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Minimization of the Noise Figure of RC Polyphase Filters

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Abstract: ***The sideband suppression ratio of RC polyphase filters is independent of source and load impedances. This property is valid for any number of stages and for any detuning between the stages. Due to this property, noise figure minimization can be done independently. Two formulas for noise figure of passive two-ports are obtained, and noise figure of polyphase filters are minimized. The noise figure of an RC polyphase filter is strongly dependent on the source resistance and capacitance. For a two-stage filter providing better than -25 dB sideband suppression over two octaves, a minimum noise figure of 10.18 dB was found.***

1. Introduction

RC polyphase filters are widely used in communication systems [1]. Their typical applications are modulation and demodulation of single sideband signals. Due to their simplicity, they are popular in IF stages of application-specific integrated circuits (ASICs). Noise in polyphase filters is qualitatively analyzed in [2], where the relation between resistor values in different stages is discussed for reducing noise. A pioneering paper on polyphase filters is [3]. Derivation of the structure and its sideband cancellation property are explained there in detail.

In a receiver system, polyphase filter usually follows the mixer stages. Therefore noise contribution of the polyphase filter to the system noise may be significant. For this reason, noise minimization of the polyphase filter is important. However, none of the mentioned publications consider terminations of polyphase filters for minimum noise figure.

In this paper we show that the sideband suppression ratio of an RC polyphase filter is independent of the impedance of the source and the load under very general conditions. As an application of this property, we show that noise figure optimization can be done independently, without degradation of the sideband suppression ratio. Two new formulas for noise figure of passive two-ports are obtained and applied for our case. It is shown that noise figure of an RC polyphase filter is strongly dependent on the source resistance and capacitance, and independent of the load impedance. As an example, noise figure of a two-stage RC polyphase filter, designed for sideband suppression lower than -25 dB over two octaves, is minimized to 10.18 dB.

2. Transfer functions of an RC polyphase filter

The one-stage RC polyphase filter is an 8-node structure shown in Fig. 2.1 [3].

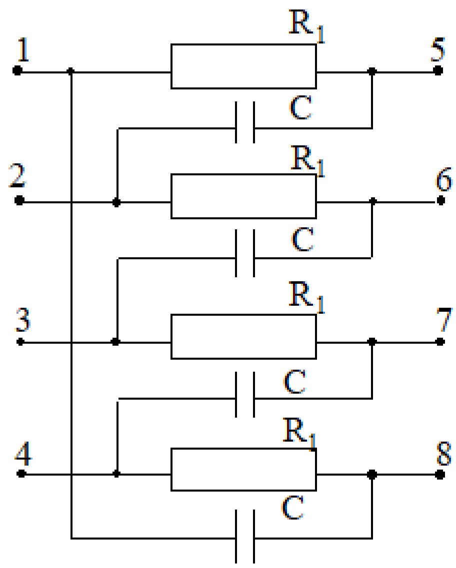

Figure 2.1: A one-stage RC polyphase filter

In Fig. 2.1, all resistors and capacitors are identical. In a multistage filter, nodes 5-6-7-8 are connected to nodes 1-2-3-4 of the next stage, respectively. Resistor and capacitor values of different stages are not necessarily identical. In this report, we consider the case when all capacitors are identical and only resistors of different stages may differ from each other.

From the many possibilities, input and output ports can be configured from the nodes in two meaningful ways (see please Fig. 2.2ab).

