and mixed-model assembly lines. Assembly lines in which more than one product model is assembled on the same line without any setup requirement between models are called as mixed model assembly line. Mixed model assembly lines offer several advantages over single-model assembly lines, including avoidance of constructing several lines, satisfied different customer demands, and minimized workforce need.

Mixed-model assembly lines provide more flexibility of responding to consumer demands on time and to reach global markets in highly competitive scenario. With the solution of assembling more than one model on each adjacent line of two-sided assembly lines, we can obtain a new line system called Mixed-model two-sidedassembly lines.

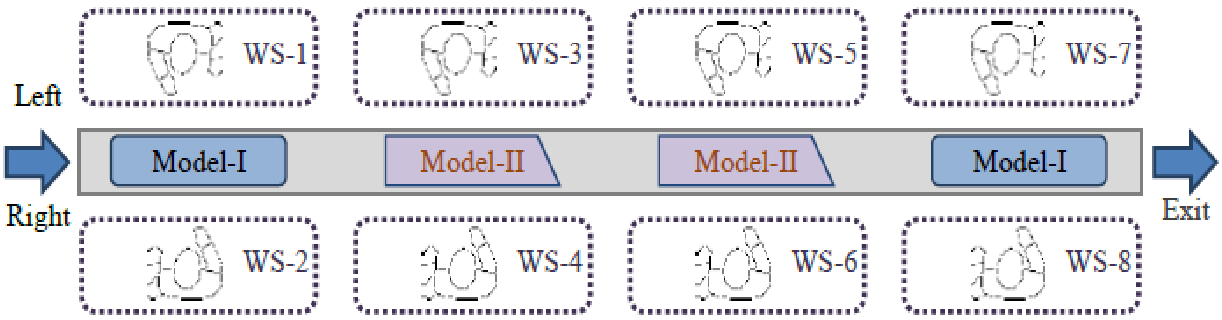

Figure 1: The structural illustration of MTALBP (Zhang et al. 2016)

2. Literature Review

(Simaria et al., 2009) presented mathematical programming model with ant colony optimization algorithm for solving two-sided mixed-model assembly line balancing problem with an objective of minimize the number of workstations of the line.(Ozcan et al., 2009) addressed TALBP with the objective of minimizes the number of mated-stations as the first objective and minimizes the number of stations as the second one for a given cycle time They presented a formal mathematical formulation for the problem and developed simulated annealing algorithm for maximizing the weighted line efficiency and minimizing the weighted smoothness index.

(Chutima et al., 2012) presents a Particle Swarm Optimization with negative knowledge (PSONK) to solve multi-objective two-sided mixed-model assembly line balancing problem with the objective of minimizing the number of mated-stations for a given cycle time. PSONK employs the knowledge of the relative positions of different particles in generating new solutions. (Aghajani et al., 2014) addressed TALBP with the objective of to minimize the cycle time for a given number of mated stations. They presented a mixed-integer programming model for robotic mixed-model two-sided assembly line balancing and developed a simulated annealing (SA) algorithm as Meta heuristic method is proposed to solve the problem.

(Rabbani et al., 2014) In this paper author presents a novel multiple U-shaped layout is proposed to deal with the mixed-model two-sided assembly line balancing (MTALB) problems with mathematical formulation of two conflicting objectives including minimizing the cycle time and minimizing the number of workstations are considered under precedence, zoning, capacity, side, and synchronism constraints and developed genetic algorithms to solve it optimally.

(Kucukkoc et al.,2014) presented a new assembly line system configuration for companies that need intelligent solutions to satisfy customized demands on time with existing resources. An agent-based ant colony optimization algorithm is proposed to solve the problem. They presented a mathematical formulation for simultaneous balancing and sequencing and developed an agent-based ant colony optimization algorithm to solve it optimally.(Yuan et al.,2015) addressed TALBP with the objective of minimizing the number of mated-stations and total number of stations for a given cycle time. A Honey bee mating optimization (HBMO) algorithm is proposed to solve this problem.

(Zhang et al., 2016) In this paper author introduces mixed-model two-sided assembly line balancing Type-II problem benefiting from the real data gathered through an industrial case study. This paper also contributes to knowledge by incorporating incompatible task groups, different from negative zoning constraints. (Kucukkoc.,2016) addressed mixed-model two-sided lines with the objective for minimizing the cycle time of the line as well as the number of workstations. A real-world problem is solved using the proposed approach and the efficiency of the line is improved. They presented a real-world problem and developed ant colony optimization algorithm to solve it optimally.

