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Simulation of the Geometric Instability of Wood Beams by Using the Newton Raphson Method

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Abstract: ***A theoretical model of nonlinear computation of wood beams section has been developed. Here, we had the opportunity to study the mechanical behavior of a console beam. The digital model uses 3D Vlasov beams. The equilibrium equations are obtained using the virtual works principles. In the literature, analytical solutions called critical loads of Euler exist to estimate the critical loads value. These analytical solutions have a restricted field of application. The beam finite element model was developed and allowed us to better appreciate the nonlinear behavior of the beam in the presence of instabilities.***

Keywords: geometric instability; critical spill load; ruin; 3D beam; Vlasov theory; incremental method

1. Introduction

In general, the modeling of material is a means of transcribing its laws of behavior through mathematical, mechanical and physical formulations. A structure under the stresses effect (bending, torsion, compression ...) usually results in a loss of geometric stability. This modifying effect of the equilibrium nature can lead to its ruin (material or geometric) has been the subject of many discussions in the past. in order, to overcome this instabilities problem presented by structures (beams and columns), the critical spill load determination becomes thus an ultimate concern because it appears here as an instability phenomenon indicator. For the study concerning (material or geometrical) instability phenomena, several approaches exist to estimate the critical spill load of wood beams and columns. Analytical solutions exist but their scope application is limited. The direct methods as for them [Batoz j.L. et al., 1990; Ayina et al, 1996; Ayina et al, 1998], does not provide information on the post-critical behavior of the beam. The incremental method itself [Ayina et al, 1985; F. MOHRI et al, 2009; H. Zarhouni, 1998; D. Dureisseix, 2014; M.Duval, 2016], allows a good predictability of the critical load based on the post critical analysis behavior of the structure.

Also remember that most of the work done by the many authors as [Ayina et al, 1985; F. MOHRI et al, 2009; H. Zarhouni, 1998; D. Dureisseix, 2014; M.Duval, 2016] using the incremental method was a base on the metal structures for both beams and plates.

These works deal the solid or reconstituted wood beams instability phenomenon. The critical spill load prediction is our main concern. In the present work, we are interested in wood beams columns of high height with cross section [Ayina, 2002].

The present works contribute to the valorization of the woods of the Congo Basin campaign. Many authors have already participated in this campaign to valorize the wood of the Congo Basin, we can cite for example [Ayina et al, 1998] work based on the torsional behavior of the wood material more precisely the Movingui and the Bilinga ; [J. Kisito Mvogo et al., 2008] based on the vibration analysis of the Congo Basin woods; [N. Manfoumbi, 2012] on Contribution to the adaptation of Eurocode 5 to tropical species in their environment, studies made in France ville in Gabon; [Oum Lissouck et al, 2014] on Multicriteria Classification and Structural Bonding of Wood Species in the Congo Basin, Limiting the Impact on Biodiversity[**J L Nsouandélé1 et al, 2016]** Experimental Determination of Volume Masses of Some Tropical Timbers Function of Their Moisture and the Temperature ... It should be recalled here that most of the work carried out by these different authors was experimental in nature. As we know, the experimental study requires large means, and the experimentation time is often very long (several weeks, several months, year, see more). We, poor world countries, do not always have well-equipped laboratories to carry out our experiments. In most cases, we are often required to travel extensively to developed countries to acquire results in state-of-the-art laboratories. The digital model that we will build will not only reduce the time of acquisition of the results but also, will allow us to have only our computer as our working material.

We propose here to present concretely a numerical modelisation of the nonlinear computation of a wood material adapted to the study of the geometrical instabilities phenomena in particular the estimation of the critical loads and the simulations of the dumping of the big beams as well as those relating to the buckling of the columns. We will highlight the definite advantages of using the non-linear geometric model developed by [AYINA et al, 1998]. It is a model whose instantaneous elemental stiffness matrix is derived from 3-D elements of Vlasov type. Numerical simulations will allow us to follow post-critical deformations after the bifurcation by updating elementary stiffness matrices through an incremental formulation.

The choice of our incremental resolution method will have to take into account several factors:

- The type of non-linearity predominant or not;
- Convergence speed and Accuracy [A.Legay et al, 2002];
- The divergence risk.

Notations

W : virtual axial displacement (following Z axis)
U : virtual displacement following X direction
V : Virtual displacement following Y direction
L : beam lenght
θ_x : rotation associated to flexion in ZY plan
ϕ : warping
ω : rectangular beam section
W_{int} :virtual work of internal forces
W_{ext} : virtual work of external forces
K_L : elastic stiffness matrix
K_T : tangent stiffness matrix
$\langle u \rangle$: Transposed nodal field vector of displacement
B : beam thickness croos-section
h : High of the beam cross-section
E : MOE
G :Sliding modulus
$\{F\}$: nodal force vector
$[C]$: behavior matrix of material.
$[B_L], [B_{NL}]$: shape functions matrix derivatives

2. Non-Linear Problems Resolving Methods

Non-linear problem solving algorithms have become a necessity in the development of complex behavioral models. It becomes essential to have reliable and efficient resolution algorithms. The classical resolution algorithms used in the finite element method are incremental and iterative algorithms, which often have convergence problems mainly related to the limit of points in loads existence , displacements or both at the same time. However, these resolution algorithms generally depend on the nonlinear problem type that one wishes to deal with.

