

# Non-Dimensional Parameters of a Membrane-Type Restrictor in an Opposed Pad Hydrostatic Bearing for High Static Stiffness

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**Abstract:** *The analysis described in this article shows that the performance of a hydrostatic bearing can be improved over a certain range of load capacity if the design parameters of the restrictor set-up are properly chosen. An opposed pad bearing is treated as a bearing frame which consist of two single pad bearings arranged on the opposite side of the frame. The dimensionless parameters include: Dimensionless membrane stiffness and Design Restriction Ratio. When the Bearing Clearance Ratio is almost constant within a particular load range, the stiffness of the bearing should theoretically approach infinity. It was derived that for such condition, the Dimensionless Membrane stiffness is constant i.e. 1.33. The design restriction ratio should be chosen differently for lower and higher loading conditions for high static stiffness.*

**Keywords:** Hydrostatic bearing, Membrane restrictor, Membrane, Opposed Pad, Diaphragm

## 1. Introduction

Hydrostatic bearings are externally pressurized bearings where a complete separation of the conjugated surfaces of a kinematic pair is maintained by means of lubricant and a pump. These bearings, due to certain advantages such as good damping characteristics, low friction and low wear, are commonly used in machine tools, measuring instruments and testing machines. As soon as the pump is turned on, the bearing system forms the fluid film. Hence, the lubrication mechanism does not require a relative motion between the two conjugated surfaces. The performance of this type of bearing mostly depends upon the type of compensation mechanism used. Generally there are two types of compensation. One is passive compensation e.g. capillary and orifice restrictors. The other is active compensation which include spools, membranes, and constant flow valves. The passive compensation restrictors generally offer a fixed resistance as it does not undergo any shape or configurational change with the change in load capacity. However actively compensated restrictors varies its resistance by changing the configuration and shape with the change in load capacity. The performance characteristics of an actively compensated restrictor is better than a passively compensated restrictor.

Although there are many design procedures for hydrostatic bearing set-up, but it is still required to improve the performance characteristics of such system even better. A bearing is said to be highly efficient if the stiffness of the system is very high i.e. the clearance is constant for a wide range of load capacity. Hence the ultimate goal of any design procedure of such system is to keep the bearing stiffness as high as possible for a range of load capacity. In 1962, Mohsin [3] introduced some commonly used design steps to improve the performance characteristics of hydrostatic bearing set-up. One way is the use of fixed-laminar flow restrictors where the bearing stiffness will be inversely proportional to the nominal clearance. But with a small nominal clearance the effect of viscosity friction becomes higher. The other way is the use of opposed pad

design. In order to enhance the load range for high static stiffness and to eliminate the contamination from surroundings, the bearing is often designed as a closed form structure made up of certain number of opposed pads. As multiple pads are taken into account, the load capacity is shared in proportion in all the pads. However, the design, manufacturing and assembly of such system is much complicated because of high precision and heavier bearing structure.

DE Gast J [4] and Cusano [5] proved that the performance characteristics of a membrane-type restrictor is much better than a fixed laminar flow restrictors. The membrane type restrictor uses a metal membrane usually called diaphragm [6] to regulate the flow. In such type of restrictor different parameters need to be controlled to maintain the bearing clearance within  $\pm 5\%$  of the average value for a wide range of load capacity. As the bearing clearance is constant in a particular range, the stiffness will be very high during that period [3].

Singh et al. [7] and Phalle et al. [8] researched about the static characteristics of multirecess hybrid journal bearing using the membrane-type restrictor as a flow divider. The finite element method was used to solve the governing equations of the lubricant flow and 3D elasticity equations of the membrane simultaneously.

C. Wang and C. Cusano [10] analysed the dynamic characteristics of a externally pressurized, circular thrust bearings membrane-type restrictors. Under static load or load composed of a static and cyclic component, the membrane-type restrictor gives better result than the fixed flow capillary restrictor. Under pure cyclic loading with increase in frequency, the capillary restrictor gives approximately same result as a variable flow membrane-type restrictor.

Y. kang and D.Peng [9] worked on the design of static stiffness of hydrostatic bearing with double action variable compensation of membrane type restrictors and self-

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compensation. The load capacity and static stiffness in thrust direction of the planar bearing is determined by solving the continuity equation. The design parameters which will lead to a negative stiffness in a particular load range should not be taken inn actual practice. The negative stiffness will induce instability to the bearing system.