(Deliceet al.,2017) presented a new modified particle swarm optimization algorithm with negative knowledge is proposed to solve the mixed-model two-sided assembly line balancing problem with minimize the number of mated stations as the first objective and minimizing the number of stations as the second one for a given cycle time. (Liet al.,2018) addressed TALBP with two objectives those are simultaneously to be optimized; one is to minimize the combination of the weighted line efficiency and the weighted smoothness index. A novel multi objective hybrid imperialist competitive algorithm (MOHICA) is to solve this problem.

Although researchers have focused on Two-sided ALB problems and, the literature review suggests that very limited number of researchers focus on the mixed model two-sided assembly line balancing problem (MTALB). MTALB problems with the objective of minimizing cost are very crucial in a number of industries and optimizing this objective is a very critical process. Hence, the main focus of this article is to optimize cost of a MTALB problem.

This article mainly presents following contributions to the research field:

1) A Mathematical Model of mixed model two-sided assembly line balancing problem is proposed with cost oriented objective that is combination of the cost of machinery, workerand transportation in mated stations of assembly line.
2) The proposed mathematical model is tested on anbenchmark problemand is solved using Lingo -17 solver to obtain the optimal solutions.
3) The results of cost function objectives are compared with the results ofstation oriented objective function. From this study, it is observed that the proposed cost oriented Efficiency P(2) that is 69.79% efficiency provides better solutions than station oriented efficiency P(1).

The rest of the paper is organized as follows: MTALBP definition is given in section 3 with objectives, assumptions and constraints. Section 4 illustrates a balancing example which is partly taken from the literature. Conclusions and future work are presented in section 5.

3. Mathematical Formulation

3.1 Overview

The main objective of the proposed model is to assign the set of tasks to the stations of two-sided assembly line so that the overall cost associated with the system is minimized. Here, objective function consists of mainly three types of costs.

The first one is associated to worker cost. In this paper every task is assigned a wage rate (Table 1).Wage rate of every station is defined as the maximum wage rate among the wage rate of tasks assigned to that station. Multiplying thecycle time of the line by the wage rate will be the wage of the worker assigned to that station.

The second one is associated to the cost of transportation facility that is directly connected to the length of the line. Length of two-sided assembly lines is based on total number of mated-stations, so costs of transportation facility are related to the total number of mated-stations.The total costs of the transportation facility (vc)per mated-station is kept fixedequal to 25 (money units per time unit).(Roshani et al., 2014)

The third one is associated to the cost of machinery. In assembly lines generallyquantity of machinery depends on the single station. The total costs of the machinery (mc)per single station is kept fixedequal to 20(money units pertime unit).(Roshani et al., 2014)

Table 1: Data of Wage rate (Roshani et al., 2014)

Tasks	Wage rate (money units pertime unit)	Tasks	Wage rate (money units per time unit)
1	3	9	4
2	6	10	8
3	2	11	2
4	3	12	4
5	8	13	10
6	4	14	2
7	7	15	6
8	10	16	6

3.2 Assumptions

The MTALB problem in this study includes the following assumptions(Ozcan &Toklu, 2009):

- Models with similar production characteristics are produced on the same two-sided assembly line
- Workers perform tasks in parallel at both sides of the line
- Some tasks may be required to be performed at one-side of the line, while others may be performed at either side of the line
- The precedence diagrams of different models are known
- Task times are deterministic and independent of the assigned station
- Parallel tasks and parallel stations are not allowed.

3.3 Mathematical modeling

Decision Variables

Symbol	Description
x_{mijk}	Binary variable indicating if task i of model m is assigned to mated-station(j,k) $\begin{cases} 1 \text{ if task i is assigned to station (j,k)} \\ 0, \text{otherwise} \end{cases}$
st_{mi}	Start time of task i for model m
Z_{ih}	Binary variable indicating precedence relationships among the tasks in the same station $\begin{cases} 1 \text{ if task i is assigned before task h in the same station} \\ 0, \text{if task h is assigned before task i in the same station} \end{cases}$
ms_j	$\begin{cases} 1 \text{ if mated} - \text{station j is utilized} \\ 0, \text{otherwise} \end{cases}$
ss_{jk}	$\begin{cases} 1 \text{ if station (j,k) is utilized} \\ 0, \text{otherwise} \end{cases}$