The methods of resolution that we will present in what follows, will be based on incremental processes. They consist of applying in successive increments a load level and to find for each given increment, the structure response. The latter is obtained after having linearized in each increment the equilibrium equations [H. Zarhouni, 1998; D. Dureisseix, 2014; M.Duval, 2016].

These incremental methods are divided into two types:

- Pure incremental methods;
- Iterative incremental methods.

2.1 Pure Incremental Method

In this method, a increment load is imposed, the tangent stiffness matrix allows to have the increase of the corresponding displacement. Indeed, the equilibrium is not corrected which leads most often to a divergence of the desired solution (see Figure 1 below).

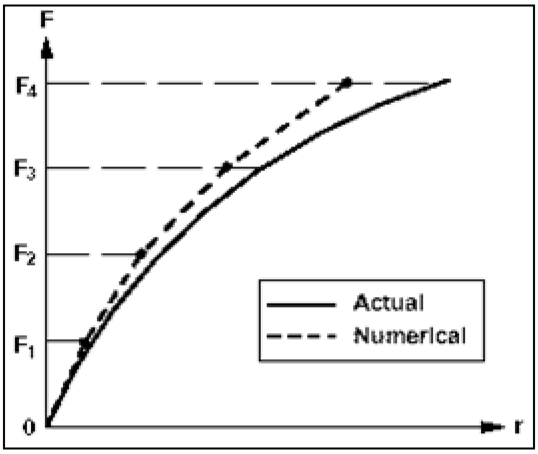


Figure 1: Pure incremental method

2.2 Incremental Iterative Method

The incremental iterative method uses the same process as the previous method. Here, a balance correction is introduced on each increment using an iterative process. This correction can be done in several ways depending on the type of stiffness matrix used (initial, tangent). Defining several incremental iterative methods, the best known of which is Newton-Raphson.

2.2.1 Newton-Raphson Tangent Stiffness Method

This Newton-Raphson method uses the tangent stiffness matrix recalculated at each iteration for the correction of the equilibrium (see figure 2 below)

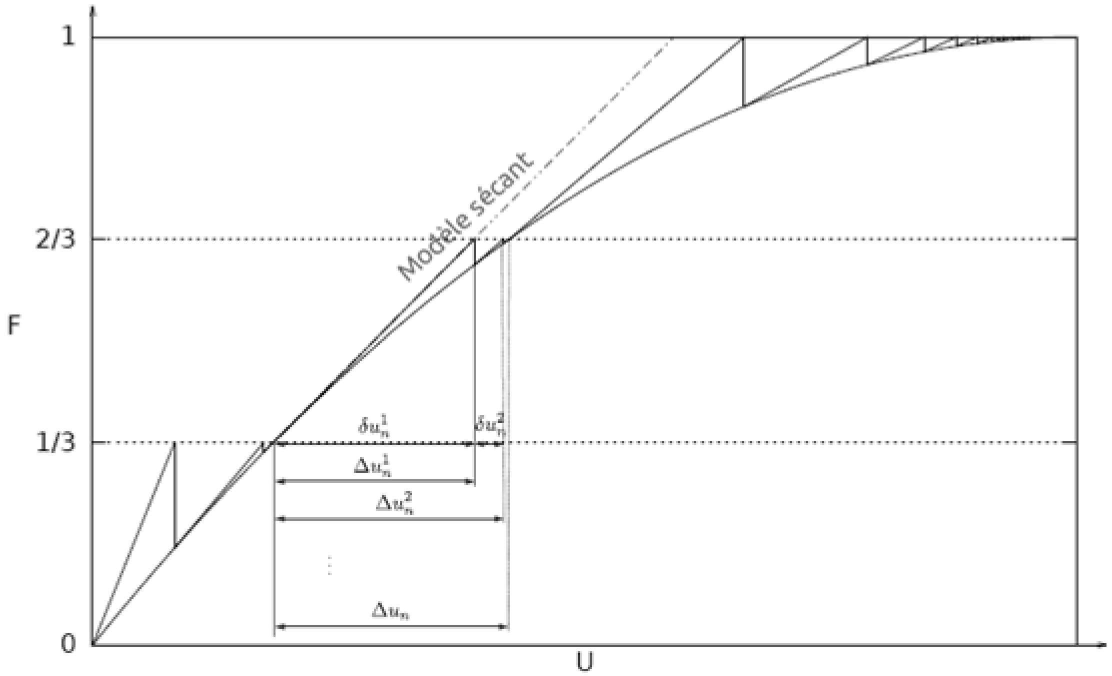


Figure 2: Newton-Raphson method, KT updated at each iteration

2.2.2 Modified Newton-Raphson Method

The modified Newton-Raphson method is identical to the previous one but uses the tangent stiffness matrix recalculated at the beginning of each increment and kept constant for all the iterations for the correction of the equilibrium (see figure3 below).