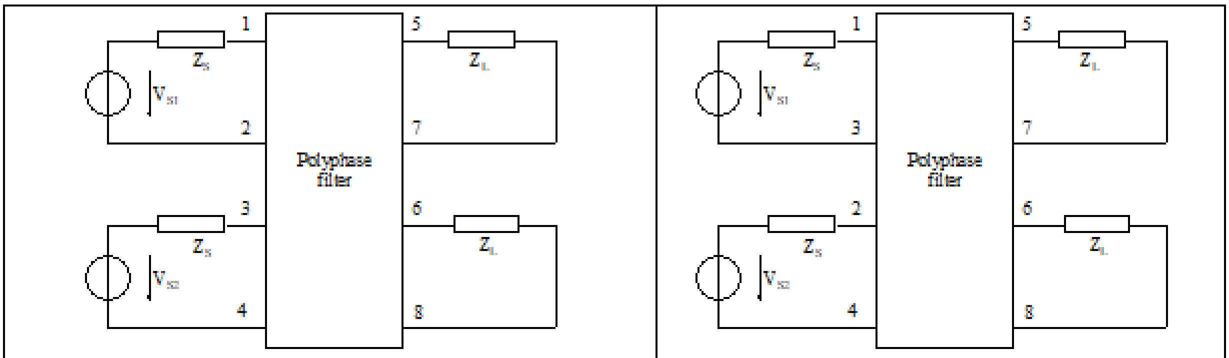

Figure 2.2 (a): Excitation for perfect amplitude match **Figure 2.2 (b):** Excitation for perfect phase match

In the arrangements of Fig. 2.2, four ports are defined (between nodes in parentheses): 1 (1-2), 2 (3-4), 3 (5-7), 4 (6-8) for Fig. 2.2a and 1 (1-3), 2 (2-4), 3 (5-7), 4 (6-8) for Fig. 2.2b. Because the arrangements are symmetrical, only two voltage gains are considered, V_{u31} and V_{u41}, that is, forward voltage gains between ports 3-1 and 4-1, respectively.

In both cases, output ports are between nodes 5-7 and 6-8. In Fig. 2.2a voltage gains V_{u31} and V_{u41} are of identical amplitude for all frequencies (amplitude match), and of 90° phase difference at one frequency value only. While in Fig. 2.2b, phase difference of V_{u31} and V_{u41} is 90° for all frequencies (phase match), and amplitudes are identical at one frequency value only (as we show in the following). Because noise figure of the configuration in Fig. 2.2b can be better than that of the other, we restrict our investigation to that case. Thus our first goal is to obtain voltage gains in Fig. 2.2b.

There is no general formula for voltage gains of arbitrary number of stages. For less than 30 dB sideband suppression, one or two stages are sufficient, thus we obtain voltage gains for one and two stage filters.

Symbolic analysis [5,6] yields the following formulas for voltage gains:

$$V_{U31,1} = \frac{Z_L}{j\omega C R_1 Z_L + 2R_1 + Z_L} \tag{2.1}$$

$$V_{U41,1} = -j\omega C R_1 \frac{Z_L}{j\omega C R_1 Z_L + 2R_1 + Z_L} \tag{2.2}$$

$$V_{U31,2} = Z_L \frac{1+\omega^2 R_1 R_2 C^2}{2R_1 + 2R_2 + Z_L + j\omega C(4R_1R_2 + 3R_1Z_L + R_2Z_L) - \omega^2 C^2 R_1 R_2 Z_L} \tag{2.3}$$

$$V_{U41,2} = -Z_L \frac{j\omega C(R_1 + R_2)}{2R_1 + 2R_2 + Z_L + j\omega C(4R_1R_2 + 3R_1Z_L + R_2Z_L) - \omega^2 C^2 R_1 R_2 Z_L} \tag{2.4}$$

where Eqs. (2.1)-(2.2) show voltage gains for one stage, the other two equations for two stages, j is the imaginary unit, ω is the angular frequency and Z_L is the load impedance. Resistor values in the first and second stages are denoted by R_1 and R_2, respectively.

Eqs. (2.1)-(2.4) are obtained by symbolic analysis because otherwise they needed a lengthy calculation. Probably the considerable amount of necessary calculations is the reason why the following result has not been found up to this time.

The ratio of the voltage gains is

$$\frac{V_{U41,1}}{V_{U31,1}} = -j\omega C R_1 \tag{2.5}$$

$$\frac{V_{U41,2}}{V_{U31,2}} = \frac{-j\omega C(R_1 + R_2)}{1+\omega^2 C^2 R_1 R_2} \tag{2.6}$$

where Eq. (2.5) is the ratio for one stage and Eq. (2.6) for two stages. We can see that the phase match is perfect for all frequencies while the amplitude match is perfect only for $\omega_1 = \frac{1}{CR_1}$ and $\frac{\omega C(R_1+R_2)}{1+\omega^2 C^2 R_1 R_2} = 1$ (that is, $\omega_1 = \frac{1}{CR_1}$ and $\omega_2 = \frac{1}{CR_2}$), respectively. As we can also see from Eqs. (2.5)-(2.6), *amplitude and phase match factors* (defined in the Appendix) *are totally independent of the source and the load impedances*.

