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Harnessing Students' Mathematical Flexibility with Cognitively Challenging Tasks

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Abstract: ***The purpose of this study is to determine the effectiveness of utilizing cognitively challenging tasks in Algebra to harness students' mathematical flexibility. This study was conducted at the University of Science and Technology of Southern Philippines (USTP) using two intact classes randomly assigned as the experimental and control group. Both groups was taught using lecture discussion method, the only difference is that the experimental group was assessed using cognitively demanding or challenging tasks while the control group was given open-ended problem solving tasks. Before and after the experiment, both groups was given the Mathematical Flexibility Test (MFT) to determine their level of mathematical flexibility in solving linear equations in one variable. Data collected were analyzed using mean, standard deviation and one-way analysis of covariance (ANCOVA). Results of the analysis revealed that the students' level of mathematical flexibility was fairly satisfactory in the experimental group while still needing improvement for the control group. The researcher then recommends that mathematics teachers across all levels may utilize cognitively challenging tasks paired with appropriate pedagogy to have greater increase of their mathematical flexibility scores. Future research may also be conducted on other assessment styles and considering other participants across all levels or from different colleges to have generalizability of the results.***

Keywords: mathematical flexibility, cognitively challenging tasks, open-ended problem-solving tasks

1. Introduction

A teacher is a facilitator of learning. With this premise, a teacher therefore does not deserve to be addressed as "teacher" unless there is discovery of a successful transmission of learning among students and that the learner has also shown indications that teaching was indeed learned. Hence, assessment is an indispensable means "for" and "of" learning. Any educational or training institution like the University of Science and Technology of Southern Philippines (USTP) shall only determine the success and failure of their existence through assessment, and the value of judgment on its results is basic for organizational and institutional improvement plans in all levels. The only way that the teacher is convinced that indeed his or her learners truly have learned is to conduct an assessment (Buenaflor, 2012).

Recently, the Philippines implemented the K-12 curriculum in the basic education and thus institutions of higher learning like the USTP also made changes with their curriculum offerings consistent with the changes in the basic education level. The curriculum reform however is also accompanied by assessment issues. Assessment needs to be integrated with, not separate from, curriculum and instruction to enhance student learning. In the context of the outcomes-based education, there is a pressing need to design assessment methods which helps promote the kind of graduates the institution would want to produce in order for them to match the need of the global workplace where certain skills are needed to compete with foreign counterparts in the job qualifications.

Flexibility in mathematical learning and mathematical thinking has been more explicitly highlighted as a cornerstone by several mathematics educators. Flexibility is, as is widely known, as one of the essential elements of creativity which is the ability of students to create qualitatively different solutions in the diverse context (Lee & Carpenter, 2015). In another definition made by Star (2017), flexibility in the education context means the ability to easily adapt or adjust to certain circumstances. This ability to adapt seems generally valuable in all aspects of mathematics teaching and learning. Flexibility in mathematical learning is a key to empowering learners to tackle complexity and uncertainty and to giving them the capacity and motive to change in the innovative era.

In this context, this study sought to determine the effectiveness of employing cognitively challenging mathematical assessment tasks to harness students' level of mathematical flexibility.

2. Literature Review

Research activities concerning mathematical flexibility were considerably credited to the work of Star since 2001 up to present although early seminal work on flexibility in mathematics education can be trace back in the work of Krutetskii (1976) and Wertheimer (1959) and was considered as an essential element of adaptive practice (Hatano & Inagaki, 1986). Also, children's flexibility in addition and subtraction was extensively studied by European researchers in 2000 and beyond. Star (2018) got interest in the study of mathematical flexibility when he was grappled with the distinction between conceptual knowledge and procedural knowledge. In his recent studies, he give two operational definitions of mathematical flexibility, that is, knowledge of multiple, standard and efficient strategies and the ability to select the most appropriate strategy which is a hint inherent to the development of flexibility. Looking back on his earlier work on mathematical flexibility, Star & Seifert (2006) in their study address the two components of flexibility, that is, students' knowledge of strategies and their

ability to implement them which involves asking students' to resolve previously completed problems but on a different strategy. They believed that that strategy students' chose to use in their first attempt on each problem was presumed to convey some information about their use of strategies and they found out that indeed it is somewhat related to their previous attempt.

