

Modeling, Analysis and Monitoring DC Motor Using Proportional-Integral Controller and Kalman Filter

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Abstract: *This paper presents a method for monitoring control systems having output that is disturbance-free and noisy output. As a model-based fault detection scheme, a good knowledge of the control system is a necessity hence the DC motor used as the control system here was first modeled as an ideal DC motor. To ensure the stability of the speed control DC motor, Lyapunov stability test for linear time invariant system is done while bode plot was used to confirm the asymptotic stability of the DC motor. Then PI controller is designed rule for stability and set point tracking using Ziegler – Nichols tuning method. Motivated by this, a Kalman filter is designed for state estimation of the noisy DC motor. The healthy and faulty DC motor outputs are then compared. Large deviation exists indicating the presence of fault. Simulation results show the DC Motor attaining steady state of 34 rev/s speed which also agrees with theoretical calculation. Also the fault detection method was able to identify incipient faults in the DC motor.*

Key words: DC motor, Kalman filter, Lyapunov Stability

1. Introduction

Condition monitoring involves measuring the system performance and comparing it with the desired value with a view to reporting any deviation for correction. This is done either using model-based or model-free method. When limit checking, plausibility tests and redundancy, are used to obtain the system performance, such approach is model-free approach. Model-based method on the other hand is carried out when a mathematical equation relating the input and output of a system is obtained for predicting the system performance at any given input. In model-based fault detection, fault is detected when a fault-sensitive signal, residual exceeds a certain value called threshold and alarm is given. To minimise false alarm, residuals are designed to make them insensitive to disturbances and noise [1]. This is achieved by setting the threshold such that it accommodates the effects created by noise and disturbances although this might compromise the sensitivity of the fault detection. Condition monitoring involves; fault detection (detecting the presence of fault), isolation (identifying faulty component) and identification (estimating the magnitude of the fault) [2].

Even though the constructional features of DC generator have little difference with DC motor the operational difference differs. DC motor converts electrical energy into mechanical energy unlike the generator that does the reverse [3]. As a machine with moving components, DC motor is prone to different types of faults and failures. Various types of faults exist in DC motor. These could either affect the input to the motor, the parameters or the output of the motor. Actuator faults affect the input and add to the output affecting the mean of the output. Parametric faults affect the system parameters, bringing multiplicative effect on the output, hence altering the variance of the output and sensor faults add to the output, affecting the mean of the output. Some common faults in DC motor are; poor commutation,

sparking at the brushes, incorrectly connected fields, problems with driven equipment, grounded field circuit, open circuited or cracked commutator risers, open circuit drive SCR, diodes and fuses, improperly functioning control circuit and control instability.

There is therefore need to monitor the DC motor and report any deviation of the performance from set standard for either correction, reconfiguration or outright replacement of the DC motor. This paper aims to model DC motor, analysis its stability and stability and set point tracking and use Kalman filter to monitor noisy DC motor and identify fault presence.

2. Literature Review

Research into condition monitoring has been on-going for a long time yet still remains one of the research areas receiving so much attention due to its importance. Attempts have been made by different researchers/writers to classify the various approaches used. Isermann [1] classified fault detection approach as whether it is with single or with multiple signal models. He also classified diagnosis into classification and inference method. Chiang et al [4] classified fault detection methods into; statistical, analytical and knowledge-based methods. Caccavale [5] put fault detection methods into model-free and model-based methods. Model-free methods do not require the model of the system to detect and isolate fault but depends on statistical analysis of data obtained from past measurements, the use of expert system, physical redundancy or spectrum analysis and other methods. Model-based systems require the model of the systems to be known. Examples include analytical redundancy and diagnostic observer.

In model-based fault detection and diagnosis, models for the healthy and faulty systems are obtained. The differences are used to generate fault-sensitive signals called residuals.

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Residuals are quantities that remain zero when there is no fault and nonzero in response to fault [6]. Different approaches were used by different researchers to model the system, hence obtain the residuals. Luh et al [6] used immune based approach for FDD in which artificial immune-like signals were formulated exhibiting the behaviour of the biological immune system. This immune system is a kind of information processing system with specific features of recognition, self-organising, memory and learning. It is used to obtain the data for estimating the optimal parameters of the system for modeling the output of the system which is compared to a known model for residual generation. Isermann [1] used parity-based approach for fault detection and isolation. He placed a known model parallel to the plant model which formed an output error similar in principle to observer-based fault detection and diagnosis discussed here. The residual shown in figure 2.1, which is the difference between plant and model output was monitored and used to form fault signatures.