Main characteristic of a single sideband modulator and demodulator is the sideband suppression ratio, that is for sinusoidal modulation, the ratio of the amplitudes of sideband voltages at the output. Assuming ideal mixers and adder in the system, we consider sideband suppression ratio as the characteristic of the filter applied.

In the Appendix we show that the sideband suppression ratio is expressed by the amplitude and phase match factors. As a consequence, *the sideband suppression ratio is also independent of the source and load impedances*. This property will appear important in noise minimization.

We state without proof, that the sideband suppression is usually dependent on asymmetry of source and load impedances, thus we have to take care for the symmetry of terminations in realization

3. Noise figure of Passive Circuits

Our goal is to minimize the noise figure of a polyphase filter. In this section we derive two formulas for noise figure of passive two-ports, and one of them is applied in the next section for noise minimization.

Due to the required symmetry for terminating impedances, it is sufficient to consider noise figure for one input and one output, the others being properly terminated (Fig. 3.1). Consequently, noise figure of passive two-ports is discussed.

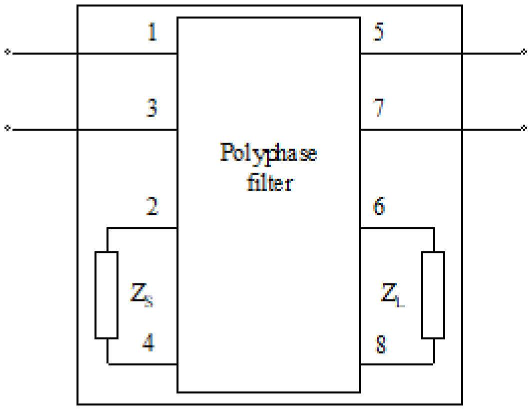

Figure 3.1: The two-port whose noise is investigated. Z_S and Z_L are adjusted synchronously to the outer terminations

We assume that impedance matrix of the investigated two-port exists. Due to reciprocity, this model consists of three complex numbers per frequency. For noise modelling, it seems more convenient using three other complex model parameters, provided that the former and the latter three are uniquely convertible to each other. Without proof we state that our model in Fig. 3.2 satisfies the mentioned requirement, that is, under the same terminations, *our model is equivalent to the impedance matrix model*.

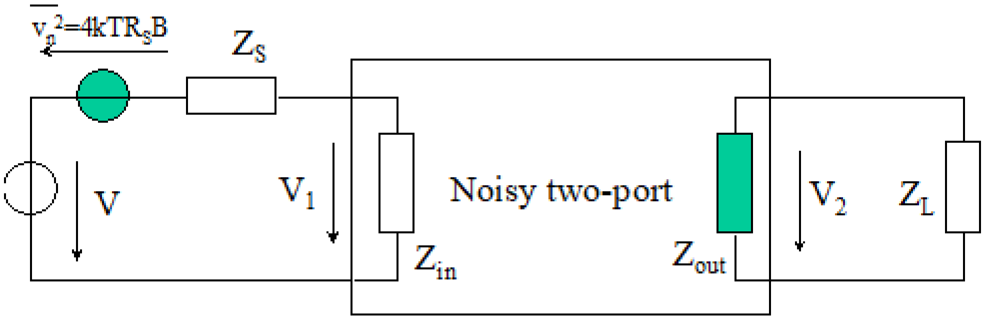

Figure 3.2: Notations for noise figure calculation. Z_S and Z_{out} are assumed to produce thermal noise. The two-port is modelled by Z_{in}, Z_{out} and the relation between V_2 and V_1

The circuit in Fig. 3.2 is considered to be in thermal equilibrium, having common noise temperature T for all noise sources.

The two-port is characterized by the input and output impedances under actual terminations and the loaded forward voltage gain. Thermal noise in the two-port is modelled by the noise of Z_{out}. The two-port is excited by a source with noisy source impedance and loaded by a noiseless load impedance.