Mariquit and Luna (2015) probe students' conceptual understanding, mathematical fluency, and mathematics anxiety through cognitive-demand mathematical tasks. In the experimental teaching, the relationship of the three factors and mental ability were investigated. To investigate and establish logical relationships, two intact groups from among freshman students in school Algebra were taken as participants and then decisive success factors were identified. This leads to the conclusion that infusing non-routine cognitive-demand mathematical tasks is effective in enhancing the participants' conceptual understanding and mathematical fluency. Furthermore, students' mental ability influences conceptual understanding and mathematical fluency but it does not affect the participants' mathematics anxiety. This study was related to the present study however the present study would not be using Algebra course. Also, the findings of the above study would be in support to the assumption of this present study that high cognitively demanding mathematical tasks as an assessment style would improve students' mathematical flexibility in which fluency and conceptual understanding are assumed to be imbedded with flexible problem solving skills.

In another unique study of Norton & Kastberg (2012), they used letter writing as a means for pre-service teachers (PSTs) to develop ability to design effective tasks, in terms of eliciting high levels of cognitive activity from students. Studies on student-dependent task analyses, by assessing the levels of cognitive demand indicated in students' responses, have demonstrated significant growth among PSTs over the course of letter-writing exchanges. They examined growth with a qualitative analysis of two PSTs who became effective at designing tasks that elicited high levels of cognitive activity. In particular, they examine how those PSTs accounted for tasks that did not elicit the kinds of activity they expected and how they adjusted their tasks to elicit higher levels of activity. They found disparity between the two PSTs' apparently successful approaches, one that fit the larger goals of the project and one that fit only the descriptions specified in the project rubric. The study affirms the potential value of letter writing projects while introducing a concern that has implications for all professional development projects. The above study pointed out the significance of letter writing activities for the prospective teachers to design high levels of cognitive activity which the present study wanted to highlight as a means to improve students' mathematical flexibility. Although, their respondents are prospective mathematics teachers, it is also relevant because designing cognitively demanding mathematical tasks need to be mastered and this present study employed and theorized to affect students' level of mathematical flexibility.

In the study conducted by McCormick (2016), she explored how primary teachers in Australia describe their efforts to integrate problem solving through their choice of task, and in particular examined the features and cognitive demand of reported tasks. Overall, direct instruction followed by application tasks, as well as real world investigation tasks, are more favorable among the participating teachers when developing problem solving proficiency in the classroom. These tasks tended to be of lower level cognitive demand (procedures without connections) requiring previously taught facts and procedures to be applied on routine problems. While real world investigation tasks have the potential to promote higher order thinking, the complexity of the chosen tasks appears to be reduced. Furthermore, participation in the Encouraging Persistence, Maintaining Challenge (EPMC) project in Australia appears to have influenced some teachers' problem solving practices to the extent that teachers are choosing and implementing challenging and unfamiliar tasks. From these findings it appears that without their participation of the EPMC project, primary teachers of Australia have a tendency to give students limited opportunities to solve challenging and unfamiliar problem solving tasks. The notion of shallow teaching continues to be evident in the reported practice, where students are asked to follow procedures without connections and solve low complexity problems with excessive repetition. Not only will students be unable to develop complete mathematical understanding, but also their ability to solve challenging and unforeseen problems in the real world will be greatly compromised. This is also true in the Philippines where students' are deprived of their learning opportunities by always giving students' low level cognitive demanding mathematical tasks and this present study would explore how high cognitive demanding mathematical tasks enhance students' level of mathematical flexibility.

3. Results

Table 1: The Mean and Standard Deviation of the Students' Achievement Test Scores

	Control Group		Experimental Group	
	Pretest	Posttest	Pretest	Posttest
Mean	6.76	7.54	5.51	11.46
Standard Deviation	4.924	4.781	4.365	4.031

Perfect Score: 40

Mean Level	Descriptive Level
31 – 40	Very High Flexibility
21 – 30	Moderately High Flexibility
11 – 20	Fair Flexibility
1 – 10	Poor Flexibility

Table 1 above presents the pre-test and post-test distribution of the mathematical flexibility scores of the control and experimental groups. It can be gleaned from the table that both groups showed poor flexibility scores before the start of the experiment and both almost remain poor after the experimental period. The experimental group had shown an increase however it only reached the fair level of mathematical flexibility. This means that both groups have shown underwhelming performance in terms of their level of mathematical flexibility despite the experimental group showing an increase but it does not guarantee that the interplay of the assessment styles can have a stronger impact

on really attaining a remarkable increase in their mathematical flexibility scores. Further, it really manifested that mathematical flexibility was difficult to develop amongst students because majority of the students in both the control and experimental groups are still very dependent on what the teachers has taught them and therefore they can only create solutions which they have experienced solving during their class sessions.