Bassani and Piccigallo (1992) [2] have provided the equations for flow rate, static load, and static stiffness for almost all type of compensation devices. Bassani (2001) analysed the compensation behaviour of double- action spool type restrictors for static characteristics of hydrostatic bearing.

Although many design parameters of membrane-type restrictors had been studied, there is no particular procedure to be followed by industry users and designers. Design of membrane type restrictors are more complex than that of fixed laminar flow restrictors like Capillary and orifice for hydrostatic bearing. In this paper, the fundamental models for membrane-type restrictor and bearing system were reviewed and used to find the optimal solution of the restrictor parameters for better performance of hydrostatic bearing set-up. It is hoped to find the key values of the design parameters before following up particular references for more detailed design on opposed pad hydrostatic bearing set-up.

**Nomenclature**

- $A_{e1}$  effective area of upper pad
  - $A_{e2}$  effective area of lower pad
  - $P_1$  upper recess pressure
  - $P_2$  lower recess pressure
  - $P_s$  supply pressure
  - $\bar{P}_1$  pressure ratio of upper recess ( $P_1/P_s$ )
  - $\bar{P}_2$  pressure ratio of lower recess ( $P_2/P_s$ )
  - $\alpha$  effective area
  - $W_1$  load capacity of upper recess ( $A_{e1} * P_1$ )
  - $W_2$  load capacity of lower recess ( $A_{e2} * P_2$ )
  - $W$  load capacity of bearing
  - $R_{r1}$  flow resistance of upper restrictor
  - $R_{r2}$  flow resistance of lower restrictor
  - $R_1$  flow resistance of upper land
  - $R_2$  flow resistance of lower land
  - $R_{01}$  flow resistance of upper land at reference configuration
  - $R_{02}$  flow resistance of lower land at reference configuration
  - 1 reference pressure ratio of upper land
  - 2 reference pressure ratio of lower land
  - $K_{r1}^*$  dimensionless stiffness of upper membrane
  - $K_{r2}^*$  dimensionless stiffness of lower membrane
  - 1 design restriction ratio of upper pad
  - 2 design restriction ratio of lower pad
- Nomenclature**
- 01 deformation ratio of upper membrane at reference configuration
  - 02 deformation ratio of lower membrane at reference configuration
  - $K$  stiffness of bearing
  - $h_0$  reference clearance of bearing
  - $h_1$  clearance of upper pad
  - $h_2$  clearance of lower pad
  - $e$  axial play eccentricity

**2. Analysis of Design Parameters**

The symmetric pad hydrostatic bearing set-up studied was shown in Fig.1. The key geometrical parameters are also shown in Fig.1 and will be discussed later. The important operating parameters are: net load capacity (W), supply

pressure ( $P_s$ ), eccentricity ( $e$ ). The flow resistance of both pads are varied due to the deflections in the respective diaphragm which are caused by the fed back bearing pressure. Hence, if the bearing load changes by some amount then the fed back bearing pressure and the diaphragm displacement will change [1].