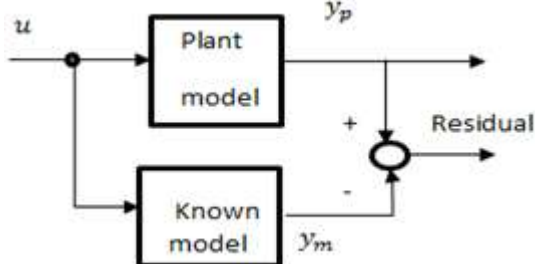


Figure 1: Residual generation

The residual was used to detect fault as usual. To isolate the faults, he formed a residual generating matrix in such a way that the elements of the residual vector are independent of one measurement each. By forming the signatures of different faults, the components that are faulty can be isolated.

State estimation by observer makes use of static observer with known input [7, 8, 9], static observer with unknown input [10, 11], and dynamic observer [12]. Static observer has a constant state matrix. A dynamic observer on the other hand, has its state matrixes varying with time. In unknown input observer, there exists additional input from high frequency un-modeled dynamics or unmeasured disturbances and control action. The observer models the system by taking its inputs and outputs, estimating the states of the system and getting the model of the system.

When a plant is linear, Kalman filter can be used for state estimation but nonlinear system is modeled using Extended Kalman (EKF). A very good option for state estimation is particle filtering which was put forward by Gerasimos G.Rigatos [12]. This is used for modeling nonlinear systems just like extended Kalman filter but it has the following advantages over EKF. EKF uses linearization hence parameters may not be optimal, while particle filter requires no linearization hence estimated parameters are optimal. Particle filter can be used for both linear and non-linear systems and for Gaussians and non-Gaussian models. The performance of the fault detection method can be tested and penalty awarded for any failure of the method to perform. This will give a guide as to whether or not an FDD method

should be used. The performance index can be calculated from

- False acceptance, where the FDD method does not report a fault as soon as it occurs
- Intermittent detection, where the same fault is reported many times over
- False alarm, where a no-fault situation is reported as fault [1, 2].

3. Methodology

3.1 Modeling the DC Motor

Figure 2 shows the electrical and mechanical parts of a DC motor. The magnet (not shown) provides the field that interacts with conductors carrying current hence producing torque in the forward direction. When in use, the load torques provides opposition to the speed of rotation of the shaft hence treated as opposing torque. The rotation of the shaft generates back emf that opposes the driving emf.

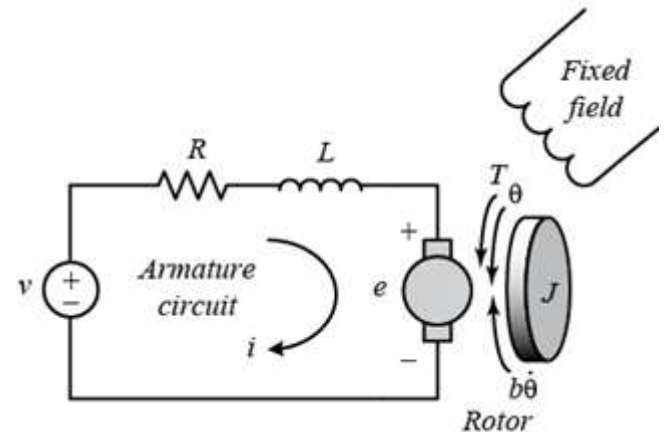


Figure 2: Electrical and Mechanical parts of a DC motor

Applying Newton's third law of motion to mechanical part of figure 2 gives equation 1.

$$J\ddot{\theta} = k_1 i - b\dot{\theta} - T_l \quad (1)$$

The electrical part gives equation 2 according to Kirchoff's Voltage law.

$$k_1 \dot{\theta} + Ri + L \frac{di}{dt} = u \quad (2)$$

J = inertia constant, $\ddot{\theta}$ = angular acceleration, b = damping coefficient, T_l = load torque, I = current magnitude, θ is the angular position, its derivative is the angular velocity and acceleration, R = resistance, L =inductance of the inductor, u =input voltage k_1 = electrical constant and k_2 = torque constant.