Table 2: The Analysis of Covariance of Students' Mathematical Flexibility Test Scores

Source of Variation	Adjusted Sum of Squares	df	Adjusted Mean Squares	F Computed	p-Value
Treatment	208.10	1	208.10	14.11	0.0037*
Error Within	958.43	65	14.75		
Total	1166.53	66			

*significant at $p<0.05$

Table 2 above presents the comparison of the students' mathematical flexibility when provided with the interplay of different assessment styles such as problem posing, cognitively demanding and error identification tasks and the controlled condition was given open-ended problem solving tasks. The analysis of covariance yielded an F-ratio of 14.11 and probability value of 0.0037 which led to the rejection of the null hypothesis at 0.05 level of significance. This implies that the mathematical flexibility scores of students exposed to the interplay of different assessment styles had a better performance than those students who experienced only open-ended problem-solving tasks in Algebra. This further implies that although the students are really having difficulty in showing an alternate solution for the given problem, the different assessment styles helped them improve slightly on their mathematical flexibility scores. This is quite encouraging for mathematics teachers to really practice these kind of tasks, most especially if non-routine problems will be considered in the learning tasks. The impact may not be very huge on students' level of mathematical flexibility but the researcher believed that when this assessment styles will be coupled with effective pedagogy, then this might have a different leap on the learning gains of students flexibility and might be on the other indicators of mathematical creativity which in the study of Roble (2018) was able to established, that is, pre-within-post problem posing and problem-based Hawaii Algebra Learning Project (HALP) model was effective in enhancing students' level of mathematical fluency, flexibility and novelty of solutions. In this study, the teaching method was only lecture-discussion which might be also a factor why the learning gains of students to improve their level of mathematical flexibility was not that very remarkable as anticipated. Furthermore, it might be also due to the fact that the quality of students who took part of this study was not that really good because these students are technology students which majority of them are not STEM graduates in their senior high school. If they are not STEM graduates, they are most likely to have very few mathematics courses and thus preparation for college Algebra may also become a problem for these students.

4. Findings

Based on the analysis, the researcher found out that before the start of the experiment, the level of mathematical flexibility of students' in both the experimental and control groups was relatively poor. After the experimental period, the control groups' mathematical flexibility score improved but remained poor while the experimental group reached the fairly flexible level. This means that the cognitively challenging mathematical tasks helped harnessed students' level of mathematical flexibility which according to many mathematics educators is very difficult to develop among students. Moreover, the level of mathematical flexibility of the experimental group was significantly different from that of the control group which are taught using open-ended problem solving tasks.

5. Conclusions and Recommendations

This study concludes that the students' level of mathematical flexibility was fairly satisfactory and needs improvement despite using cognitively demanding mathematical tasks. This validates the claim of many mathematics educators that mathematical flexibility was indeed difficult to develop most especially on students' with poor foundational skills in mathematics. The researcher then recommends that mathematics teachers across all levels may utilize cognitively challenging tasks paired with appropriate pedagogy to have greater increase of their mathematical flexibility scores. Future research may also be conducted on other assessment styles and considering other participants across all levels or from different colleges to have generalizability of the results.

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Author Profile

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Dennis B. Roble is a faculty and a researcher of the Department of Mathematics Education of the College of Science and Technology Education of the University of Science and Technology of Southern Philippines (USTP). He finished his Bachelor of Science in Mathematics at Xavier University-Ateneo de Cagayan and both his Master's degree in Teaching Mathematics and Doctor of Philosophy in Mathematics Education at USTP as a CHED scholar. He has also attended an autumn school in the University of Wurzburg, Germany last October 2018. He is actively involved in personal, collaborative and institutionally funded researches. He was also able to publish research articles in both CHED recognized and internationally peer-reviewed journals and disseminate research outputs in international conferences. Recently he has presented his research papers in Germany and Taiwan.