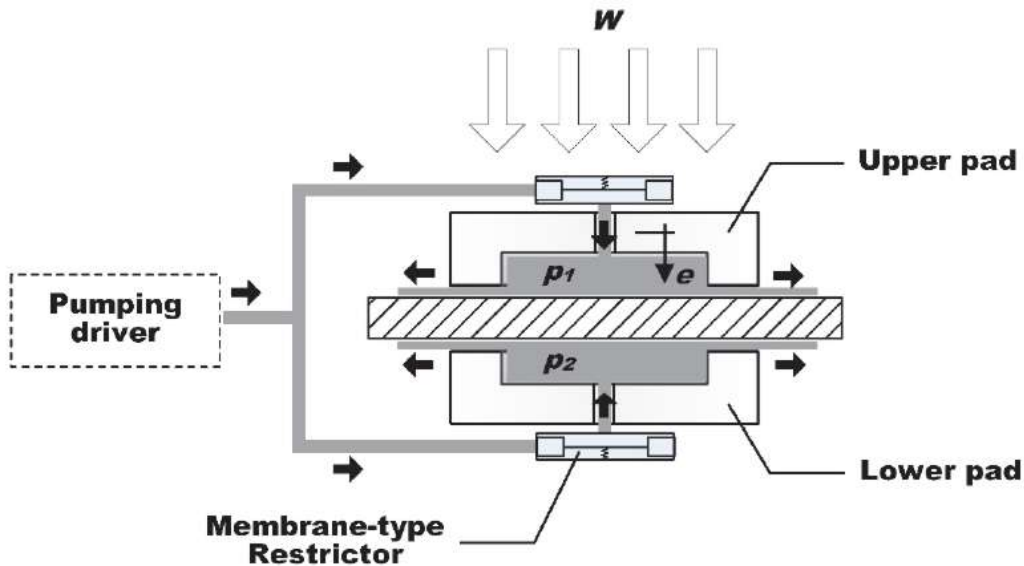


Figure 1: A sketch plot for Hydrostatic opposed pad bearing

The multi pad bearings are studied by applying to each pad the equations already developed for the single pads and by summing together the effects of all the pads, after expressing the pad clearances as function of displacement of moving member [2].

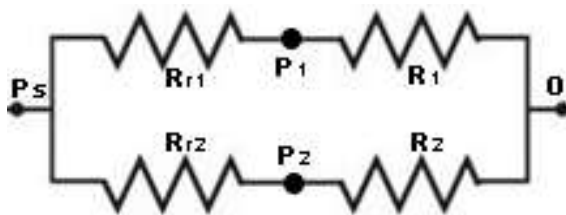


Figure 2: Equivalent Electrical Circuit of the Bearing Set-up

The net load capacity of an opposed pad bearing is derived by considering the load capacity of both upper and lower pads [2].

$$W = W_1 - W_2 = P_1 A_{e1} - P_2 A_{e2} \quad (1)$$

The net load capacity can be converted into a dimensionless load capacity using the following equation:

$$\frac{W}{P_s A_{e1}} = \bar{P}_1 - \frac{\bar{P}_2}{\alpha} \quad (2)$$

The flow resistance of the upper restrictor can be written as follow [1]:

$$R_{r1} = R_{01} \frac{1-\alpha}{\alpha} \bar{P}_1 - \frac{1}{K_{r1}^*} \frac{1}{1-\alpha} (1 - \bar{P}_1)^{-3} \quad (3)$$

Eqn (3) can be written can be written in another form as:

$$\frac{R_{r1}}{R_1} = \frac{R_{01}}{R_1} \frac{1-\alpha}{\alpha} \bar{P}_1 - \frac{1}{K_{r1}^*} \frac{1}{1-\alpha} (1 - \bar{P}_1)^{-3} \quad (4)$$

From the flow rate equation for single pad bearing set-up it was derived that [1]:

$$\frac{R_{r1}}{R_1} = \frac{1}{\bar{P}_1} - 1 \quad (5)$$

It is useful to take the origin of displacement in such a way as to split the axial play into two equal parts. Thus the

equations for eccentricity and dimensionless clearance can be written as [2]:

$$\frac{e}{h_0} = 1 - \quad (6)$$

$$\frac{h_1}{h_0} = 1 - \quad (7)$$

$$\frac{h_2}{h_0} = 1 + \quad (8)$$

The flow resistance of the upper pad is inversely proportional to the cube of the upper clearance, hence the following equation is valid [2]:

$$\frac{R_{01}}{R_1} = \frac{h_1^3}{h_0^3} = 1 -^3 \quad (9)$$

Inserting Eqn (5) and Eqn (6) into Eqn (4), the relationship between upper pad load capacity and bearing eccentricity can be written as:

$$\frac{1}{\bar{P}_1} - 1 = 1 -^3 \frac{1-\alpha}{\alpha} \bar{P}_1 - \frac{1}{K_{r1}^*} \frac{1}{1-\alpha} (1 - \bar{P}_1)^{-3} \quad (10)$$

With similar considerations the relationship between lower pad load capacity and bearing eccentricity can be written as:

$$\frac{1}{\bar{P}_2} - 1 = 1 +^3 \frac{1-\alpha}{\alpha} \bar{P}_2 - \frac{1}{K_{r2}^*} \frac{1}{1-\alpha} (1 - \bar{P}_2)^{-3} \quad (11)$$

The values of  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_{01}$ ,  $\alpha_{02}$  can be derived considering each lower pad and upper pad separately as single pad bearing set-up. The equations to find the values of these parameters are as follows [1]:

$$\alpha_{01} K_{r1}^* = 1 - \quad (12)$$

$$\alpha_{01} = 1 - \frac{1}{1-\alpha} \quad (13)$$

By choosing the value of the dimensionless membrane stiffness and design restriction ratio, the value of the load capacity of the both pads can be calculated for a given range of eccentricity.

