

Forecasting of Time Series Using Fuzzy Logic and Particle Swarm Optimization Algorithm

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Abstract: During the last few decades, fuzzy time series have been developed so as to better the exactness of forecasting. In this study, we suggest a hybrid algorithm of fuzzy time series and particle swarm optimization (PSO) algorithm to solve the forecasting problem. Such algorithm is considered a very effective and a recent method. It is inspired by 'birds' flight and communication behaviors. The used algorithm assigns the length of each interval in the universe of discourse, and the degree of membership values, and updates weights. The selected data sets are used to clarify the suggested method, and then compare the forecasting exactness with another method, that used a hybrid model based on the statistical model (ARIMA) and Artificial Neural Network (ANN). The results show that the suggested algorithm is more exact in forecasting time series, and can compete well with other methods.

Keywords: Forecasting, PSO, Fuzzy Time Series

1. Introduction

It is commonly agreed that forecasting plays an important and significant role in our daily life. It is widely used for making crucial decisions about the future. Although, there are many widely known methods of forecasting, the problems of forecasting they cannot solve in which the historical data are available in linguistic form. Fuzzy time series allows to overcome this drawback [1]. However, fuzzy time series are not just limited to linguistic values, and can be used for the prediction of numerical values too [2]. In the design of fuzzy logic it is possible to create fuzzy sets which are described using linguistic labels and have a membership level with values between [0,1] to characterize real world phenomena [3]. The forecasting of time series sometimes uses the statistical model, ARIMA. This method needs expert criteria, stationary data, correlation graphs analysis, model diagnostic checking, etc. These procedures may be complex, so it is important to study some new techniques that can find in the prediction tasks [3].

This study shows the results of forecasting using fuzzy logic and (PSO) for predicting the time series of cancer in Yemen. The PSO method is used to better the forecast using fuzzy logic, as this can help in finding a good solution to a complex problem in a way as to obtain the smallest forecast error for the time series. The suggested algorithm assigns the length of each interval in the universe of discourse and degree of membership values, simultaneously. Kuo *et al* [4] improved their method for forecasting enrollments based on the fuzzy time series and PSO. Most existing methods in fuzzy time series assume that the intervals in the universe of discourse have the same length.

Processing numerical data is one of the limitations of the linear models. A linear model may not be effectively constructed if the relationship between the variable is not linear. Also if multivariables are used in such a model for forecasting then calculating the parameters of the model is a difficult issue. Furthermore, the strong relationship among these variables may result in large errors. Being nonlinear, the fuzzy logic systems (FLS) have been combined with time series models to get rid of the problems occur with

linear data and can also handle linguistic chaos and ambiguity. Autoregressive (AR), Multiple Regressions, Exponential Smoothing and Autoregressive Integrated Moving Average (ARIMA) are a few examples of time series models that have been combined with FLS. These Fuzzy time series models are used by many researchers to solve linear and nonlinear problems. Since the introduction of FLS, it has successfully been applied to a lot of time series problems including stock and FOREX indexes [5] [6] [7] [8], enrollments [9] [10] [11], temperature [12], weather forecasting [13] [14], and disease diagnosing [15].

2. Literature Review

Forecasting is an organizational way of predicting future events and situations by evaluating the past values of a variables. Forecasting helps to overcome the distrust of future. In fact forecasting, of fuzzy time series not only covers the statistical quality but also the linguistic confusion analysis as well. There are number of studies that tackle the idea of fuzzy time series from different angles. For example, the idea of fuzzy time series based on the historical enrollments of the University of Alabama is practiced. Moreover in [9] the time invariant fuzzy time series model and the time-variant fuzzy time series model based on the fuzzy set theory for forecasting the enrollments of the University of Alabama is presented in [10][9]. Moreover, a heuristic model of fuzzy time series model has been developed to improve forecasting in [16]. Weighted models are suggested in [6], to resolve the recurrence and weighting issues in forecasting of fuzzy time series. Such models discuss the similarity to the weight functions in local regression models; though, both are dissimilar. The local regression models focused on fitting using a small amount quality of the data; however, the weighted fuzzy time series models made fuzzy relationships using the promising data from the whole database. PSO is integrated with fuzzy time series model in [26] for forecasting the time series of two numerical data sets, the enrollment of the university of Alabama and sales volume of products. The hybrid algorithm of fuzzy time series and PSO assigned the length of each intervals of discourse and degree of membership values, simultaneously. The results indicated that the

suggested hybrid algorithm can compete well with similar methods.