The noise figure is defined as

$$F = \frac{SNR_{in}}{SNR_{out}} \tag{3.1}$$

where

$$SNR_{in} = \frac{P_{s\,in}}{P_{n\,in}}, \qquad SNR_{out} = \frac{P_{s\,out}}{P_{n\,out}} \tag{3.2ab}$$

and we denoted the signal and the noise with subscripts s and n, respectively.

The signal power at the input is, assuming sinusoidal signal with amplitude V_1:

$$P_{s\,in} = \frac{1}{2}\frac{|V_1|^2}{R_{in}} \tag{3.3}$$

The noise power at the input is the noise power coming from the impedance Z_S :

$$P_{n\,in} = \frac{1}{R_{in}}\left(\left|\sqrt{4kTR_S B}\,\frac{Z_{in}}{Z_S+Z_{in}}\right|^2\right) = \frac{4kTB}{R_{in}}\,\frac{R_s|Z_{in}|^2}{|Z_s+Z_{in}|^2} \tag{3.4}$$

In Eq. (3.4), k is the Boltzmann constant ($\cong 1.3807 \cdot 10^{-23}$ Joule/°K), T is the noise temperature and B is the noise bandwidth.

The signal power at the output is

$$P_{s\,out} = \frac{1}{2}\frac{|V_2|^2}{R_L} \tag{3.5}$$

and the noise power is

$$P_{n\,out} = \frac{1}{R_L}\left(\left|\sqrt{4kTR_{out}B}\,\frac{Z_L}{Z_{out}+Z_L}\right|^2\right) = \frac{4kTB}{R_L}\,\frac{R_{out}|Z_L|^2}{|Z_{out}+Z_L|^2} \tag{3.6}$$

By combining the equations above, the noise figure is obtained:

$$F = \frac{1}{|V_u|^2}\,\frac{|Z_s+Z_{in}|^2}{R_s|Z_{in}|^2}\,\frac{R_{out}|Z_L|^2}{|Z_{out}+Z_L|^2} \tag{3.7}$$

In Eq. (3.7), $V_u = V_2 / V_1$ is the loaded voltage gain. We can see from Eq. (3.7) that if $|Z_S| << |Z_{in}|$ and $|Z_L| >> |Z_{out}|$ then the formula can be simplified as

$$\boxed{F = \frac{1}{|V_u|^2}\frac{R_{out}}{R_s}} \tag{3.8}$$

as it was obtained by [4]. Eq. (3.8) is useful for quick approximations in system design.

In Fig. 3.3 we compare the noise figure values obtained by the circuit analysis program APLAC [7], to those of Eq. (3.8). Also, In Fig. 3.4 we compare analyzed noise figure to that of Eq. (3.7) when the above conditions are not fulfilled. Agreement is good in both cases.

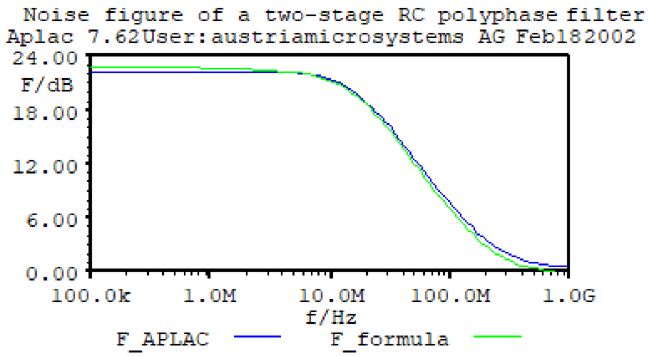

Figure 3.3: Comparison of Eq. (3.8) to an analysis result. R_S is adjusted to meet conditions for Eq. (3.8)

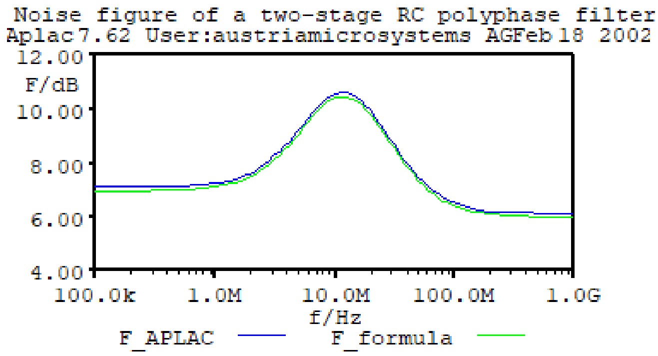

Figure 3.4: Comparison of Eq. (3.7) to an analysis result when the conditions of Eq. (3.8) are not met (noise matching)

4. Noise Figure Minimization of the polyphase Filter

In this Section, we use Eq. (3.7) in minimization of the noise figure of a polyphase filter. The noise minimum is found with respect to Z_S. Following this step, noise figure for the next stage (summing buffer) can be minimized with varying Z_L. In the last two steps, the invariance of the sideband suppression ratio with respect to Z_S and Z_L is exploited.