Notations	
Symbol	Description
I	Set of all assembly tasks
N	Total no. of tasks
J	Set of all mated-stations
M	Set of all models
i	Index of assembly task; $i = 1,2,\dots\ I$
j	Index of station; $j = 1,2,\dots\ J$
m	Index of model; $m = 1,2,\dots\ M$
k	Index of mated-station direction; $\begin{cases} 1 \text{ indicates a left direction} \\ 2 \text{ indicates a right direction} \end{cases}$
(j,k)	Index of station j and the associated mated-station direction k
$P(i)$	Set of immediate predecessors of task i
$S(i)$	Set of immediate successors of task i
t_{mi}	Completion time of task i for model m
μ	Large positive number
CT	Cycle time
w_{mi}	Wage rate assigned to task i for model m
$d1_i$	$\begin{cases} 0 \text{ if task } i \text{ is a right} - \text{side} \\ 1 \text{ otherwise} \end{cases}$
$d2_i$	$\begin{cases} 0 \text{ if task } i \text{ is a left} - \text{side} \\ 1 \text{ otherwise} \end{cases}$
vc	Total costs of the transportation facility
mc	Total costs of the machinery
R^+	Positive real number

Objective Function

$$\text{Minimize} Z = Z1 + Z2 + Z3 \quad (1)$$

$$Z1 = \sum_{j=1}^{J}\sum_{k=1}^{2} ct * \text{Max}(w_{mi} * x_{mijk}) \quad (2)$$

$$Z2 = vc * \sum_{j=1}^{J}\sum_{k=1}^{2} ss_{jk} \quad (3)$$

$$Z3 = mc * \sum_{j=1}^{J} ms_j \quad (4)$$

Objective (1) minimizes overall cost (Z) associated with the system. Itconsists of three parts those are dealing with three types of costs. The first part (Z1) indicate worker cost Second part (Z2) indicatetransportation cost between the mated stationand third part (Z3)indicate cost of machinery associated to a station

Constraints

$$\sum_{j=1}^{J}\sum_{k=1}^{2} x_{mijk} = 1, \forall i \in I\ \forall m \in M \quad (5)$$

$$\sum_{j=1}^{J}(d1_i * x_{mij1} + d2_i * x_{mij2}) = 1, \forall i \in I\ \forall m \in M \quad (6)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{2} j * x_{mhjk} - \sum_{j=1}^{J}\sum_{k=1}^{2} j * x_{mijk} \le 0$$

$$\forall a,b \in I, b \in p(a)\ \forall m \in M \quad (7)$$

$$\sum_{k=1}^{2} x_{mijk} * (st_{mi} + t_{mi}) \le j * ct\ \forall i \in I, j \in J\ \forall m \in M \quad (8)$$

$$\sum_{k=1}^{2}(x_{mijk} * (j-1) * ct) \le st_{mi}\ \forall i \in I, j \in J\ \forall m \in M \quad (9)$$

$$st_{mh} - st_{mi} + \mu(1 - \sum_{k=1}^{2} x_{mijk}) + \mu(1 - \sum_{k=1}^{2} x_{mhjk}) \ge t_{mi}$$

$$\forall i,h \in I, i \in p(h), \forall m \in M, \forall j \in J \quad (10)$$

$$st_{mh} - st_{mi} + \mu(1 - x_{mijk}) + \mu(1 - x_{mhjk}) + \mu(1 - z_{ih}) \ge t_{mi}$$

$$\forall i,h \in I, i \notin p(h), h \notin p(i), \forall m \in M, \forall j \in J \quad (11)$$

$$st_{mi} - st_{mh} + \mu(1 - x_{mijk}) + \mu(1 - x_{mhjk}) + \mu * z_{ih} \ge t_{mh}$$

$$\forall i,h \in I, i \notin p(h), h \notin p(i), \forall m \in M, \forall j \in J \quad (12)$$

$$x_{mijk} = x_{pijk}\ \forall i \in I, \forall j \in J, \forall k \in K, \forall m,p \in M \quad (13)$$

$$\sum_{m=1}^{M}\sum_{i=1}^{N} x_{mijk} - N * ss_{jk} \le 0 \forall j \in J\ \forall k \in K \quad (14)$$

$$\sum_{m=1}^{M}\sum_{i=1}^{N} x_{mijk} - ss_{jk} \ge 0 \forall j \in J\ \forall k \in K \quad (15)$$