Taking Laplace transform of equation (2) and collecting like terms with simplification gives equations 3 and 4.

$$K_1 s\theta(s) + Ri(s) + Lsi(s) = U(s) \quad (3)$$

$$(Ls + R)i(s) = U(s) - K_1 s\theta(s) \quad (4)$$

Taking Laplace transform of equations (1) and collecting like terms with simplification gives equations 5 and 6.

$$Js^2\theta(s) = K_2 i(s) - bs\theta(s) - T_l(s) \quad (5)$$

$$(Js^2 + bs)\theta(s) = K_2 i(s) - T_l \quad (6)$$

Making $i(s)$ the subject in equation (4) and substituting in equation (6) gives the equation 7.

$$(Js^2 + bs)\theta(s) = \frac{K_2}{Ls+R}U(s) - \frac{K_1K_2S}{Ls+R}\theta(s) - T_l(s) \quad (7)$$

Multiplying through by $Ls + R$ and collecting terms in $\theta(s)$ from equation (7) gives equation 8.

$$[(Js^2 + bs)(Ls + R) + K_1K_2s]\theta(s) = K_2U(s) - (Ls + R)T_l(s) \quad (8)$$

The output position is therefore given as in equation 9.

$$\theta(s) = \left[\frac{K_2}{JLs^3 + (JR + bL)s^2 + (bR + K_1K_2)s} U(s) \right] - \left[\frac{Ls + R}{JLs^3 + (JR + bL)s^2 + (bR + K_1K_2)s} T_l(s) \right] \quad (9)$$

Figure 3 shows the open loop transfer function of the DC motor.

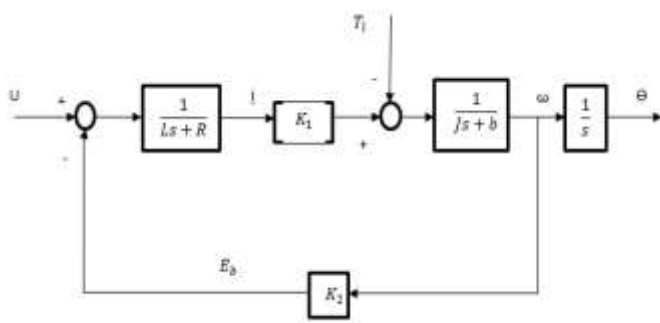


Figure 3: Open loop transfer function of the DC motor

Taking $J=2 \text{ Kg}m^2/s$, $b=1 \text{ Js/rad}$, $K_1=2 \text{ Vs/rad}$, $K_2=20 \text{ J/A}$, $u=100 \text{ V}$, $L=0.5H$, $R=10\Omega$, $T_l=30 \text{ Nm}$ and putting into (9) gives the position as equation 10.

$$\Theta(s) = \frac{20}{s^3+20.5s^2+50s}U(s) - \frac{0.5s+10}{s^3+20.5s^2+50s}T_l(s) \quad (10)$$

Speed $\omega(s)$ is obtained by differentiating position to give equation 11.

$$\omega(s) = \frac{20}{s^2+20.5s+50}U(s) - \frac{0.5s+10}{s^2+20.5s+50}T_l(s) \quad (11)$$

With a constant load torque of 30Nm and a step input voltage of 100V , combining the controlled input u and the disturbance input, T_l equation 11 gives equation 12.

$$\omega(s) = \frac{-15s+1700}{s^2+20.5s+50} \quad (12)$$

3.2 Controller Design

DC motor output speed has to be controlled to a set point for monitoring the output hence identifying any major deviation as fault. Hence in this section, a PI controller is designed to ensure close loop stability and set point tracking. The transfer function of a PI controller is given as equation 13

$$\text{Let } C = K_p + \frac{K_i}{s} = \frac{sK_p + K_i}{s} \quad (13)$$

Taking equation (12) as G , the close loop transfer function is given as equation 14 and simplifies to equation 15.

$$H(s) = \frac{\left(\frac{sK_p + K_i}{s}\right) \left(\frac{-15s+1700}{s^2+20.5s+50}\right)}{1 + \left(\frac{sK_p + K_i}{s}\right) \left(\frac{-15s+1700}{s^2+20.5s+50}\right)} \quad (14)$$

$$H(s) = \frac{-15K_p s^2 + (1700K_p - 15K_i)s + 1700K_i}{s^3 + (20.5 - 15K_p)s^2 + (1700K_p - 15K_i + 50)s + 1700K_i} \quad (15)$$