Let us consider first the one-stage filter. Analytic expressions for V_u, Z_{in} and Z_{out} are

$$V_{u,1} = \frac{Z_L}{j\omega CR_1Z_L + Z_L + 2R_1} \tag{4.1}$$

$$Z_{in,1} = \frac{j\omega CR_1Z_L + Z_L + 2R_1}{j\omega CZ_L + j\omega CR_1 + 1} \tag{4.2}$$

$$Z_{out,1} = \frac{j\omega CR_1Z_S + Z_S + 2R_1}{j\omega CZ_S + j\omega CR_1 + 1} \tag{4.3}$$

With these expressions substituted into Eq. (3.7), the noise figure is at resonance $\omega_1 = 1/CR_1$:

$$F_1 = \frac{R_S^2 + 2R_SR_1 + X_S^2 - 2X_SR_1 + 2R_1^2}{R_1R_S} \tag{4.4}$$

where $Z_S = R_S + jX_S$. Noise figure at resonance is independent of Z_L as expected. The noise figure has a minimum at

$$Z_S = R_1 + jR_1 \tag{4.5}$$

However, in IC realization, inductive termination is not allowed. With $X_S = 0$, the noise figure has a minimum at

$$R_S = \sqrt{2}R_1 \tag{4.6}$$

Its value is

$$\min(F_1) = 2 + 2\sqrt{2} \tag{4.7}$$

that is, 6.838 dB.

We can observe at this point that *noise match is different from power match*. From Eq. (4.2) it follows that power match at the input is dependent on Z_L. However, noise match is independent of Z_L as we have seen.

Now let us continue with a two-stage filter. Expressions for V_u, Z_{in} and Z_{out} are

$$V_{U,2}=Z_L\frac{1+\omega^2R_1R_2C^2}{2R_1+2R_2+Z_L+j\omega C(4R_1R_2+3R_1Z_L+R_2Z_L)-\omega^2C^2R_1R_2Z_L} \tag{4.8}$$

$$Z_{in,2}=\frac{2R_1+2R_2+Z_L+j\omega C(4R_1R_2+3R_1Z_L+R_2Z_L)-\omega^2C^2R_1R_2Z_L}{1+j\omega C(R_1+3R_2+2Z_L)-\omega^2C^2(R_1R_2+R_1Z_L+R_2Z_L)} \tag{4.9}$$

$$Z_{out,2}=\frac{2R_1+2R_2+Z_S+j\omega C(4R_1R_2+R_1Z_S+3R_2Z_S)-\omega^2C^2R_1R_2Z_S}{1+j\omega C(3R_1+R_2+2Z_S)-\omega^2C^2(R_1R_2+R_1Z_S+R_2Z_S)} \tag{4.10}$$

Substituting Eq. (4.8)-(4.10) into Eq. (3.7), the noise figure at center frequency $\omega_C=\frac{1}{C\sqrt{R_1R_2}}$ (this frequency corresponds to the local maximum of the sideband suppression) is obtained as follows:

$$F_2=\frac{1}{4}\frac{2R_s^2(R_1+3R_2)+R_s(R_1^2+14R_1R_2+R_2^2)+4R_1R_2(3R_1+R_2)}{R_sR_1R_2} \tag{4.11}$$

where both terminations are considered as resistive. The noise figure is independent of the load as before. The noise figure has a minimum at

$$R_S=\sqrt{\frac{2R_1R_2(3R_1+R_2)}{R_1+3R_2}} \tag{4.12}$$

The value of the minimum noise figure is a complicated function of R_1 and R_2, we suggest using Eq. (4.11) instead.