Hsu *et al.* [12] suggested a modified turbulent PSO method for the temperature prediction and the Taiwan Futures Exchange (TAIFEX) forecasting, based on the two-factor high order fuzzy time-invariant series. Kuo *et al.* [4] improved their method for forecasting enrollments based on the fuzzy time series and PSO. Most existing methods in fuzzy time series assume that the intervals in the universe of discourse have the same length.

3. Methodology

For forecasting, we collected the data of cancer patients in Ibb governments in Yemen for the period from 2010 to 2017 as a sample, for the experiment. We used the fuzzy logic and PSO algorithm for the same sample and we divided the period into sub-groups, we found the relation and inferred the laws. First, we applied the process of forecasting. Then, we updated the groups according to MSE and AD and we re-carried out the calculation; therefore, the result became better. After that, we used PSO algorithm for bettering the process of forecasting and we updated it to be more exact and faster for approximation. Finally, we compared the forecasting exactness with ANN and ARIMA time series methods suggested by Al-Badani [17]. In this study, we developed a new integrated method to assign the length of intervals and membership value, simultaneously. We suggested an algorithm to get the best forecasted value. PSO finds a near optimal solution for length of intervals and degree of membership values by updating the partials based on AMD objective function and their pbest and gbest in each iteration.

Fuzzy Time Series Definitions:

Fuzzy time series was first presented and defined by Song and Chissom [18]. A brief overview of the fuzzy time series definitions in the literature is included within the forecasting procedure is described as follows:

Definition1: Let U be the universe of discourse, where $U = \{u_1, u_2, \dots, u_m\}$ and let A be a fuzzy set in the universe of discourse U defined as follows:

$$A = fA(u_1)/u_1 + fA(u_2)/u_2 + \dots + fA(u_m)/u_m \quad (1)$$

wherein fA is the membership function of the fuzzy set A such that $fA: U \rightarrow [0, 1]$ and $fA(uk)$ represents the grade of membership of uk .

Definition2: Let $Y(t)$ ($t = 0, 1, 2, \dots$) a subset of a real number, be the universe of discourse on which fuzzy sets $fj(t)$, $j=1,2,\dots,n$ are defined. $F(t)$ is a collection of $fj(t)$, then $F(t)$ is called a fuzzy time series on $Y(t)$ ($t = 0, 1, 2,\dots$). Therefore, $F(t)$ is a linguistic variable and $fj(t)$ as the possible linguistic value of $F(t)$.

Definition3: If there exists a fuzzy relationship $R(t-1,t)$, such that $F(t) = F(t-1) * R(t-1, t)$ then $F(t)$ is said to be caused by $F(t-1)$; wherein y is an Max–Min composition operator. Considering a fuzzy logical relationship $Ai \rightarrow Aj$, where $Ai = F(t-1)$ and $Aj = F(t)$, Ai and Aj are the left and right hand sides of the fuzzy logical relationship, respectively.

Definition4: If $R(t-1,t)$ is independent of t , then $F(t)$ is considered as a time-invariant fuzzy time series; otherwise, $F(t)$ is a time variant fuzzy time series whether it is caused by $F(t-1), F(t-2), \dots,$ and $F(t-m)$, ($m > 0$). In this forecasting method, the relation can be expressed as the fuzzy relational equation:

$$F(t) = F(t-1) * R^w(t-1, t) \quad (2)$$

where w is the number of months which the forecast is being affected.