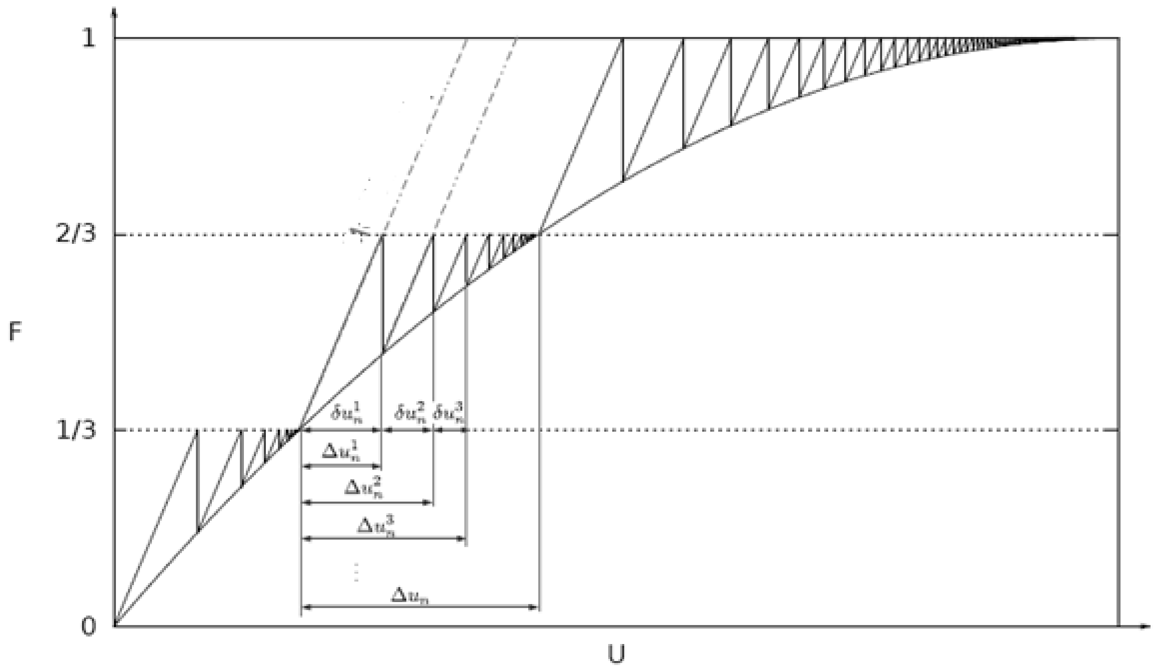


Figure 3: Modified Newton-Raphson method, KT is constant on each increment

Indeed, in the pure incremental method, the equilibrium is not corrected which often leads to a divergence of the sought solution this problem can be avoided by using very small increments, which makes the method heavy.

The modified Newton-Raphson method has a less rapid convergence than the previous one, but it has the advantage of keeping the stiffness matrix constant for each increment, which makes it possible to have a significant gain in the calculation time.

Newton-Raphson's method with tangential rigidity has a very fast convergence, but its main disadvantage lies in the computation time of the updating of the stiffness matrix tangent to each iteration.

At the introduction, we specified that the choice of a method in computation in the calculation of the structures should take into account several criteria namely the type of predominant nonlinearity and the speed of convergence. In the rest of our present work, we will use the Newton-

Raphson iterative method with tangent rigidity because it is better adapted to our problem. Currently, the Newton-Raphson tangent stiffness resolution method is the basis of the most popular resolution algorithms for solving non-linear structure problems [H. Zarhouni, 1998; A.Legay et al, 2002; O.Boudrioua et al, 2002; D. Dureisseix, 2014; M.Duval, 2016]. Note that in this method, we always win in terms of the number of iterations resulting in rapid convergence.

3. Methodology

After building our 3D beam element Vlasov type, we will bring out the kinematics and equilibrium equations, then we will establish the main stiffness matrices of our problem.

3.1 Kinematic Equations

Let Hencky's beam [Ayina et al, 2000] below move in space in a dual XYZ and global coordinate system xyz [Ayina, 1999].