We note that in case of $R_1=R_2$, Eq. (4.12) reduces to Eq. (4.6), and

$$\min(F_2)=4+4\sqrt{2} \tag{4.13}$$

that is, 9.85dB. This is the lower limit for noise figure of two-stage RC polyphase filters.

Comparing Eq. (4.7) to Eq. (4.13), it is easy to find the general formula for lower limit noise figure of n stages:

$$\boxed{\min(F_n)=2^n(1+\sqrt{2})} \tag{4.14}$$

We have not proved this formula in general but checked its validity by computer simulation for n=1,2…5.

5. Example: Optimum noise figure of a detuned two-stage polyphase filter

As an example, we minimize the noise figure of a two-stage detuned polyphase filter. Input data for the design is the capacitance value C=8pF (due to estimated silicon area), the center frequency f_c=10MHz and the sideband suppression at center frequency S_{max}=-25dB. The following equations are solved for R_1 and R_2:

$$f_C=\frac{1}{2\pi C\sqrt{R_1R_2}} \tag{5.1}$$

$$S_{max}=20\log\left(\frac{R_1+R_2-2\sqrt{R_1R_2}}{R_1+R_2+2\sqrt{R_1R_2}}\right) \tag{5.2}$$

The result is R_1=1.227kΩ, R_2=3.226kΩ. Applying Eq. (4.12) and (4.11) for noise matching, the source resistance for minimum noise figure is R_S=2.239kΩ, and the minimum noise figure is F_{min}=10.18dB. Sideband suppression of this filter as a function of frequency is shown in Fig. 5.1. Noise figure at the center frequency as a function of the source resistance and capacitance are given in Fig. 5.2 and 5.3, respectively.

It can be shown that filter bandwidth is not independent of f_c and S_{max}. Therefore the capacitance value as an input variable cannot be replaced by an equation for bandwidth.

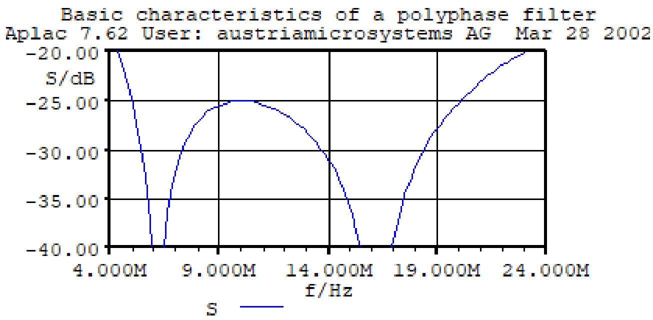

Figure 5.1: Sideband suppression of the noise matched filter

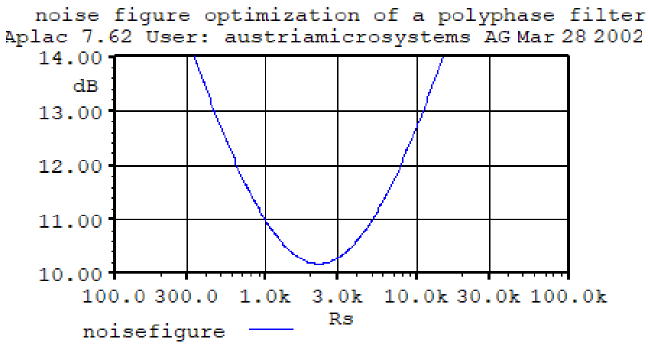

Figure 5.2: Noise figure as a function of the source resistance

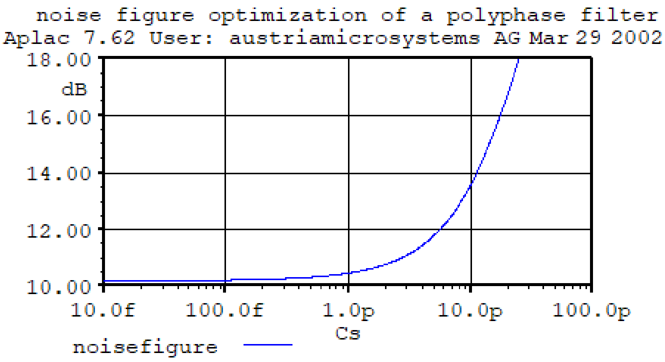

Figure 5.3: Noise figure as a function of source capacitance

6. Conclusions

Some results on noise figure minimization of one- and two-stage polyphase filters have been obtained. In Section 2 voltage transfer functions have been given. In Section 3 two formulas for noise figure of passive two-ports are shown. In Section 4 noise figure of one- and two-stage polyphase filters are minimized. It was shown that the sideband suppression ratio is independent of the source and load terminations, thus noise figure can be minimized without degradation of the sideband suppression. In Section 5 we applied our results in designing and noise matching of a two-stage detuned polyphase filter.