Definition5: Relationships with the same fuzzy set on the left hand side can be further grouped into a relationship group. Relationship groups are also referred to as fuzzy logical relationship groups or FLRG 's in short. Suppose there are relationships such that $A_i \rightarrow A_{j1}$, $A_i \rightarrow A_{j2}, \dots, A_i \rightarrow A_{jn}$ then they can be grouped into a relationship group as table 1:

$$A_i \rightarrow A_{j1}, A_i \rightarrow A_{j2}, \dots, A_i \rightarrow A_{jn}$$

$$A_2, A_2, A_3, A_1 \rightarrow A_1$$

$$A_2, A_3, A_1, A_1 \rightarrow A_3$$

$$A_3, A_1, A_1, A_3 \rightarrow A_5$$

$$A_1, A_1, A_3, A_5 \rightarrow A_4$$

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$$A_5, A_6, A_4, A_6 \rightarrow A_6$$

The same fuzzy set cannot appear more than once on the right hand side or the relationship group. The term relationship group was first introduced by Chen in [19].

Forecasting:

The used algorithm below for the forecasting of cancer is based on (2010 to 2016) time series production data. In the following steps, it will be demonstrated how the model is used to forecast cancer. Actual cancer data are taken from the period 2010 to 2016 as shown in table 1. Finally, forecasting process looks as follows:

Process 1: Count the variables of the historical data. The variable V_t of the data between time $t(dt)$ and $t-1 (dt-1)$ is counted as $V_t = dt - dt-1$ ($t = 2, 3, \dots, n$).

Process 2: Define the discourse universe of. Find the maximum (D_{max}) and the minimum (D_{min}) among all V_t . The discourse universe U is, then, defined as $U = [D_{min} - D1, D_{max} + D2]$ where $D1$ and $D2$ are two proper positive numbers.

Process 3: In this study we consider a fuzzy number so as to fuzzify the intervals. In this process, appropriate interval length, membership values, degree and weights should be assigned by PSO.

Process 4: Fuzzify the variables of historical cancer data. If the variable V_t is within the scope of uj , it belongs to fuzzy set A_j . All of the variables must be classified into the corresponding fuzzy sets as table 1.

Table 1: Fuzzified historical forecast Cancer

Month	Actual value	Forecast	Fuzzy sets Cancer	Month	Actual value	Forecast	Fuzzy sets Cancer
Jan	38	38	A9	Nov	34	34	A8
Feb	41	40.40654	A10	Des	24	26	A6
Mar	31	30	A7	Jan	33	35.59726	A8
Apr	33	34	A8	Feb	31	30	A7
May	28	29.31572	A7	Mar	31	30	A7

Jun	27	25.0954	A6	Apr	25	26	A6
Jul	30	28.02377	A7	May	29	30	A7
Aog	26	24.42038	A6	Jun	43	43.09762	A10
Seb	54	54.3085	A13	Jul	41	42	A10
Oct	36	38.30182	A9	Aog	37	38	A9

Process 5: Calculate the fuzzy time series $F(t)$ at window base w . The operation matrix $Ow(t)$ and the

$$C(t)=F(t-1)=[C_1 C_2 \dots C_m] \quad (3)$$

Criterion matrix $C(t)$ are selected to count the fuzzy forecasted $F(t)$.

where C_j indicates the membership value at the interval uj within fuzzy set A_i .

Then, $F(t)$ can be calculated as the maximum of every column in $R(t)$ as follows:

$$F(t)=[\max_{k=1, \dots, w-1}\{R_{k1}\} \dots \max_{k=1, \dots, w-1}\{R_{km}\}] \quad (4)$$

Process 6: Forecasted value. Suppose there are k non-zero values corresponding to intervals u_1, u_2, \dots, u_k , with their midpoints m_1, m_2, \dots, m_k , respectively. Similar to Liu et al. [20] defuzzification method, we use weighted average method to calculate the defuzzified C_{vt} :

$$C_{vt} = \frac{\sum_{i=1}^m f_i m_i}{\sum_{i=1}^m f_i} \quad (5)$$

The forecasted value F_{vt} at time t is counted as follows:

$$F_{vt} = C_{vt} + d_{t-1} \quad (6)$$

There are various measurements used to evaluate time series forecasting [21]. Mean Square Error (MSE), Root Mean Square Error (RMSE), and Mean Absolute Percentages Error (MAPE) are described in (1), (2), and (3) respectively [22][23].