Hence the static stiffness of the opposed pad bearing can be estimated as [2]:

$$K = \frac{dW}{de} = \frac{P_s A_{e1}}{h_0} \frac{d\bar{P}_1}{d} \frac{\bar{P}_2}{\alpha} \quad (14)$$

The static stiffness of the opposed pad bearing is inversely proportional to the nominal clearance of the set-up. The smaller the nominal clearance, the greater will be the bearing stiffness. The power losses will decrease, but the manufacturing of such system will be more complex as accuracy will be more [2].

### 3. Analysis of Results

In the previous section the equations for load capacity, dimensionless clearance ratio and stiffness were derived for the opposed pad hydrostatic bearing set-up. The equations from single pad membrane type restrictor set-up were used to find the bearing parameters for high static stiffness.

In this section the performance characteristics of an opposed pad hydrostatic bearing set-up were plotted and examined. Several different combination of bearing parameters were used to get the best solution. The performance characteristics of a hydrostatic bearing includes the following two plots:

- Clearance Ratio Vs. Dimensionless Load Capacity
- Dimensionless Bearing Stiffness Vs. Dimensionless Load Capacity

As the opposed pad consists of two pads, hence the size of the pads can be different. The size effect of the pads on the bearing set-up is discussed for three different cases.

- 1) Effective area is 0.5 i.e.  $A_{e1} = 0.5 * A_{e2}$
- 2) Effective area is 1.0 i.e.  $A_{e1} = 1.0 * A_{e2}$
- 3) Effective area is 2.0 i.e.  $A_{e1} = 2.0 * A_{e2}$

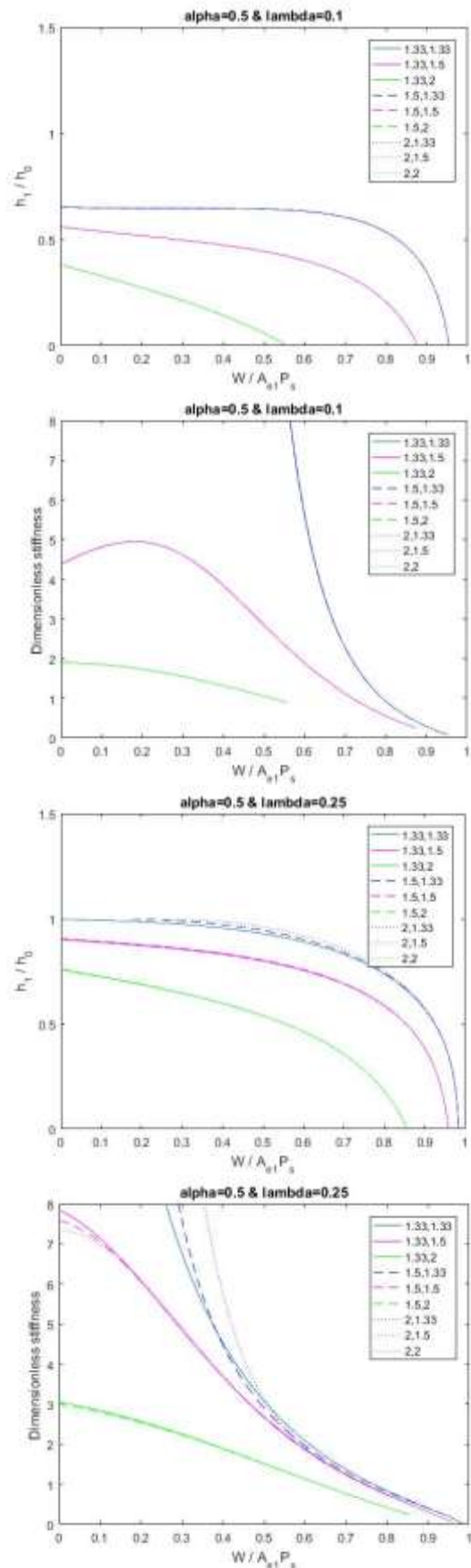
The design restriction ratio 0.1, 0.25, 0.5 were taken equally for both upper and lower pads. Because of limitations in the practical system further lower and higher values of this parameters are not taken into considerations.