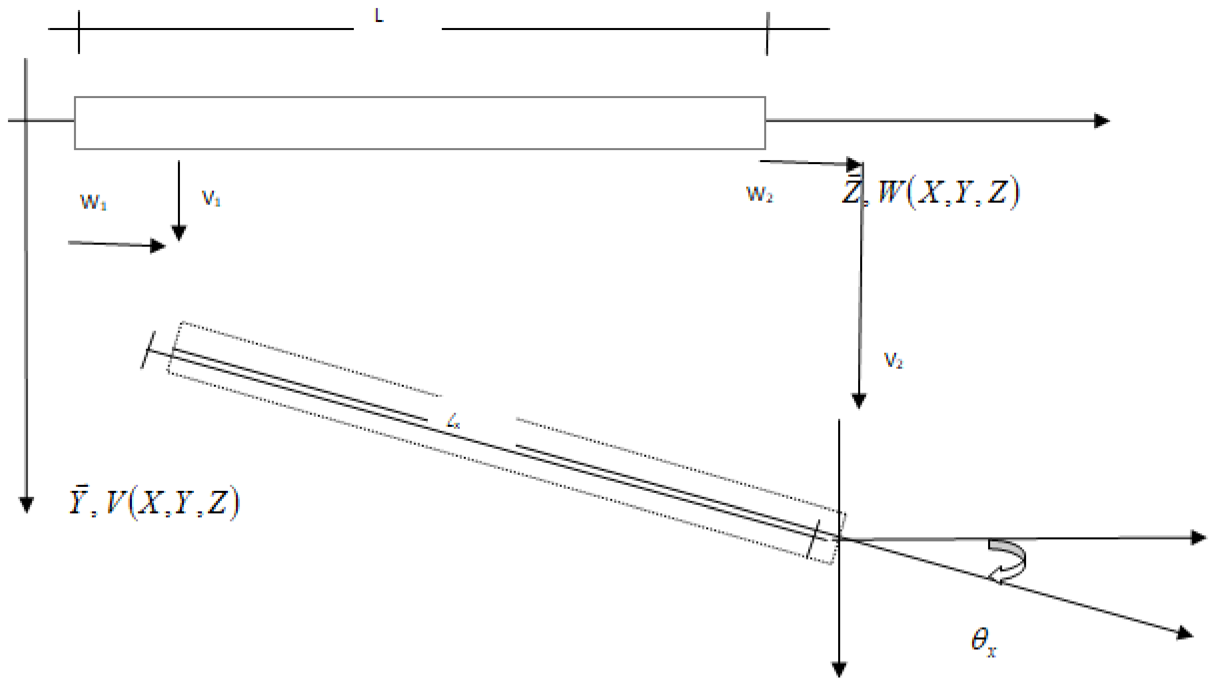


Figure 4: field of displacements of a point P of the beam

The Vlasov theory gives us the virtual field displacements of point P (x, y) of a material from the virtualized generalized displacements of the center of gravity O (0,0) of the cross section.

$$\begin{pmatrix} du_p \\ dv_p \\ dw_p \end{pmatrix} = \underbrace{\begin{pmatrix} du - y\phi \\ dv - x\phi \\ dw - xu' - yv' - \omega\phi \end{pmatrix}}_{\text{Linear part}} + \underbrace{\begin{pmatrix} \frac{-x(u'^2+\phi'^2)}{2} - \frac{yu'v'}{2} - \omega u'\phi \\ \frac{-xu'v'}{2} - \frac{y(v'^2+\phi'^2)}{2} - \omega v'\phi \\ -xv'\phi + yu'\phi \end{pmatrix}}_{\text{non-linear part}} \quad (1)$$

The linear part

$$\begin{pmatrix} du - y\phi \\ dv - x\phi \\ dw - xu' - yv' - \omega\phi \end{pmatrix} = \begin{pmatrix} du \\ dv \\ dw \end{pmatrix} + \begin{pmatrix} u' \\ v' \\ \phi \end{pmatrix} \wedge \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega\phi \end{pmatrix} \quad (2)$$

$$\frac{\partial \vec{OP}}{\partial t} = \vec{\Omega} \wedge \vec{OP} + \left[\frac{\partial u(P)}{\partial t}\right] \quad (3)$$

Equation (2) is the kinematics equation of the problem. The term $\frac{\partial \vec{OP}}{\partial t} = \vec{\Omega} \wedge \vec{OP} + \left[\frac{\partial u(P)}{\partial t}\right]$ is the equation that governs the description of the motion of a rigid body in Eulerian description. The term $(dw - xu' - yv' - \omega\phi)$ is the linear component of axial displacement caused by torsion and $(-xu' - yv')$ specifically represents the numerical quantity which translates the respective contributions of inflections along the x and y directions. The linear part of relation (2) is the analytical expression of virtual kinematic relations associated with the warping of thin-walled sections usually known as Vlassov's theory.