Our results have been achieved through an extensive usage of symbolic analysis programs. Large amount of the necessary computation effort, especially in noise figure minimization of two-stage filters, is a reason why these results have not been revealed before.

The independence of the sideband suppression from source and load are proved here for one- and two stages. This property was observed as generally valid for arbitrary number of stages by computer simulation, however.

7. Acknowledgement

The author is grateful to Dr. G. Schultes who let him know the simplified formula for noise figure [4]. P. Thurner is also acknowledged who took the author's attention to the symbolic analysis program SapWin [5].

8. Appendix: The Sideband Suppression Ratio

The analyzed polyphase filter is applied in single sideband modulators and demodulators. The amplitude and phase mismatches result in finite sideband suppression. In the following, this relation is shown in the example of a single sideband modulator (Fig. 8.1).

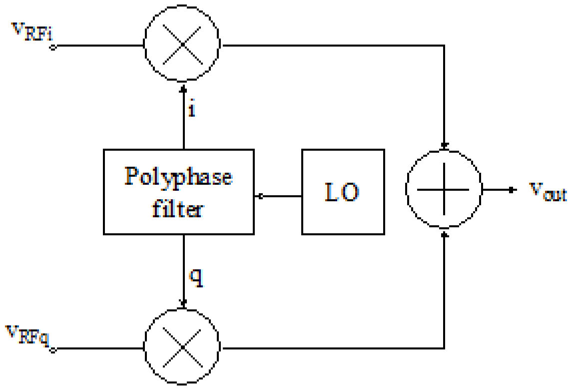

Figure 8.1: Single sideband modulator

We assume the in-phase and quadrature RF voltages in perfect amplitude and phase match:

$$v_{RFi} = V_{RF}\cos(\omega_{RF} t) \tag{8.1}$$

$$v_{RFq} = V_{RF}\cos\left(\omega_{RF} t + \frac{\pi}{2}\right) \tag{8.2}$$

We take into account errors in LO signals only:

$$v_{LOi} = V_{LOi}\cos(\omega_{LO} t) \tag{8.3}$$

$$v_{LOq} = V_{LOq}\cos\left(\omega_{LO} t + \frac{\pi}{2} + \varphi\right) \tag{8.4}$$

where V_{LOi} may differ from V_{LOq} causing amplitude mismatch, and φ may differ from zero causing phase mismatch. In the following, we derive relation between sideband suppression, amplitude match and phase match. With no loss of generality, V_{RF}=1 Volt is chosen.

The output voltage of the modulator can be written as

$$v_{OUT} = V_U\cos[(\omega_{RF}+\omega_{LO})t-\varphi_U] + V_L\cos[(\omega_{RF}-\omega_{LO})t-\varphi_L] \tag{8.5}$$

where U and L denote the upper and lower sidebands, respectively. If the mixers, the adder, and both the amplitude and phase matches are perfect ($V_{LOi}=V_{LOq}$ and $\varphi=0$), then $V_U=0$ and $V_L=V_{LOi}$. Therefore we define (upper) sideband suppression in dB as follows:

$$S = 20\log\left(\frac{V_U}{V_L}\right) \tag{8.6}$$

Using trigonometric identities in Eqs. (8.1)-(8.5), sideband suppression can be expressed as

$$S = 20\log\sqrt{\frac{[1-m\cos(\varphi)]^2+[m\sin(\varphi)]^2}{[1+m\cos(\varphi)]^2+[m\sin(\varphi)]^2}} \tag{8.7}$$

where

$$m = \frac{V_{LOq}}{V_{LOi}} \tag{8.8}$$

is the amplitude match and φ is the phase match. For perfect match, m=1 and φ=0, resulting infinitely large negative S.