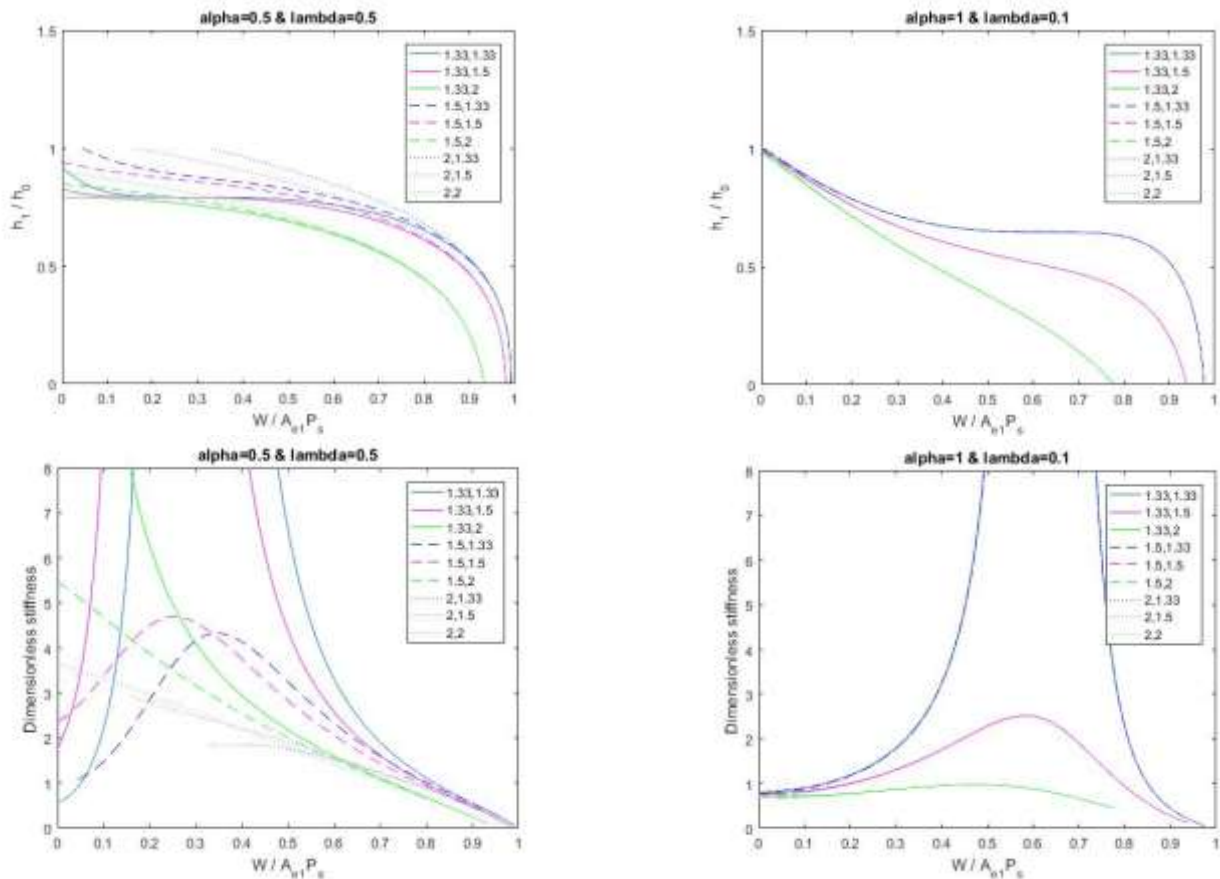
As the eccentricity varies, the bearing clearance in both the pads changes. Hence, the pressure ratio  $\bar{P}_1$  and  $\bar{P}_2$  change according to Eqn (10) and Eqn (11) respectively. A total of nine combination between  $\bar{P}_1$  and  $\bar{P}_2$  is possible for three dimensionless membrane stiffness (1.33, 1.5, 2) of each pad. The eccentricity is varied from 0 to 1 to determine the pressure ratio and hence the net load capacity of the bearing.