Using Ziegler – Nichols tuning rule, K_i is kept constant while K_p is increased until an oscillation starts when $K_p = 0.1756$, which is the ultimate gain K_u and the period of oscillation, P_u is 0.3838s . The controller parameters are calculated as;

$$K_p = 0.45K_u = 0.45 * 0.1756 = 0.079 \quad (16)$$

$$K_i = \frac{1.2 K_p}{P_u} = \frac{1.2 * 0.079}{0.3838} = 0.247 \quad (17)$$

Putting equations (16) and (17) into (15) gives the close loop transfer function as equation 18.

$$H(s) = \frac{-1.185s^2 + 131s + 420}{s^3 + 19.32s^2 + 181s + 420} \quad (18)$$

Figure 4 shows the armature voltage, back emf and the open loop step response of the DC motor.

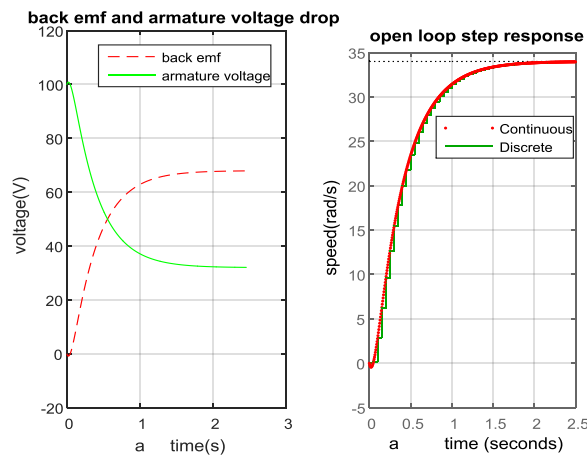


Figure 4: Back emf, armature voltage and open loop response of the DC motor

From figure 4a, the back emf builds up from zero value when the DC motor is switched on to a steady state value of $68 (34 * 2) \text{ V}$, while the armature voltage decreases as time increases until a steady state value is attained. At any time, the sum of these two voltages gives the input voltage of 100V . Figure 4b shows the responses for both continuous and discrete time transfer functions for the speed control DC motor. It has a time constant of 0.421s and a settling time of 1.78s . It shows that the steady state rotational speed is 34 rad/s and occurs when current attains its steady state value and the driving torque is zero. Hence the voltage drop across the inductor becomes zero due to no change in current.

At steady states, $\frac{di}{dt} = \frac{d^2i}{dt^2} = 0$, hence using the values provided above reduces (1) and (2) to 19.

$$20i = w + 30 \quad \text{and} \quad 100 = 10i + 2w \quad (19)$$

Solving equation (19) simultaneously, gives the steady state speed as 34rad/s , which agrees with the simulation result.

3.3 Stability Analysis.

Although for linear time invariant (LTI) systems such as DC motor, the Eigen values of the A matrix give sufficient information to test for stability, a Lyapunov stability test and Bode diagrams will be used to confirm this. The eigen values of the A matrix are -2.6102 and -47.8898 where the dominant pole is -2.6102 . Both poles are in the left hand plane hence the DC motor is stable. The simulation results shows that bounded input voltage of 100V gives bounded speed output of 34 rad/s confirming stability in open loop.

The Lyapunov algebraic equation is given as

$$A^T P + P^T A = -Q \quad (20)$$

Where P and Q are symmetric matrixes and are positive definite. The task here is, given matrix Q, we are to show that matrix P can be found to be positive definite. That is find $P = [p1 \ p2; p2 \ p3]$ with $p1, p2$ and $p3$ all real and positive, given that $Q=[1 \ 0; 0 \ 1]$ for the DC motor with $A = [-0.5 \ 10; -50 \ -10]$. The equation below is solved for P.

$$\begin{bmatrix} -0.5 & 10 \\ -50 & -10 \end{bmatrix}^T \begin{bmatrix} p1 & p2 \\ p2 & p3 \end{bmatrix} = \begin{bmatrix} p1 & p2 \\ p2 & p3 \end{bmatrix} \begin{bmatrix} -0.5 & 10 \\ -50 & -10 \end{bmatrix} \quad (21)$$

This gives $P = \begin{bmatrix} 0.29 & 0.01 \\ 0.01 & 0.06 \end{bmatrix}$ which is positive definite, hence the DC motor is asymptotically stable.