The resulting deformation field is given by:

$$\begin{pmatrix} \varepsilon_{xp} \\ \varepsilon_{yp} \\ \varepsilon_{zp} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ w' - xu'' - yv'' - \omega\phi'' \\ 0 \\ 0 \\ 2x\,\phi' \end{pmatrix}}_{\text{linear part}} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \frac{1}{2}(u'^2 + v'^2) - xv''\phi + yu''\phi + \frac{1}{2}(x^2 + y^2)\phi'^2 \\ 0 \\ -v'\phi \\ u'\phi \end{pmatrix}}_{\text{non-linear part}} \quad (4)$$

3.2 Stiffness Matrices

The virtual works principle allows us to write:

$$dW_{\text{int}} = dW_{ext} \quad (5)$$

The discretized expression of the complete form of the virtual works principle in updated Lagrangian description developed. when we express the displacements, the deformations and their variations in the usual discretized forms used in the finite element method, the complete form of the equilibrium equations of a structure discretized in updated Lagrangian description is given by the equation system(6) below translating the equilibrium equation .

$$\left({}^{t}k_{\alpha\beta}+{}^{\tau}k^{1}_{\alpha\beta}+{}^{\tau}k^{2}_{\alpha\beta}+{}^{\tau}k^{3}_{\alpha\beta}+{}^{\tau}k^{4}_{\alpha\beta}\right)_{t}u_{\alpha}=\left({}^{t}P_{\beta}-{}^{t}F_{\beta}\right) \quad (6)$$

The ${}^{t}K_{\alpha\beta}$ matrix is the elastic stiffness matrix that is classical at the date t while the sum of the ${}^{\tau}K^{1}_{\alpha\beta}$, ${}^{\tau}K^{2}_{\alpha\beta}$, ${}^{\tau}K^{3}_{\alpha\beta}$ matrices is the incremental stiffness matrix of the large displacements, as a function of the increase in nodal displacements between the date t and the date τ

For a beam element with 2 nodes A and B, the field vector of the transposed nodal displacements is written as follow:

$$\langle u\rangle = \langle U_A, V_A, W_A, \theta_{xA}, \theta_{yA}, \theta_{zA}, \phi_A, U_B, V_B, W_B, \theta_{xB}, \theta_{yB}, \theta_{zB}, \phi_B \rangle \quad (7)$$

The tangent stiffness instantaneous matrix $\left[{}_{k}K_{T}^{(j)}\right]$ at the time τ corresponds to an intermediate position at the end of the first interaction with respect to the initial configuration 0Γ.

It is obtained by the formula:

$$\left[{}_{k}K_{T}^{(j)}\right] = \frac{\partial\left\{{}_{k}^{j}F_{\text{int}}\right\}}{\partial\left\{{}_{k}u^{(j)}\right\}} \quad (8)$$

$$\left.\begin{aligned} \left[{}_{k}K_{T}^{j}\right] &= \left[{}_{k}K_{j}^{j}\right] + \frac{\partial\left[K_{s}^{j}\right]}{\partial\left[{}_{k}u^{(j)}\right]}\left\{{}_{k}q^{(j)}\right\} \\ \left[{}_{k}K_{T}^{j}\right] &= \left[{}_{k}K_{0}\right]+2\left[{}_{k}K_{1}^{j}\right]+2\left[{}_{k}K_{2}^{j}\right]+4\left[{}_{k}K_{3}^{j}\right]+2\left[{}_{k}K_{4}^{j}\right] \end{aligned}\right\} \quad (9)$$

where:

$$\left.\begin{aligned} \left[{}_{k}K_{0}\right] &= \int_{k_v}\left[{}^{k}B_{L}\right]^{T}\left[C\right]\left[{}^{k}B_{L}\right]d^{k}v \\ \left[{}_{k}K_{1}^{(j)}\right] &= \int_{k_v}\frac{1}{2}\left[{}^{k}B_{NL}^{(j)}\right]^{T}\left[C\right]\left[{}^{k}B_{L}\right]d^{k}v \\ \left[{}_{k}K_{2}^{(j)}\right] &= \int_{k_v}\frac{1}{2}\left[{}^{k}B_{L}^{(j)}\right]^{T}\left[C\right]\left[{}^{k}B_{NL}\right]d^{k}v \\ \left[{}_{k}K_{3}^{(j)}\right] &= \int_{k_v}\frac{1}{4}\left[{}^{k}B_{NL}^{(j)}\right]^{T}\left[C\right]\left[{}^{k}B_{NL}^{(j)}\right]d^{k}v \\ \left[{}_{k}K_{4}^{(j)}\right] &= \int_{k_v}\frac{1}{2}\left[{}^{k}B_{NL}^{(j)}\right]^{T}\left[C\right]\left(\left[{}^{k}B_{L}\right]+\frac{1}{2}\left[{}^{k}B_{NL}^{(j)}\right]\right)d^{k}v \end{aligned}\right\} \quad (10)$$