**Table 1:** Bearing parameters used for analysis

Bearing Parameter	Values Taken for Simulation
$\alpha$	0.5, 1, 2
$K_{r1}^*$	1.33, 1.5, 2
$K_{r2}^*$	1.33, 1.5, 2
$\lambda$	0.1, 0.25, 0.5
$e$	0 to 1

Fig.3 shows the performance characteristics of the Hydrostatic Bearing when lower effective area was adopted for different design restriction ratio.



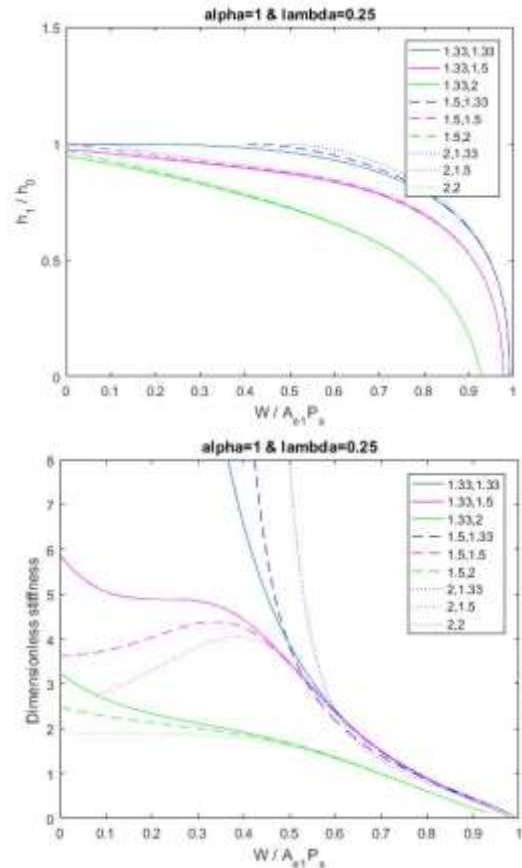


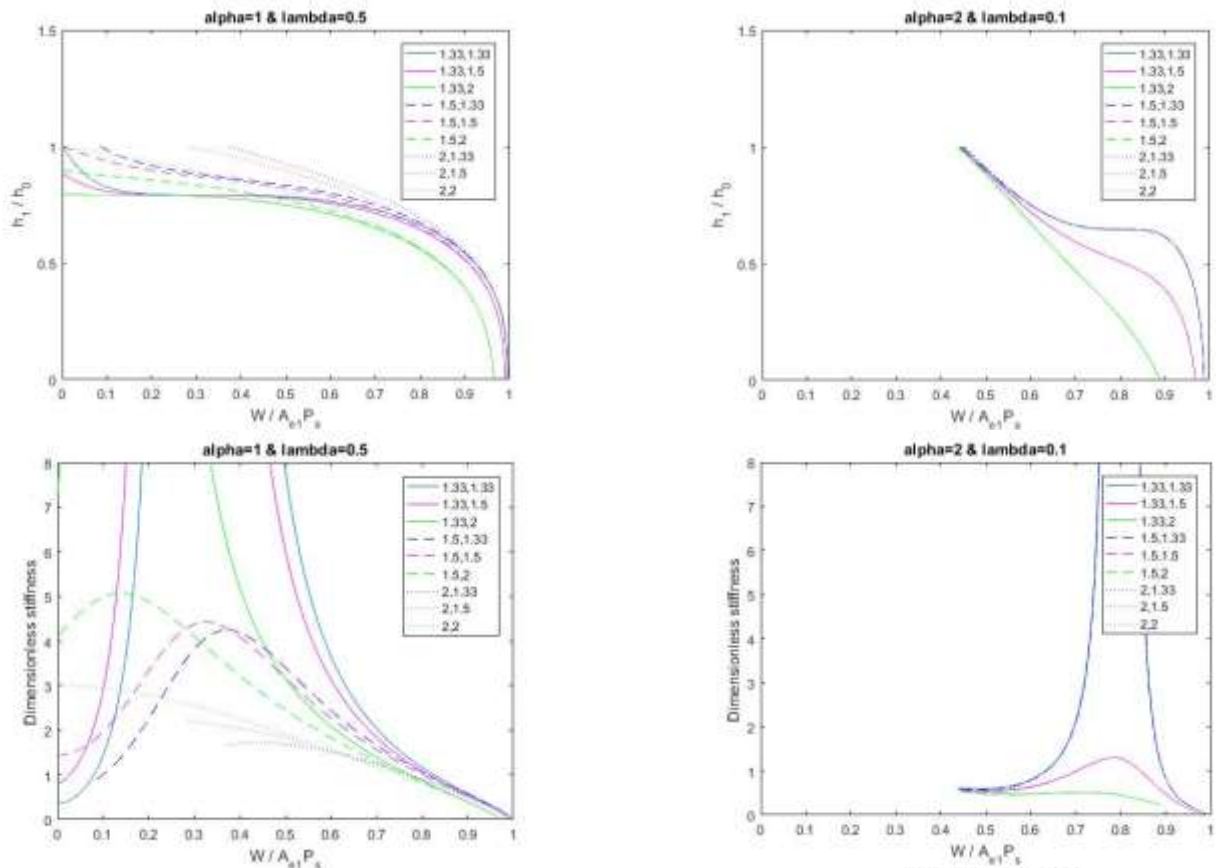
**Figure 3:** Effect of dimensionless membrane stiffness of both pads on clearance ratio and dimensionless bearing stiffness, as a function of dimensionless load capacity when effective area of 0.5 was adopted for different values of design restriction ratio