Contour plot of the negative of sideband suppression as a function of amplitude match m and phase match φ is shown in Fig. 8.2.

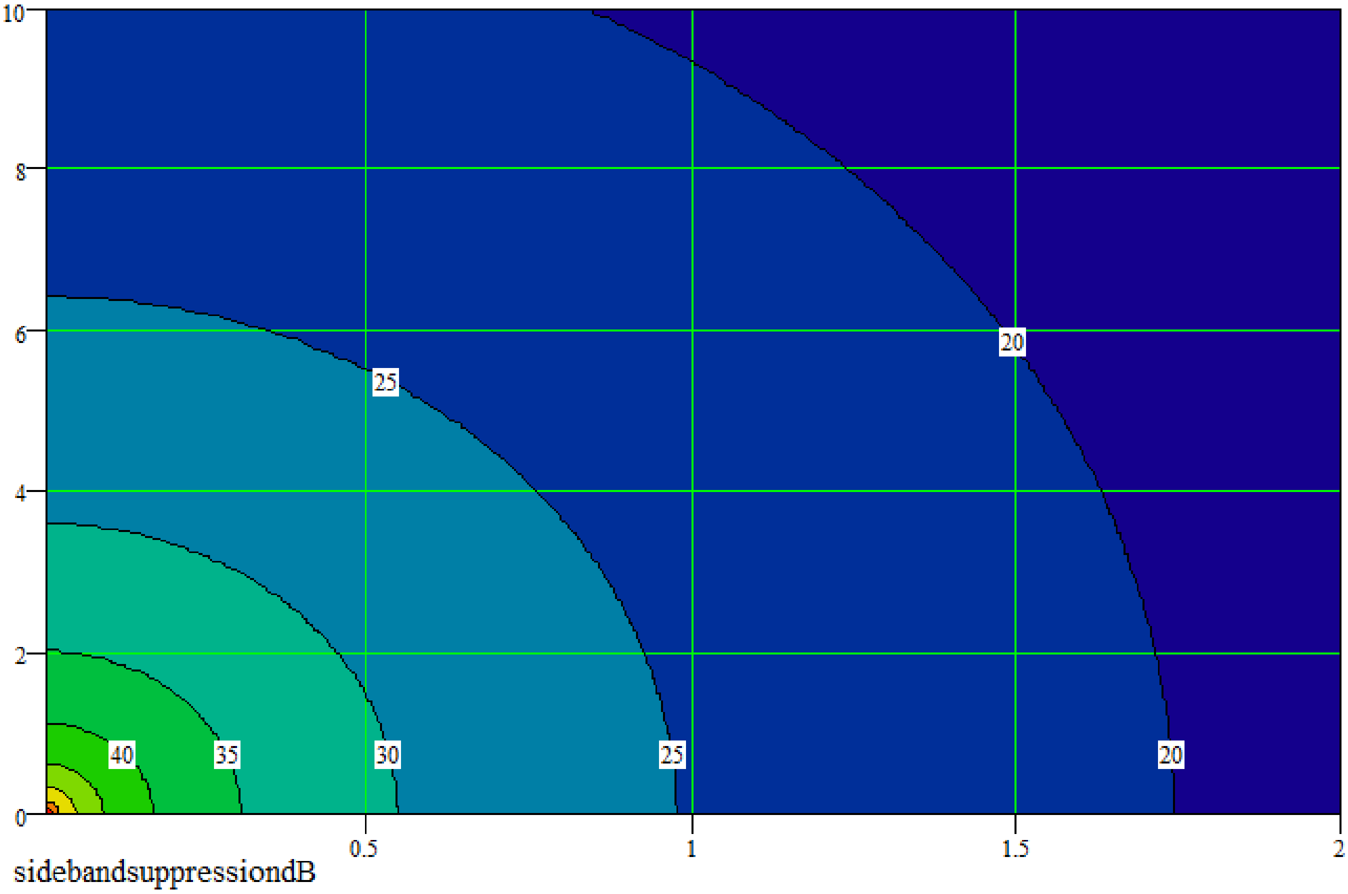

Figure 8.2: Contour plot of negative of sideband suppression in dB. Horizonztal axis: amplitude match in dB, vertical axis: phase match in degrees

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