4. Digital Approach Results and Discussions

We recall that the load that causes the lateral bifurcation of the beam or the buckling of the column is called the critical load. To observe it experimentally, we proceed as follows; from deformation state Γ_0 corresponding to an initial charge P_0, load increments p_i are gradually increased to previous loads, thus defining configuration states Γ_i If the bifurcation takes place in the configuration state Γ_{i_0}, so Γ_{i_0-1} is the critical configuration state and the critical load is equal to

$$\mathbf{P}_{cr} = P_0 + \sum_{n=1}^{i_0-1} p_n$$

The non-linear curve of forces / displacements or of stresses /deformation resulting from this operation previously (progressive addition of incremental loads) described thus illustrates the geometric instability phenomenon .

On a numerical level, we will use the tangent stiffness Newton-Raphson method. In the following lines, we will briefly explain the main steps of this method.

Calculations steps:

1) We solve the static problem $[K_L]\{u\} = \{F\}$ and determined $\{u\} = [K_L]^{-1}\{F\}$
2) The tangent stiffness matrix is calculated $[K_T(u)^j] = [K_L] + [K_1(u)^j] + 2[K_2(u)^j] + 4[K_3(u)^j] + 2[K_4(u)^j]$
3) We constructed the incremental load vector ΔF
4) We are constructed $\{F\}_{j+1} = \{F\}_j + \Delta F$

5) We calculate $\{u\}_{j+1}=\{u\}_j+\Delta u$
6) Actualization of the tangent stiffness matrix $[K_T(u)^j]$ using Sherman-Morrison formula[M.Duval, 2016]

5. Application

The table1 above illustrates the simulations parameters used for the digital processing, resulting from a console beam of Movingui (Distemonanthus Benthamianus)

Table 1: Simulations Parameters

Désignations	*Numerical values*
L beam column Length	1,46 m
B width of the cross-section of the beam	0,01 m
h Height of the beam cross-section	0,1 m
E Elasticity modulus	17333 MPA
G Sliding modulus	894 MPA

Application to a Console Beam

Figure (4) below shows a console beam

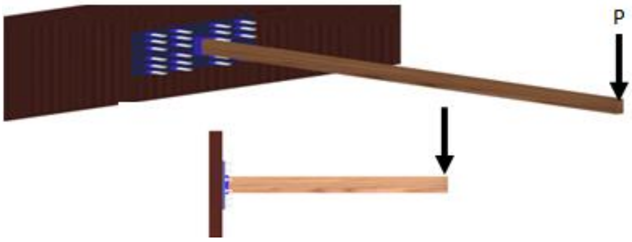


Figure 4: Console beam

The figure (5) below is the modeling of our discretized console beam into four finite elements is five nodes.

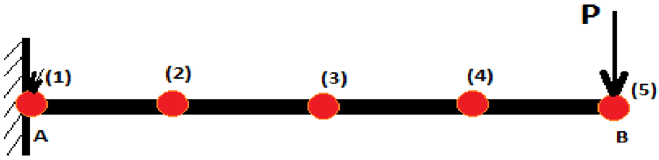
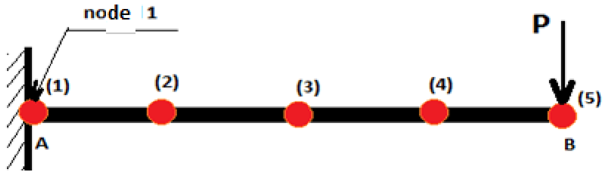


Figure 5: Modeling of theconsole beam discretized into four elements

By applying the finite element method to the incremental Newton-Raphson method, we obtain the following results.