The following points can be concluded from the Fig.3

- The influence of dimensionless bearing stiffness of the lower pad i.e.  $K_{r2}^*$  is more compared to  $K_{r1}^*$ . The effect of  $K_{r1}^*$  almost negligible for lower values of design restriction ratio like 0.1 and 0.25.
- For all the cases of design restriction ratio, when both dimensionless stiffness is 1.33, the plot is almost horizontal for a certain range of load which is as expected.
- The bearing fails early i.e. the clearance ratio becomes zero with a smaller value of design restriction ratio.
- When the value of  $K_{r2}^*$  is more, the bearing may fail early as concluded from the above plots.
- When  $K_{r1}^* = K_{r2}^* = 1.33$  and design restriction ratio is 0.25, the clearance ratio is almost 1 for a load capacity range of 0 to 0.6. But when  $K_{r1}^* = K_{r2}^* = 1.33$  and design restriction ratio is 0.1, the clearance ratio value decreases by a large amount as compared to the previous case but remains constant throughout the load capacity range of 0 to 0.6.

Fig.4 shows the performance characteristics of the Hydrostatic Bearing when effective area of one was adopted for different design restriction ratio.



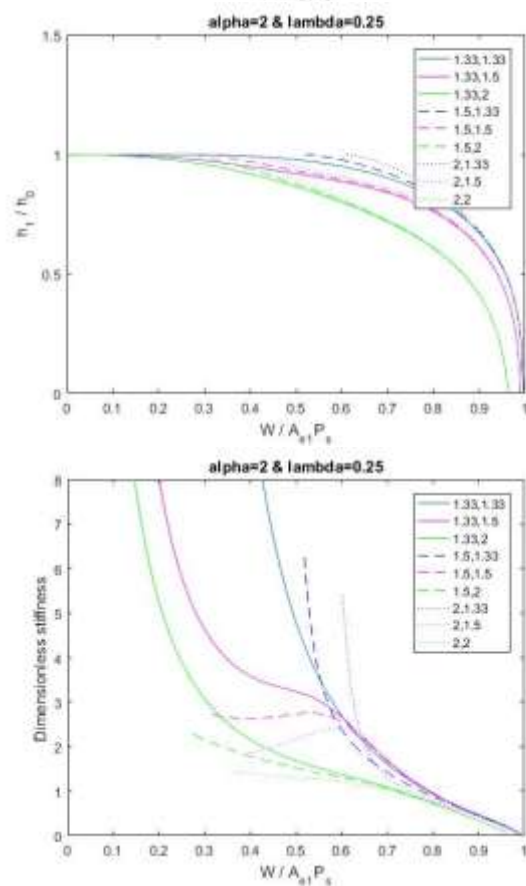


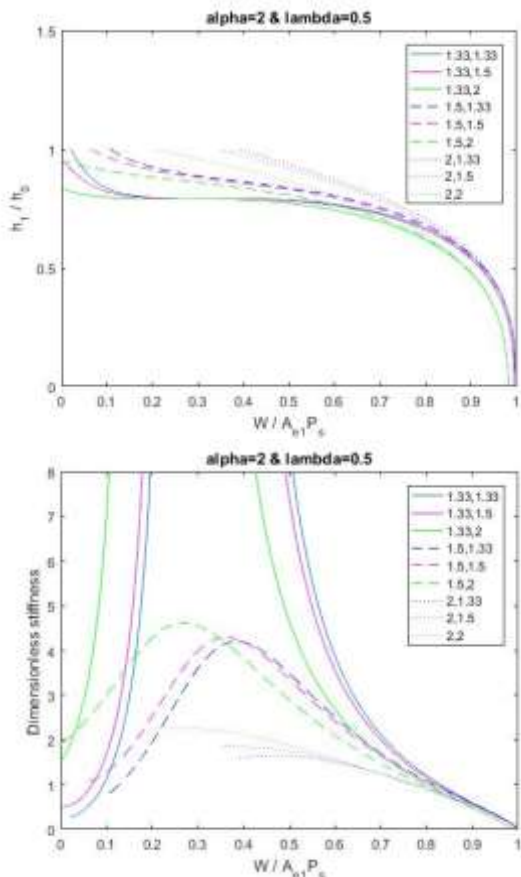
**Figure 4:** Effect of dimensionless membrane stiffness of both pads on clearance ratio and dimensionless bearing stiffness, as a function of dimensionless load capacity when effective area of 1 was adopted for different values of design restriction ratio

The following points can be concluded from the Fig.4

- The effect of dimensionless stiffness of lower pad is still higher than the dimensionless stiffness of upper pad. The effect of  $K_{r1}^*$  is almost negligible for lower value of design restriction ratio.
- The average value of clearance ratio decreases with decrease in design restriction ratio. This result can be easily seen comparing Fig. with Fig.
- An increase in effective area increases the load capacity range without bearing failure.
- The curve with  $K_{r1}^* = K_{r2}^* = 1.33$  and design restriction ratio = 0.25 still gives best result i.e. the clearance ratio is constant over a wide range and hence the stiffness is very high in this range.

Fig.5 shows the performance characteristics of the Hydrostatic Bearing when higher effective area was adopted for different design restriction ratio.