Figure 5 shows the Bode diagram for the closed loop DC motor. With a gain margin of 22.7 dB and phase margin of 132 degrees which are all positive showing that the Bode diagram further confirms the stability of the DC motor.

The Bode diagram shows that though the close loop DC motor is stable, the system is close to instability as the gain margin is very small. This near instability was shown more by the open loop system having gain margin of 9.28 dB and phase margin of 18.7 degrees which are smaller than the close loop.

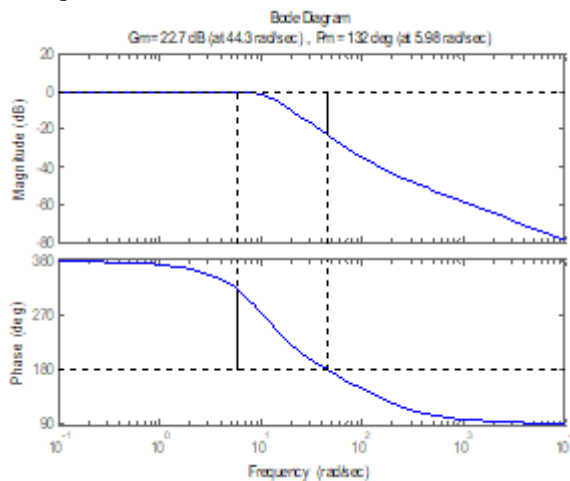


Figure 5: Bode diagram

3.4 Fault Detection in DC Motor

Once a stable system can be modeled, it can be controlled to set point and monitored for fault detection. Section 3.3 already shows that the DC motor can be modeled and its stability guaranteed. The motor can then be controlled to set point and monitored using observer. Figure 6 shows the schematic of observer-based fault detection indicating actuator, sensor and parametric faults.

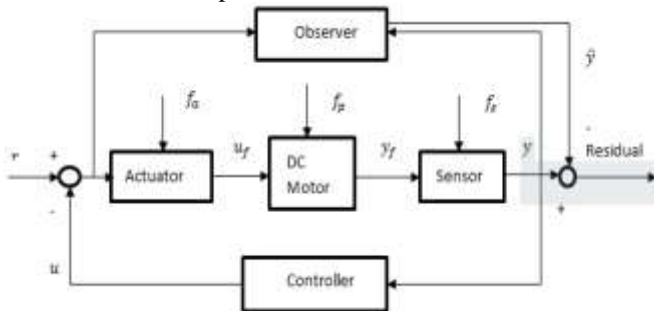


Figure 6: Observer-based fault detection and diagnosis

When the DC Motor becomes faulty, difficulty arises in measuring its states accurately. In such a case, an observer

can be used for state estimation. An observer takes the input and output of a system and estimate the state of the system. When the estimation error is zero, the state of the observer and the system become equal, hence if difference exists between the system output and the observer output, it is due to fault or noise. If the DC motor is corrupted by noise, an optimal estimator (Kalman filter) will be used to obtain the state estimates. The, equation for a DC motor with noise in the state and output is given as equation 21.

$$\dot{X} = Ax + Bu + n_i, \quad y = Cx + n_o \quad (21)$$

where, n_i and n_o are the state and output noise respectively.

The state estimate is given by equation 22.

$$\dot{\hat{X}} = A\hat{X} + Bu + L(y - C\hat{X}) \quad (22)$$

Where L is the Kalman gain. Taking error, $e = x - \hat{x}$ and control law, $= -K\hat{x}$, equation 21 and 22 become

$$\dot{X} = (A - BK)x + BKe + n_i \quad (23)$$

$$\dot{\hat{X}} = (A - BK - LC)\hat{x} + Ly \quad (24)$$

The error dynamics which is; $\dot{e} = \dot{X} - \dot{\hat{X}}$ becomes

$$\dot{e} = (A - LC)e + n_i - Ln_o \quad (25)$$

The optimal estimator can be represented in state space taking X and e as states variables as

$$\begin{bmatrix} \dot{X} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} X \\ e \end{bmatrix} \quad (26)$$