6. Results



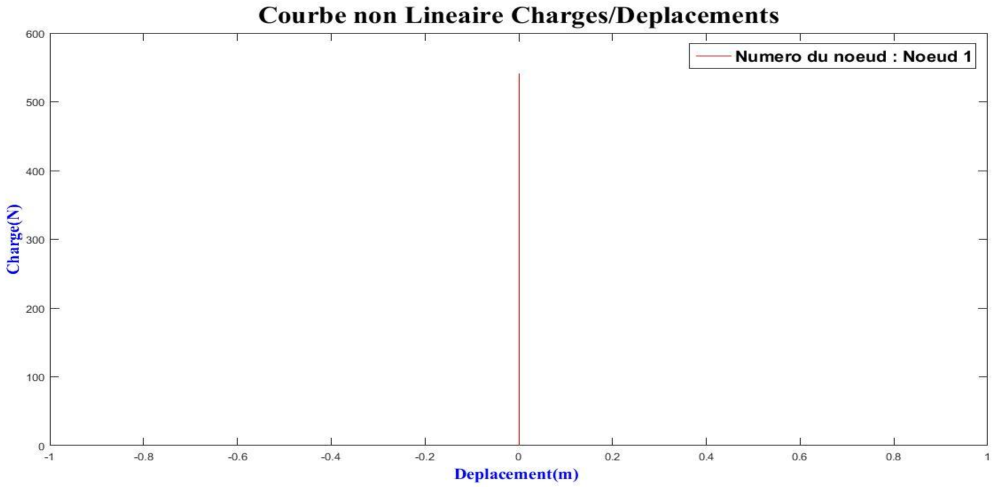
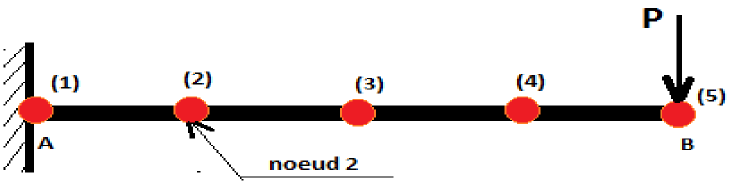


Figure 6: nonlinear curve loads / displacements, illustration of the load / displacement curve, at node 1, Number of iterations 243, spill load of problem 541N

Illustration de la courbe charges/déplacements, au nœud 1, Nombre d'itérations 243, charge de déversement du problème 541N.



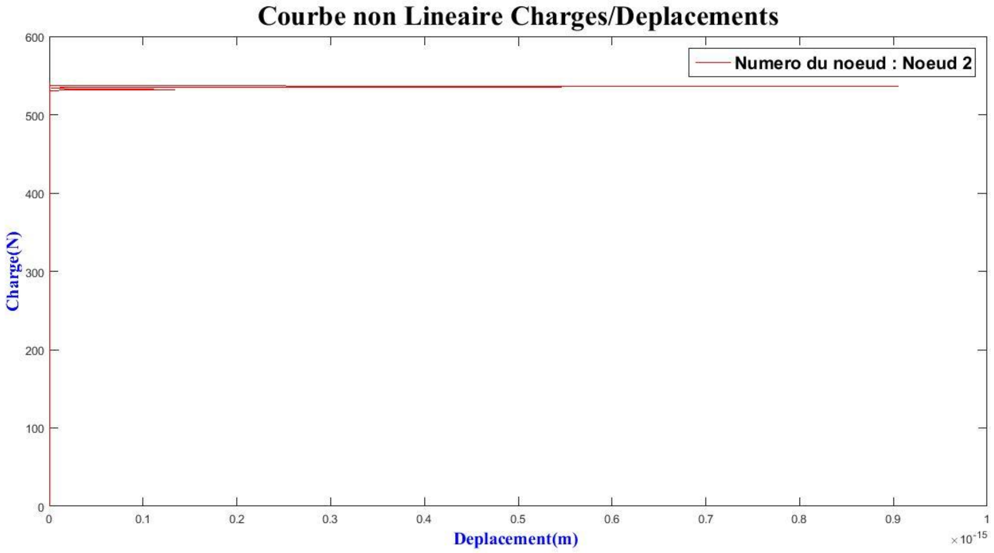
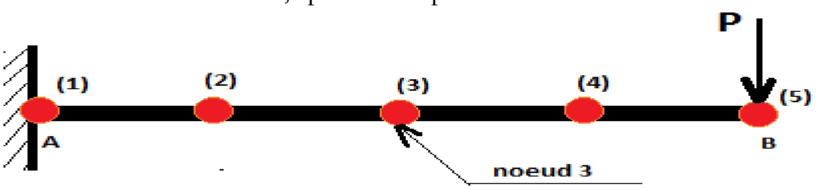


Figure 7: nonlinear curve loads / displacements, illustration of the load / displacement curve, at node 2, Number of iterations 243, spill load of problem 541N