$$\sum_{k=1}^{2} ss_{jk} - 2 * ms_j \le 0 \forall j \in J \quad (16)$$

$$\sum_{k=1}^{2} ss_{jk} - ms_j \ge 0 \forall j \in J \quad (17)$$

$$x_{mijk} \in \{0,1\} \; \forall\, i \in I, \forall\, m \in M, \forall\, j \in J, \forall\, k \in K \quad (18)$$

$$ss_{jk} \in \{0,1\} \; \forall\, j \in J \; \forall\, k \in K \quad (19)$$

$$ms_j \in \{0,1\} \; \forall\, j \in J \quad (20)$$

$$z_{ih} \in \{0,1\} \; \forall\, i, h \in I, i \notin p(h), h \notin p(i) \quad (21)$$

$$st_{mi} \in R^{+} \; \forall\, i \in I, \forall\, m \in M \quad (22)$$

Constraint (5) states that all tasks are assigned to only one station. Constraint (6) ensures the left or right-side assignment of the task. Constraint (7) is the precedence constraint, which ensures that precedence relationships among the tasks are not violated. Constraints (8) and (9) ensure that the start time of the task is within the time limit of the station on which it is assigned. Constraints(10) to (12) are specially designed for a TALBP. Constraint (10) will be active when task h is a precedence of task i and are assigned at same mated station on opposite sides otherwise the constraint will not be active. When this holds, the constraint is applied to$st_{mi} - st_{mh} \geq t_{mh}$ which ensures that task h is assigned before task i. Constraints (11) and (12) become active when tasks h and i do not have any precedence relationship and are assigned on the same station (j,k). If $I$is assigned earlier than P , then constraint (11) become $st_{mh} - st_{mi} \geq t_{mi}$; if not, then constraint (12) becomes $st_{mi} - st_{mh} \geq t_{mh}$ .Constraint (13) ensure the assignment of a task on same station for all the models. Constraints (14) and (15) ensure that $ss_{jk} = 1$ if any task is assigned to station $(j,k)$and 0 otherwise. Constraints (16) and (17) ensure that $ms_j = 1$ if any task is assigned to mated-station $j$and 0 otherwise. Constraints (18) to (22) are the binary constraints.

4. Solution Approach and Results

4.1 Benchmark Problem

In this section we use benchmark problem data (Kucukkoc et al., 2014)to solve mixed model two-sided assembly line balancing problem (MTALB)problem as depicted in Table 2.In this table, there are 16 tasks with their preferred side (Left, Right and Either).Further, it showsimmediate predecessor and task processing time for both the models A and B.

Table 2: Data of benchmark problem (Kucukkoc et al., 2014)

Task No	Side	Processing Time (Time-units)		Immediate Predecessor(s)
		Model A	Model B	
1	E	6	0	-
2	E	5	2	-
3	L	2	0	1
4	E	0	9	1
5	R	8	0	2
6	L	4	8	3
7	E	7	7	4,5
8	E	4	3	6,7
9	R	0	5	7
10	R	4	1	7
11	E	6	3	8
12	L	0	5	9
13	E	6	9	9,10
14	E	4	5	11
15	E	3	8	11,12
16	E	4	7	13

4.2 Performance comparison

In this paper MTALBP is solved by LINGO 17 solver, and the results of task assignment are shown in figure 2 and figure 3. In figure 2, there are 5 mated stations in the optimal solution.

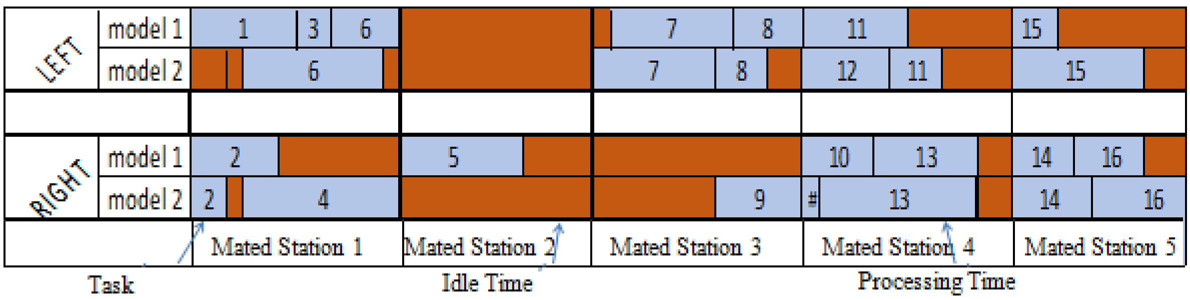

Figure 2: Optimal task assignment of station oriented P(1) problem

As can be seen in the figure 3, there are 5mated stations in the optimal solution. Take station 1 for example to explain how tasks are assigned. Tasks 1, 3, and 6 are assigned to left side mated-station for model 1and task 6are assigned to left side of mated-station for model 2 , and tasks 2 and 5 are assigned to right side mated-station for model 1and tasks 2 and 4 are assigned to right side mated-station for model 2.