The $n_i - Ln_o$ term is eliminated if the Kalman gain, L is chosen to minimise the cost function

$$J = E \left[\int_0^\infty (x^T Qx + u^T Ru) dt \right] \quad (27)$$

E is the expectation or mean of the given value. For the stochastic DC motor considered here, the state and output noise is assumed to be Gaussian-distributed, hence the state covariance and output covariance are given as

$$\text{Cov}(n_i n_i^T) = \bar{Q} \quad \text{and} \quad \text{Cov}(n_o n_o^T) = \bar{R} \quad (28)$$

If \bar{P} is obtained by solving the algebraic Riccati equation

$$A^T \bar{P} + \bar{P} A - \bar{P} C^T \bar{R}^{-1} C \bar{P} + \bar{Q} = 0, \quad (29)$$

Then the optimal gain or Kalman gain L is given as

$$L = \bar{P} C^T \bar{R}^{-1} \quad (30)$$

\bar{P} and \bar{R} determine how close the measured value is to the true value. Large \bar{R} relative to \bar{P} indicates highly noisy, hence more attention on estimation while the reverse means measured result is almost accurate. \bar{Q} dictates set point tracking. High value of \bar{Q} increases set point tracking but brings high noise presence in the estimator output. The controlled input to the DC motor and the measured output for a faulty DC motor become;

$$u_f = u + f_a \quad \text{and} \quad y = y_f + f_s \quad (31)$$

The DC motor therefore becomes;

$$\dot{X} = Ax + Bu + Bf_a + f_p$$

$$y = Cx + Du + Df_a + f_s$$

Figure 7 shows how this approach detected incipient fault at 5 and 10 s. The presence of noise output ensures that the response is not smooth. Such could result in false alarm. But in this case that the magnitude of the incipient fault was big compared to that of the noise and disturbance, it is easy to detect the fault presence. Comparing the fault detection

ability for two of the states of the DC motor, current and speed, figure 8 shows the result. It can be seen from figures 8a and 8c that for a healthy DC motor, the outputs of the observer and the system match save few little disturbances or model error.

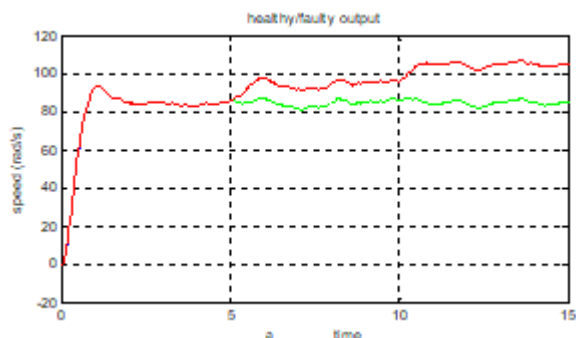


Figure 7: Healthy and faulty DC motor outputs.

With a faulty system, discrepancy exists between the outputs as shown in figure 8b. It can be seen from figure 8 that when current is monitored for fault detection, difficulty arises in differentiating between faulty DC motor and healthy DC motor as in figure 8d.

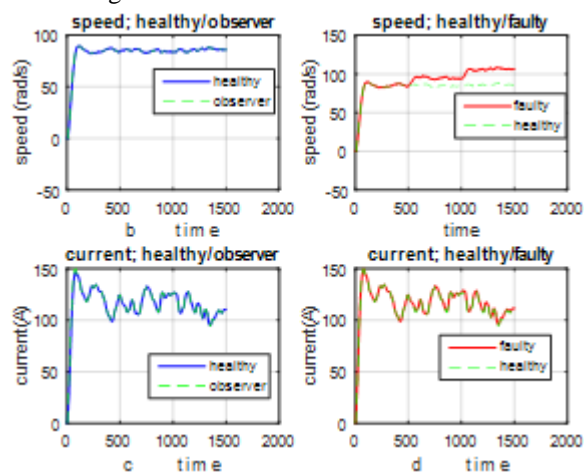


Figure 8: Healthy, observer and faulty DC motor states

4. Conclusion

A method for monitoring control systems with stochastic, Gaussian-distributed output was analysed. The control system used is DC motor subjected to noise, assumed to be Gaussian-distributed. An ideal DC motor was first modeled and the transfer function and the open loop behaviour examined for stability and set point tracking. The motor was found to be stable from the eigen values of the state matrix, Lyapunov stability test and the Bode plot. Observer in form of Kalman filter, an optimal estimator was designed for states estimation to confirm the ability of the above method to detect fault. The Kalman filter showed presence of fault.

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