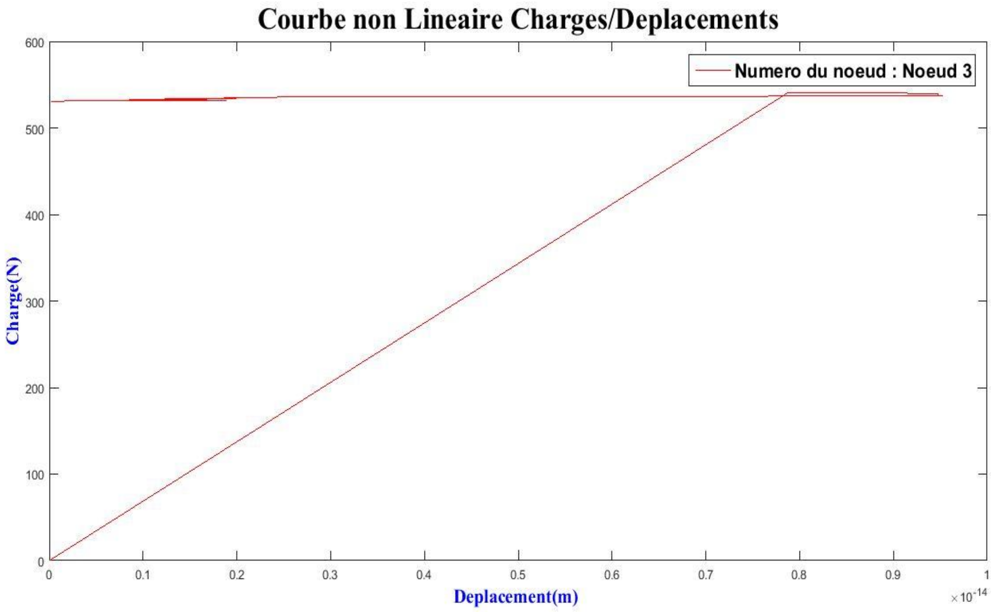
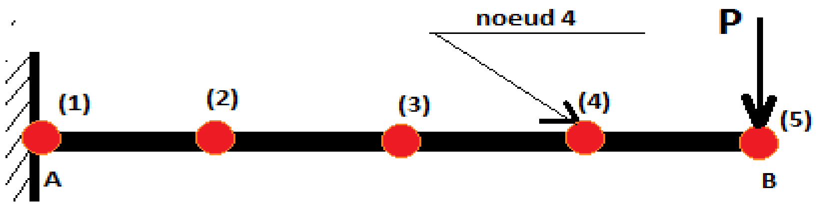


Figure 8: nonlinear curve loads / displacements, illustration of the load / displacement curve, at node 3, Number of iterations 243, spill load of problem 541N



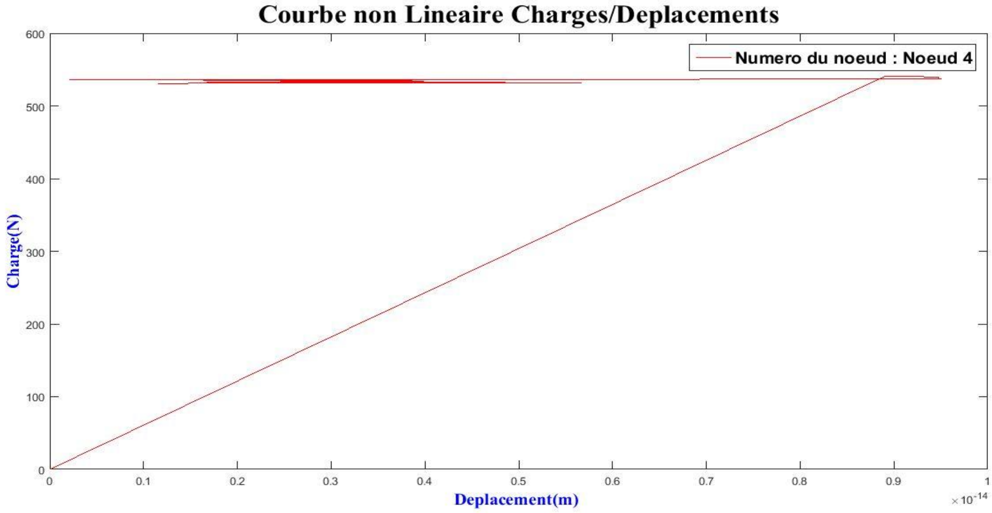


Figure 9: nonlinear curve loads / displacements, illustration of the load / displacement curve, at node 4, Number of iterations 243, spill load of problem 541N

7. Results and Analysis

1) At node 1, at 541N load , the beam breaks suddenly
2) At node 2, the beam begins to present geometric instabilities from 541N.
3) At node 3 and node 4, the load / displacement curve first shows linear behavior. Then the curve becomes one which afterwards stabilizes at 541N. after the loss of stability occurs.
4) The critical load of the problem is 541N.

8. Conclusion

The Vlasov type 3D beam model with croos section was presented. The equilibrium equations are derived from the virtual works principle. We have had the opportunity to calculate the tangent stiffness matrix this in the hypothesis of large displacements. The relevance of the finite element method to the study of geometrical instability phenomena has been put to the design. The figures obtained allowed us to analyze the post-critical behavior of the beam with instabilities.

We plan to continue this work by integrating the notions of creep and defects because the very complex aspect of the wood material by the nature of its constitution, its structure (orthotropy and hypotheses of heterogeneity with or without defects) and finally the nature loading suggests using all these different conventional approaches for calculating critical loads.

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