$$MSE = \frac{1}{N} \sum_{i=1}^n (forecast_i - actual_i)^2 \quad (7)$$

$$RMSE = \sqrt{MSE} \quad (8)$$

$$MAPE = \frac{1}{N} \sum_{j=1}^n \left| \frac{T_j - Y_j}{T_j} \right| \quad (9)$$

where T_j is the expected value, Y_j is the output, and N is the total number of samples.

Particle swarm optimization:

Particle swarm optimization (PSO) is an evolutionary and population method which is based on the stochastic optimization technique proposed by Eberhart and Kennedy [24, 25]. PSO was first introduced to optimize various continuous nonlinear functions and requires only primitive and simple mathematical operators[26]. In this method, each solution is like a bird in the search space, called 'particle'. All particles in PSO have fitness values, evaluated by fitness functions. Each particle has also a velocity that determines its flight direction. In the PSO, particles fly in search space following particles with best solution. Firstly, the PSO consists of a randomly produced population and velocity. Then, the velocity is dynamically adjusted at each step according to the experience by itself and its colleagues as given by Eq(10)[26].

The new particle position is found by adding the new velocity to the current position Eq(11).

$$V_{i,t+1} = w \cdot V_{i,t} + R1 \cdot C1(Pi - Xi,t) + R2 \cdot C2(Pg - Xi,t) \quad (10)$$

$$Xi,t+1 = Xi,t + Vi,t+1 \quad (11)$$

Where i is the i th particle; Xi,t is the position of particle i in iteration t ; Vi,t is the velocity of particle i in iteration t ; Pi is the best previous position of particle i so far ($pbest$) and Pg is the best previous position among all the particles ($gbest$). w is inertial weight and its function is to balance global and local exploitations of the swarm. The most applicable way of using inertia weight is linear decreasing [27], which is determined as follows:

$$w = w_{max} - ((w_{max} - w_{min}) / iter_{max}) \cdot iter \quad (12)$$

Where w_{max} is the initial value of weighting coefficient; w_{min} , the final value of weighting coefficient; $iter_{max}$, maximum number of iterations; and, $iter$ is the current iteration. $C1$ and $C2$ are two learning factors which control the influence of $pbest$ and $gbest$ on the search process and are usually set 2 to cover the whole region of $pbest$ and $gbest$. $R1$ and $R2$ are two random numbers within the range of [0, 1].

In this study, after the weights have been optimized via PSO, the 'blanks' in the 'Then' part can be filled. The partially completed if-rules from table 2 are shown in fully completed form in table 2.

Table 2: Generated if-then rules in chronological order

No Rules	if-then rules	Then	Wight's			
1	if $x_1 \in A_2$ and $x_2 \in A_2$ and $x_3 \in A_3$ and $x_4 \in A_1$	then	W1=0.29658	W2= -0.13442	W3=0.23372	W4=0.74985
2	if $x_1 \in A_2$ and $x_2 \in A_3$ and $x_3 \in A_1$ and $x_4 \in A_1$	then	W1=0.48306	W2=0.21997	W3=0.69238	W4=0.35498
3	if $x_1 \in A_3$ and $x_2 \in A_1$ and $x_3 \in A_1$ and $x_4 \in A_3$	then	W1=0.61283	W2=0.41238	W3=0.21561	W4=0.55086
n	if $x_1 \in A_n$ and $x_2 \in A_n$ and $x_3 \in A_n$ and $x_4 \in A_n$	then	W1= n	W2= n	W3= n	W4= n

The process of PSO algorithm is as follows:

Process1: Initialization of a population of particles with random positions and velocities.