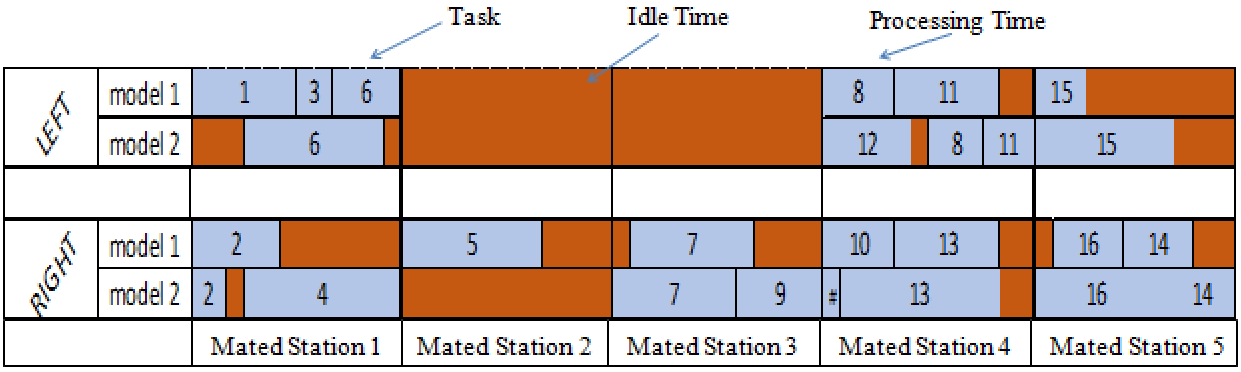

Figure 3: Optimal task assignment of cost oriented P(2) problem

According to the solution obtained by LINGO-17 solver, the idle time of left side mated-station appears between the processing of tasks 6 and 8 for model1 and tasks 6 and 12 for model2 are more uniform for worker rest and other works. In fact, the idle time can be rearranged to the end of processing of task 6 without violating precedence relationship. Table 3.indicate that cost orientedefficiency P(2) is 69.79% on the other hand station orientedEfficiency P(1) is 62.04 %.

Table 3: Efficiency comparison result of problems

Problem	Efficiency
Station oriented Efficiency P(1)	62.04%
Cost oriented EfficiencyP(2)	69.79%

Table 4 is the summary result for the application of the model to solve problem that indicate total cost of cost oriented assembly lineP(2) is less as compare to station oriented assembly lineP(1). Total worker cost is less in cost oriented assembly line P(2) as compare to station oriented assembly line P(1) and total no. of single station is also less in cost oriented assembly lineP(2).

Table 4: Performance comparison of problems

S.N.	Mated station	Single station	Station oriented P(1)		Cost oriented P(2)	
			Task assigned	Wage rate	Task assigned	Wage rate
1	1	1 2	1,3,6 2,4	8 6	1,3,6 2,4	8 6
2	2	1 2	- 5	- 3	- 5	- 3
3	3	1 2	7,8 9	10 8	- 7,9	- 10
4	4	1 2	11,12 10,13	3 9	8,11,12 10,13	6 9
5	5	1 2	15 14,16	15 12	15 14,16	15 12
Total Worker cost			888		828	
Total no. of mated station			5		5	
Total no. of single station			9		8	
Total cost			1193		1113	

5. Conclusions and Future Research

In this paper, we presented a new mathematical model for solving the mixed model two-sided assembly line balancing with cost oriented objective function that actually minimizing cost. Solutions obtained by LINGO-17 solver for objectives are provided and comparative analysis shows that cost oriented Efficiency P(2) that is 69.79% efficiencyhigher than station orientedefficiency.

In futuremixed model two-sided assembly line balancing can be developed for stochastic approach and meta-heuristic, such as tabu search algorithm and simulated annealing algorithm, ant colony optimization algorithm can be applied to solve mixed model two-sided assembly line balancing problem based on cost oriented objective function.

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