Stability Analysis of Mathematical Model

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Abstract: ***As was mentioned in the complex integration, it is often necessary in control theory to know if a system is stable or not. It is unstable if terms exist in the solution for the output that either increase without limit or oscillate with ever increasing amplitude. Thus terms of the form e^t , t^2 , t sin(nt) , in the solution would lead to an unstable system, but those of the form e^{-t}, sin(nt) , te^{-2t}, cos(nt) would indicate a stable system. With this problem we are going to investigate same model for investigation of this phenomena, related to stable or otherwise.***

1. Introduction

In the field of applied mathematics, the classical channel or tube flow seem to have important applications. Stability analysis has been recently studies by many authors because of the growing industrial applications importance. Analysis of stability of a disturb system seems to be an active field of research, Stuart [1], discussed the effect of magnetic field upon stability of corresponding MHD flow. It was found that the magnetic field has a powerful stabilizing influence on the disturb system.

Maysoon [2] investigate mathematical models (ratio-dependent) for two systems, one without time delay and the second with time delay.

It was found that the two models are stable. Maysoon, Saad, [3], present control and synchronization with known and unknown parameters, and found that there are two cofactors that have an effect on determining any case to achieve the control, the two cofactors are proposed in the control and the matrix that produce from the time derivative of Lyapunov function. In adding, they found some weakness cases in Lyapunov stability theory.

R. Matousek, *Member*, *IAENG*, I. Svarc, P. Pivoňka, P. Osmera, M. Seda Proceedings of the World,[4]. They discuss three methods for stability analysis of nonlinear control systems are introduced in this contribution: method of linearization, Lyapunov direct method and Popov criterion. Since stability analysis of nonlinear control systems is difficult task in engineering practice, these methods are made easier and tabulated.

2. The Model and Governing Differential Equations

Consider a straight circular tube of radius a in a fully developed laminar flow. The longitudinal velocity u at a distance r from the center line of the tube is given by the following Figure.

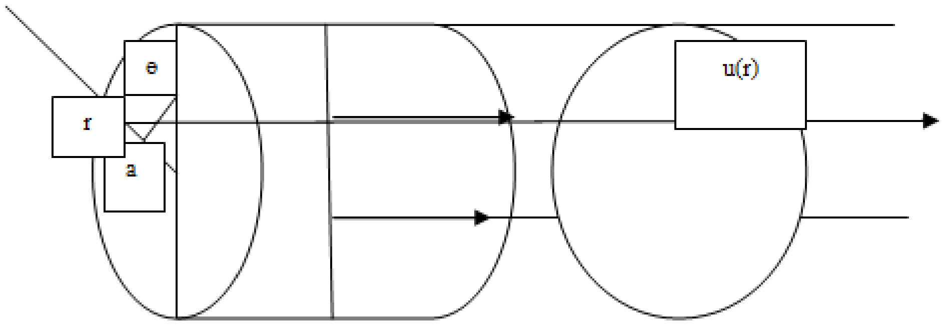

Figure (*): The axes in the tube and the distribution of the velocity in Poiseuille flow

If we consider the tube as in Figure (*), the Navier - Stoke equations can be written in the form

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{v}{a^2}\left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{z}\frac{\partial u}{\partial z}\right) \quad \text{--- 1}$$

Where z = r/a, ρ is the fluid density and v is the kinematic viscosity. The boundary condition of the no slip at the wall of the tube is

$$u = 0 \text{ at } z = 0. \quad \text{--- 2}$$

The longitudinal pressure gradient in this place will be taken to satisfy

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = G\cos wt \quad \text{--- 3}$$

Where G is a constant, $\frac{2\pi}{T}$ is the angular frequency and T is the period of oscillation.
The longitudinal velocity has the form

$$u(z,t) = f(z)\, e^{iwt} \quad \text{--- 4}$$

where physical significance is attributed only to the real part.
Using equation (4), equation 1, becomes

$$\frac{d^2 f}{dz^2} + \frac{1}{z}\frac{df}{dz} - \frac{iwa^2}{v} f = -\frac{Ga^2}{v} \quad \text{--- 5}$$

Which, when G= 0, is a form of Bessel's equation and the solution is

$$F(z) = -i\,\frac{G}{w}\left\{1 - \frac{J_0\left(i^{3/2} z\sqrt{\frac{wa^2}{v}}\right)}{J_0\left(i^{3/2}\sqrt{\frac{wa^2}{v}}\right)}\right\} \quad \text{--- 6}$$

$$F(z) = -i\,\frac{G}{w}\,e^{iwt}\left\{1 - \frac{J_0\left(i^{3/2}\sqrt{i}\; z\sqrt{\frac{wa^2}{v}}\right)}{J_0\left(i^{3/2}\sqrt{i}\sqrt{\frac{wa^2}{v}}\right)}\right\} \quad \text{--- 7}$$

The non – dimensional quantity $\frac{wa^2}{v} = \frac{a^2}{v} / \frac{1}{w}$ is a measure of the ratio of the time taken for viscosity to smooth out transverse variation in vorticity to the period of the impose oscillation. The discussion of equation (7) for w, is a tedious, owing to the Bessel function with a complex argument, but two limiting cases , namely very low and very high frequency, i. e. $\frac{wa^2}{v} \ll 1 \;\; and \;\; \frac{wa^2}{v} \gg 1$ respectively, can easily be investigated.

1) Case of low frequency, that is $\frac{wa^2}{v}$ << 1, equation (7) becomes, on taking the real part

$$u(z,t) = \frac{Ga^2}{4v}\{1 - z^2\}\cos wt. \quad \text{--- 8}$$

This follows from a series expansion of the Bessel function. The result of this output case is indicating stability.

2) Case of high frequency

In the case of very high frequency, that is $\frac{wa^2}{v} \gg 1,$ the quantity $\frac{J_0\left(i^{3/2} z\sqrt{\frac{wa^2}{v}}\right)}{J_0\left(i^{3/2}\sqrt{\frac{wa^2}{v}}\right)}$ must be very small in the core flow.

Then, taking the real part

$$U(z,t) = \frac{G}{w}\sin wt. \quad \text{--- 9}$$

Equation (9) supports the fact that this equation indicates the system is stable as well.

3. Conclusion

The problem is concern about the stability which is related to control systems. We solve a Navier Stoke equations, through a straight circular tube of radius a, and the output result is included the trigonometric functions, coswt and sinwt, which are indicate a stability of the system.

References

[1] J.T.Suart. On the stability of viscose flow between parallel plates in the presence of co-planar magnetic field. Proc.Roy. Soc. A221, 189, (1954).

[2] Aziz, and Al-Azzawi, S.F. (2015) Control and Synchronization of a Modified Hyperchaotic Pansystem via Active and Adaptive Control Techniques. *Computational and Applied Mathematics Journal*, 1, 151-155.

[3] Maysoon M. Aziz, Saad Fawzi AL-Azzawi. Using Feedback Control Methods to Suppress a Modified Hyperchaotic Pan System. *Computational and Applied Mathematics Journal.* Vol. 1, No. 3, 2015, pp. 97-106.

[4] R. Matousek, *Member, IAENG*, I. Svarc, P. Pivoňka, P. Osmera, M. Seda Proceedings of the World. Simple Methods for Stability Analysis of Nonlinear Control Systems, Congress on Engineering and Computer Science 2009 Vol II, WCECS 2009, October 20-22, 2009, San Francisco, USA .