**Figure 5:** Effect of dimensionless membrane stiffness of both pads on clearance ratio and dimensionless bearing stiffness, as a function of dimensionless load capacity when effective area of 2 was adopted for different values of design restriction ratio

The following points can be concluded from the Fig.5

- The effect of dimensionless stiffness of lower pad is higher than the dimensionless stiffness of upper pad when dimensionless load capacity exceeds from 0.4.
- When the design restriction ratio is small the dimensionless clearance ratio exceeds one when the dimensionless load capacity is below 0.4.
- The curve with  $K_{r1}^* = K_{r2}^* = 1.33$  and design restriction ratio = 0.25 still gives best result i.e. the clearance ratio is constant over a wide range and hence the stiffness is very high in this range.
- With increase in effective area, the chance of bearing failure at lower loading condition is eliminated.
- When the design restriction ratio increases from 0.25 to 0.5, the average dimensionless clearance ratio decreases. However the bearing still can be operated in a dimensionless load capacity range of 0.1 to 0.7 where the bearing clearance is almost constant and the bearing stiffness is very high.

#### 4. Conclusion

The paper describes guide lined for the design of membrane-type restrictor with opposed pad configuration. The analysis described in the paper shows that a high static stiffness of the bearing set-up is attainable if the dimensionless design parameters of the restrictor were properly chosen. The parameters include: Dimensionless membrane stiffness and

Design restriction ratio. When the deformation vs. load relationship of the restrictor matches the theoretical one, the stiffness of the bearing should approach infinity in a wide range of load capacity.

The stiffness of the membrane is approximately constant within its working range. Hence, this type of variable flow restrictor can only provide infinite stiffness to the bearing set-up at a specific loading condition. The actual design of membrane to attain high static stiffness for the entire range of loading is difficult due to material and structural limitations of the set-up.

The variation of clearance ratio and dimensionless bearing stiffness with the variation of load were plotted for different combination of design restriction ratio, effective area and dimensionless membrane stiffness of both the pads. It was derived that a design restriction ratio of 0.25 and dimensionless bearing stiffness of 1.33 for both the pads is the optimum values for a high static stiffness in a dimensionless load capacity range of 0 to 0.6 approximately.

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