Process 2: Evaluation of the objective values of all particles, put $pbest$ of each particle equal to its current position, and put $gbest$ equal to the position of the best initial particle.

Process 3: Updating the velocity and position of particles according to Eqs. (10) and (11).

Process4: Evaluating the objective values of all particles.

Process 5: Comparing each particle's current objective value with its $pbest$ value. If the current value is better, update $pbest$ with the current position and objective value.

Process 6: Assigning the best particle of the current whole population with the best objective value. If the objective value is better than that of $gbest$, update $gbest$ with the current best particle.

Process 7: If it is not, go back to Step3.

After the weights have been optimized via PSO, the 'blanks' in the then part can be filled. The partially

completed if-rules from table 2 are shown in fully completed form in table 2.

Comparison of Forecasting Results

In this section, we compare the forecasting results of different method on historical data of cancer time series. A comparison of mean square errors (MSE) of different existing methods is shown on table 3.

Table 3: A comparison of mean square error (MSE) of Cancer production forecast

Models	MSE	MAE	RMSE	MAPE
Hybrid ANN and ARIMA	0.028711	0.344533	0.169443	0.202654
The suggested cancer of forecasting systemHybrid Fuzzy-pso	0.00037	0.0113	0.0007	0.1029

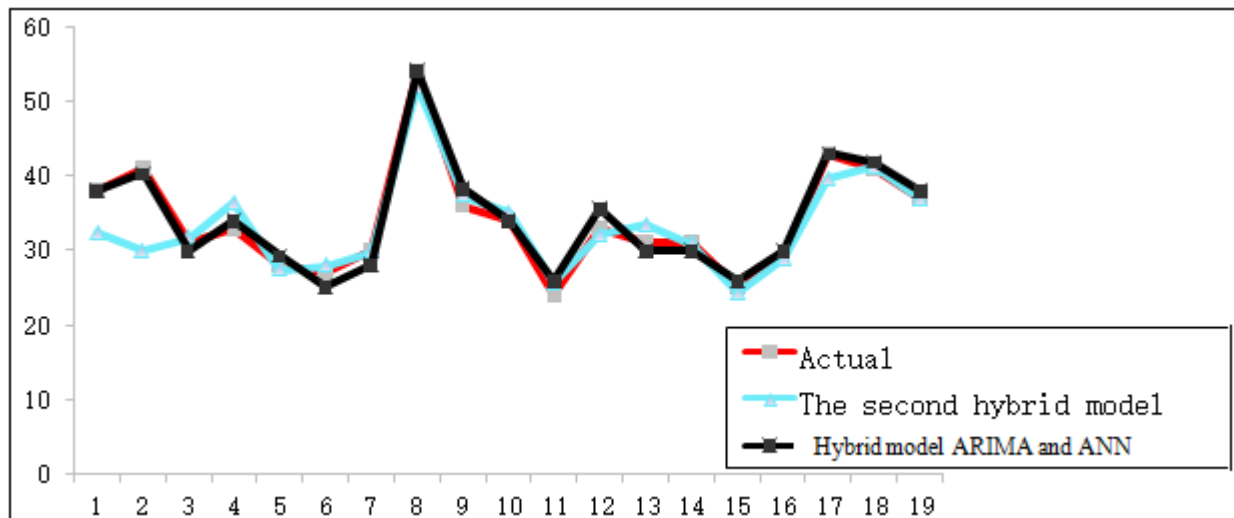


Figure 1: Actual cancer productions vs. forecasted cancer

4. Conclusion

The motivation of the implementation of fuzzy time series in different cancer forecast is to support the development of decision support system in cancer system, one of the real life problems falling in the category having uncertainty in known and unknown parameters. The past experiences reveal that the cancer system is a complex process and hard to model by the mathematical formulations. The historical time series cancer data used in the present study is taken from cancer patients in Ibb governments in Yemen. The other goal was to compare AREMA method with various fuzzy time series methods. It is observed that it produces more accurate results in comparison AREMA and ANN hybrid.

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